Veering and modes aberration of structures subjected to in-plane loadings according to linearized and full nonlinear formulations

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Abstract

This research work was originated and inspired by a presentation made by Professor Arthur W. Leissa at ISVCS 9, Courmayeur, Italy, on July 2013 [1]. According to his talk, authors realized that the interesting phenomena related to mode aberration have rarely been investigated in the recent years.

During service and due to the nature of applied loadings, structural components, such as stiffeners, panels, ribs and boxes in aerospace constructions, for example, are subjected to stress fields. Those stresses, and especially compression ones, may significantly modify the equilibrium state of the structures and, thus, affect their dynamic response, eventually in a catastrophic manner. For this reason, the evaluation and the analysis of the natural frequencies and mode shapes changing as the elastic system is subjected to operational loads is of primary interest.

By employing a refined beam model with higher-order capabilities, this work, thus, investigates the eigenvalue loci veering, crossing, coalescence and eventual buckling events due to mode degeneration of metallic and composite structures undergoing pre-stress states and, eventually, large displacements and rotations. The proposed models are based on the Carrera Unified Formulation (CUF), according to which each theory of structures (either 1D or 2D) can be expressed as a degenerated form of the three-dimensional equilibrium equations in a hierarchical manner [2]. As an example, 1D beam theories can be formulated from the three-dimensional displacement field ($\mathbf{u}$) as an arbitrary expansion of the generalized unknowns ($\mathbf{u}_\tau$); i.e.,

$$\mathbf{u}(x,y,z) = F_\tau(x,z)\mathbf{u}_\tau(y), \qquad \tau = 1,2,....,M \tag{1}$$

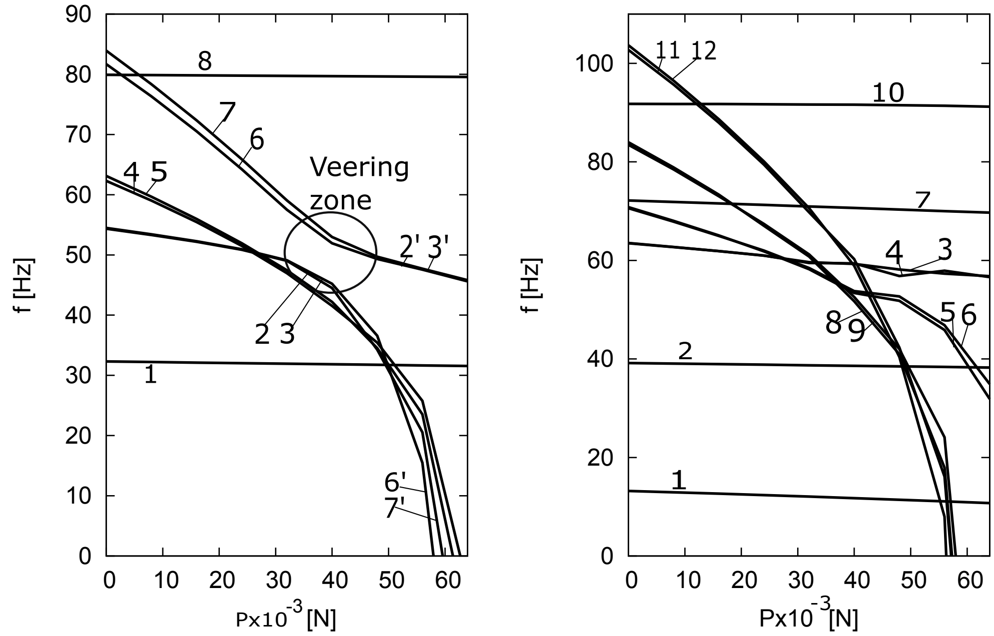
where F_τ are generic functions on the beam cross-section domain, M is the number of expansion terms, and τ denotes summation. Depending on the choice of F_τ and the number of expansion terms M, different classes of beam models can be formulated and, thus, implemented in a straightforward manner. Namely, in this work, low- to higher-order beam models with only pure displacement variables are implemented by utilizing Lagrange polynomials expansions of the unknowns on the cross-section.

Give a structure subjected to a pre-stress state σ_0, it can be easily demonstrated that the linearization of the virtual variation of the work of internal strains can be approximated as follows:

$$\delta(\delta L_{\text{int}}) \approx < \delta\varepsilon^T \sigma > + < \delta(\delta\varepsilon^T)\sigma_0 > \tag{2}$$

or, in other words,

$$\mathbf{K}_T^{ij\tau s} \approx \mathbf{K}^{ij\tau s} + \mathbf{K}_{\sigma_0}^{ij\tau s} \tag{3}$$



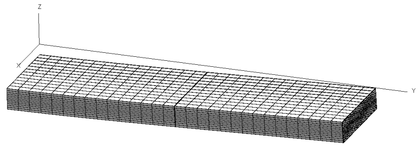
(a) Lamination A

(b) Lamination B

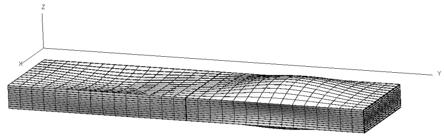
Figure 1: Mode aberration of a composite box beam subjected to compression.

In Eq. (2), ε and σ are the vectors of strain and stress components, whereas $< (\cdot) >= \int_V (\cdot) \mathrm{d}V$. In contrast, Eq. (3) shows that the *fundamental nucleus* (FN) of the tangent stiffness matrix, $\mathbf{K}_T^{ij\tau s}$, can be approximated as the sum of the FNs of the linear stiffness, $\mathbf{K}^{ij\tau s}$, and the geometric stiffness, $\mathbf{K}_{\sigma_0}^{ij\tau s}$. According to CUF and in the framework of the finite element method, as in the case of this work, finite element arrays of generally refined structural models can be formulated in a straightforward manner by expanding the FNs versus the indexes $(\tau, s = 1, \cdots, M)$ and $(i, j = 1, \cdots, N)$, where N is the number of nodes of the finite element employed. For the sake of brevity, the derivation and the complete expressions of the FNs in Eq. (3) are not given here, but they can be found in [2] and [3]. Once the global tangent stiffness matrix $\mathbf{K}_T$ is known, the natural frequencies and mode shapes of the structure can be evaluated by solving the usual eigenvalue problem $\left(\mathbf{K}_T - \omega^2 \mathbf{M}\right) \mathbf{u} = 0$, where $\mathbf{M}$ is the mass matrix. However, it is important to underline that Eqs. (2) and (3) are based on the fundamental hypotheses that the equilibrium state is linear and the structure undergoes infinitesimal strains and displacements/rotations [4].

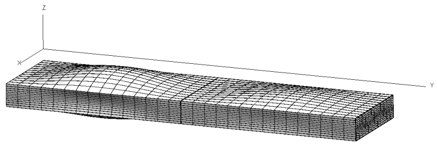
For representative purposes, a numerical example is shown here. We consider a cantilever single-cell, two-bay composite box beam subjected to compression load P. The box is made of two layers of carbon/epoxy material on each flange. In lamination A, the fibre orientation is 0 deg in the top and bottom flanges and ± 15 deg in the right and left flanges. On the other hand, in lamination B, an angle-ply lamination ± 45 deg is employed for all the flanges. Figure 1 shows the variation of the natural frequencies for the first important modes and for different values of the load P. Also, for the sake of completeness, Fig. 2 depicts some mode shapes of the box in the case of $P = 0$. The analysis clarifies that, independently of the lamination angles, buckling occurs approximately for the same compression loading. Nevertheless, Fig. 1 shows that, in the case of lamination A, veering phenomena appear as a consequence of severe mode aberrations. Moreover, Fig. 2 demonstrates the importance of employing adequate structural models when dealing with this kind of analysis, especially if composite and thin-walled structures are considered. From this point of view, due to its higher-order and enhanced capabilities, CUF is a good candidate for mode aberration investigations.



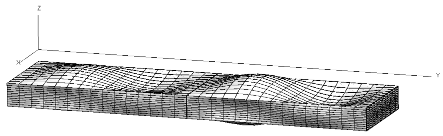
(a) Mode 1



(b) Mode 2



(c) Mode 3



(d) Mode 4

Figure 2: Representative mode shapes of the composite box strucrure. Lamination A.

As a final comment, it must be underlined that the hypotheses according to which the approximation in Eq. (2) holds may be too much limiting in the case of problems that involve moderate or large displacements, e.g. for analyses and investigations that go beyond the first buckling load. In this case, ~~and by~~ assuming that the nonlinear equilibrium state is reached with infinitesimal and consecutive load steps (i.e., dynamic effects are not accounted for), the tangent stiffness to be employed for the linear, free-vibration eigenvalue problem comes from the following expression of the internal strain energy:

$$\delta(\delta L_{\text{int}}) =< \delta\epsilon^T \delta\sigma > + < \delta(\delta\epsilon^T)\sigma_0 > \tag{4}$$

or rather

$$\mathbf{K}_T^{ij\tau s} = \mathbf{K}^{ij\tau s} + \mathbf{K}_{T_1}^{ij\tau s} + \mathbf{K}_{\sigma_0}^{ij\tau s} \tag{5}$$

where $\mathbf{K}_{T_1}^{ij\tau s}$ represents the contribution due to the *secant* stiffness and $\mathbf{K}_{\sigma_0}^{ij\tau s}$ takes into account both the linear and geometrical nonlinear stress components. Accordingly, the differences between linearized and full nonlinear vibration problems will be discussed during the *11th International Symposium on Vibrations of Continuous Systems*.

References

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