A free-vibration thermo-elastic analysis of laminated structures by variable ESL/LW plate finite element

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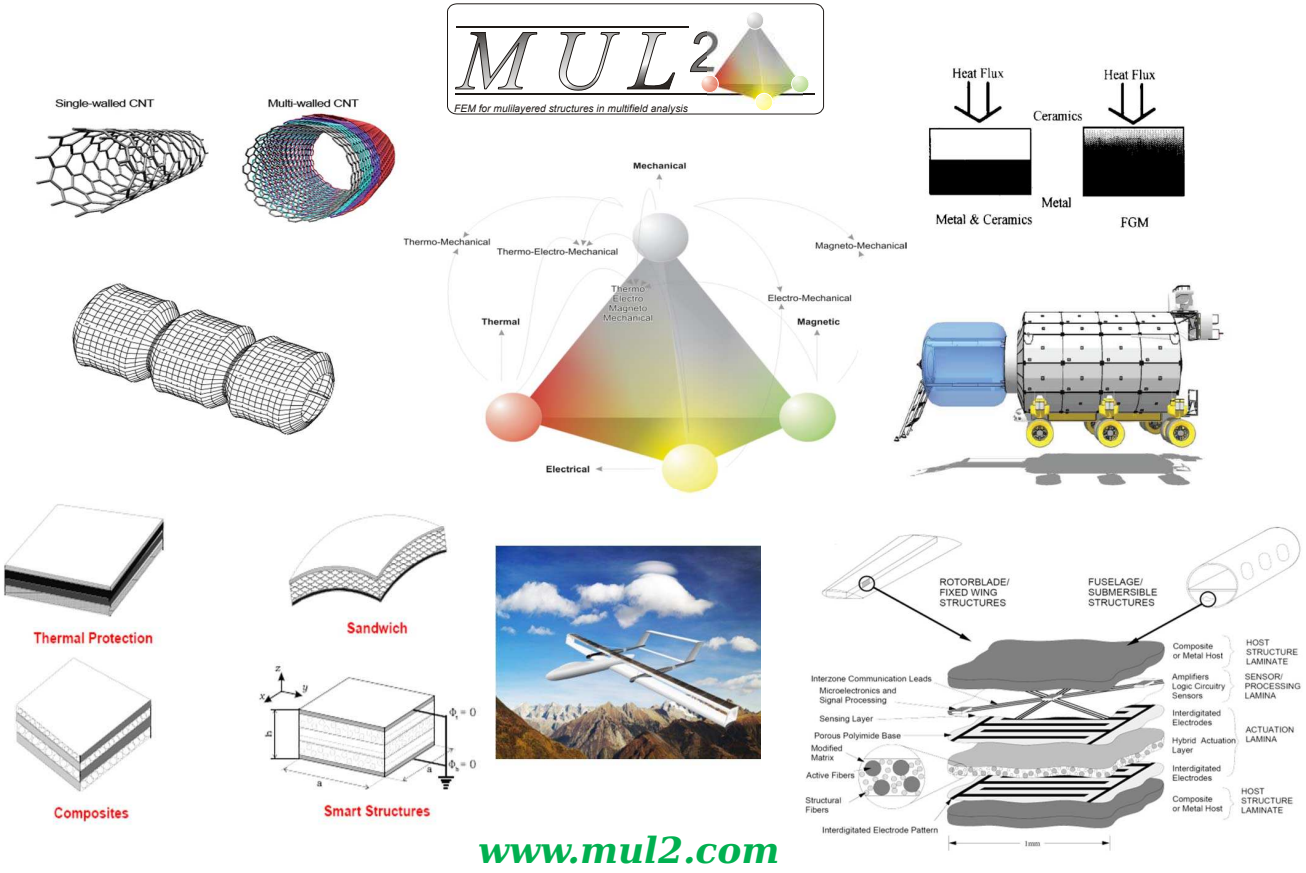
MULtilayered structures **www.mul2.com** **MUL**tifield interaction



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Unified Formulation

2D approximation of **mechanical displacements** and **temperature** using the *thickness functions*

$$\begin{cases} u^k(x,y,z) = F_0(z)\, u_0^k(x,y) + F_1(z)\, u_1^k(x,y) + ... + F_N(z)\, u_N^k(x,y) \\ v^k(x,y,z) = F_0(z)\, v_0^k(x,y) + F_1(z)\, v_1^k(x,y) + ... + F_N(z)\, v_N^k(x,y) \\ w^k(x,y,z) = F_0(z)\, w_0^k(x,y) + F_1(z)\, w_1^k(x,y) + ... + F_N(z)\, w_N^k(x,y) \\ \Theta^k(x,y,z) = F_0(z)\, \Theta_0^k(x,y) + F_1(z)\, \Theta_1^k(x,y) + ... + F_N(z)\, \Theta_N^k(x,y) \end{cases}$$

in compact form:

$$\boldsymbol{u}^k(x,y,z) = F_\tau(z)\boldsymbol{u}_\tau^k(x,y) \;\; ; \;\; \delta\boldsymbol{u}^k(x,y,z) = F_s(z)\delta\boldsymbol{u}_s^k(x,y) \;\; ; \;\; \tau, s = 0, 1, ..., N$$

$$\Theta^k(x,y,z) = F_\tau(z)\Theta_\tau^k(x,y) \;\; ; \;\; \delta\Theta^k(x,y,z) = F_s(z)\delta\Theta_s^k(x,y) \;\; ; \;\; \tau, s = 0, 1, ..., N$$

Taylor Polynomials

$$\boldsymbol{u}^k = F_0 \boldsymbol{u}_0^k + F_1 \boldsymbol{u}_1^k + \ldots + F_N \boldsymbol{u}_N^k = F_\tau \boldsymbol{u}_\tau^k$$

$$\Theta^k = F_0 \Theta_0^k + F_1 \Theta_1^k + \ldots + F_N \Theta_N^k = F_\tau \Theta_\tau^k$$

$$\tau = 0, 1, \ldots, N$$

$$F_0 = (z)^0 = 1\,;\; F_1 = (z)^1 = z\,; \ldots;\; F_N = (z)^N$$

Legendre Polynomials

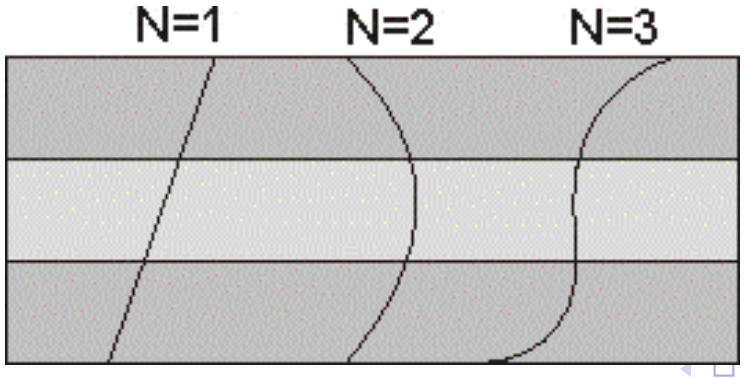
$$\boldsymbol{u}^k = F_t \boldsymbol{u}_t^k + F_b \boldsymbol{u}_b^k + F_r \boldsymbol{u}_r^k = F_\tau \boldsymbol{u}_\tau^k$$

$$\Theta^k = F_t \Theta_t^k + F_b \Theta_b^k + F_r \Theta_r^k = F_\tau \Theta_\tau^k$$

$$\tau = t, b, r \quad ; \quad r = 2, ..., N$$

$$F_t = \frac{P_0 + P_1}{2}\;;\; F_b = \frac{P_0 - P_1}{2}\;;\; F_r = P_r - P_{r-2}$$

Equivalent Single Layer Approach



Layer Wise

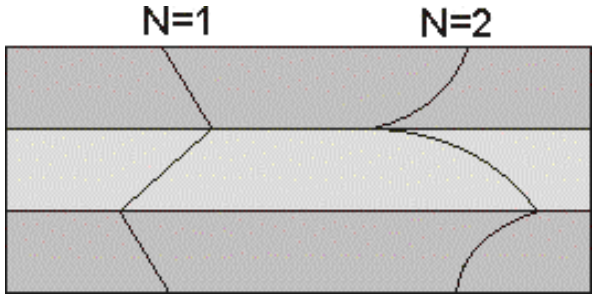
Legendre polynomials expansion:

$$\boldsymbol{u}^k = F_t \boldsymbol{u}_t^k + F_b \boldsymbol{u}_b^k + F_r \boldsymbol{u}_r^k = F_\tau \boldsymbol{u}_\tau^k$$

$$\Theta^k = F_t \Theta_t^k + F_b \Theta_b^k + F_r \Theta_r^k = F_\tau \Theta_\tau^k$$

$$\tau = t, b, r \quad ; \quad r = 2, ..., N$$

$$F_t = \frac{P_0 + P_1}{2} \; ; \; F_b = \frac{P_0 - P_1}{2} \; ; \; F_r = P_r - P_{r-2}$$



Interlaminar continuity condition:

$$u_t^k = u_b^{k+1} \quad ; \quad k = 1, n_l - 1$$

Variable-Kinematic

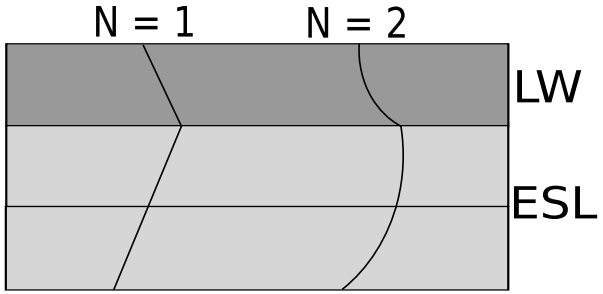
Legendre polynomials expansion:

$$\boldsymbol{u} = F_t \boldsymbol{u}_t + F_b \boldsymbol{u}_b + F_r \boldsymbol{u}_r = F_\tau \boldsymbol{u}_\tau$$

$$\Theta = F_t \Theta_t + F_b \Theta_b + F_r \Theta_r = F_\tau \Theta_\tau$$

$$\tau = t, b, r \quad ; \quad r = 2, ..., N$$

$$F_t = \frac{P_0 + P_1}{2} \; ; \; F_b = \frac{P_0 - P_1}{2} \; ; \; F_r = P_r - P_{r-2}$$

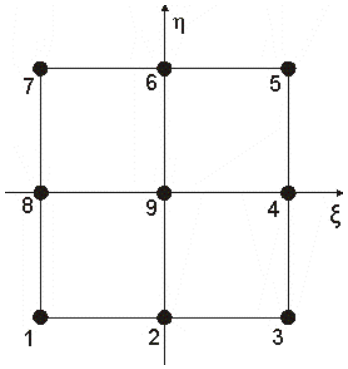


Interlaminar continuity condition is guaranteed in specified zones

Finite Element Method

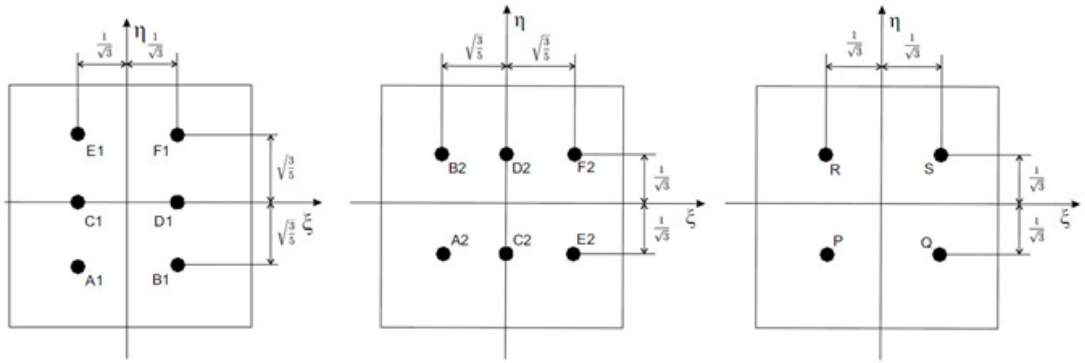
Approximation of **variables** in the reference midplane surface using the *Langrangian* shape functions:

$$\boldsymbol{u}_\tau = N_i(\xi,\eta)\,\boldsymbol{u}_{\tau i}$$



MITC

To overcome the problem of the ***membrane and shear locking***, the strain components are calculated using a specific interpolation strategy:



ϵ_{xx} γ_{xz} ϵ_{yy} γ_{yz} ϵ_{xy}

For example:

$$\epsilon_{xx} = N_{A1}\epsilon_{xx_{A1}} + N_{B1}\epsilon_{xx_{B1}} + N_{C1}\epsilon_{xx_{C1}} + N_{D1}\epsilon_{xx_{D1}} + N_{E1}\epsilon_{xx_{E1}} + N_{F1}\epsilon_{xx_{F1}}$$

Governing Equations and Fundamental Nucleus

Principle of Virtual Displacements (PVD) for mechanical problems

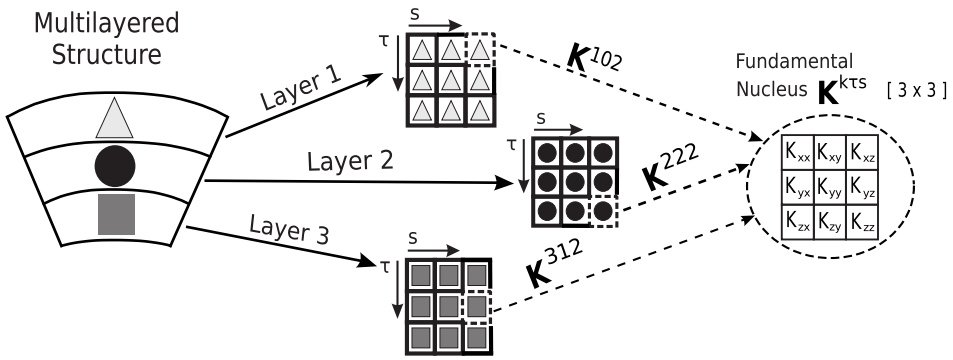
$$\int_V \delta \boldsymbol{\epsilon}^{k^T} \boldsymbol{\sigma}^k \, dV = \delta L_e$$

Governing equations in compact form:

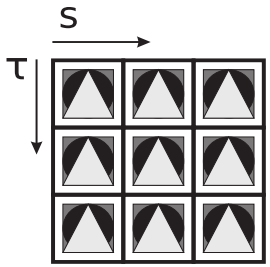
$$\delta \boldsymbol{u}^{k\tau i} : \boldsymbol{K}^{k\tau sij} \boldsymbol{u}^{ksj} = \boldsymbol{P}^{k\tau i}$$

where $$\boldsymbol{K}^{k\tau sij} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}^{k\tau sij}$$

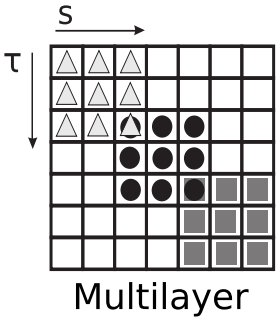
Assembling Approaches



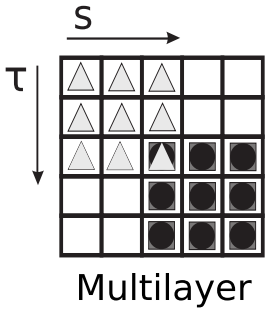
Equivalent-Single-Layer Assembling



Layer-Wise Assembling



Variable-Kinematic Assembling



Case 1 :

{layer1}{layer2,layer3}

Partially coupled thermo-mechanical problems

Static Analysis

Principle of Virtual Displacements

$$\int_V \delta\boldsymbol{\epsilon}^{k^T}\boldsymbol{\sigma}^k \, dV = \delta L_e$$

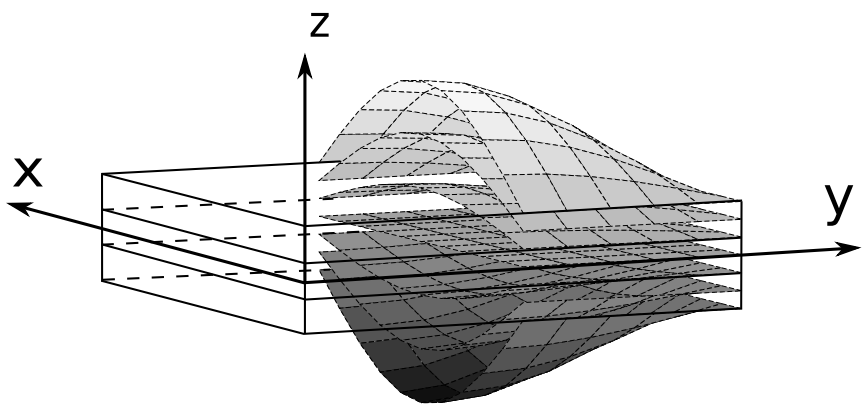
$$\boldsymbol{\sigma}^k = \boldsymbol{\sigma}_u^k - \boldsymbol{\sigma}_\Theta^k = \boldsymbol{C}^k\boldsymbol{\epsilon}^k - \boldsymbol{\lambda}^k\Theta^k$$

$$\boldsymbol{\lambda}^k = \boldsymbol{C}^k\boldsymbol{\alpha}^k$$

$$\int_V \delta\boldsymbol{\epsilon}^{k^T}\boldsymbol{\sigma}_u^k dV = \int_V \delta\boldsymbol{\epsilon}^{k^T}\boldsymbol{\sigma}_\Theta^k dV$$

$$\Theta(x,y,z) = \Theta(z) sin\left(\frac{mx\pi}{a}\right) sin\left(\frac{ny\pi}{b}\right)$$

$\Theta(z)$ is assumed linear



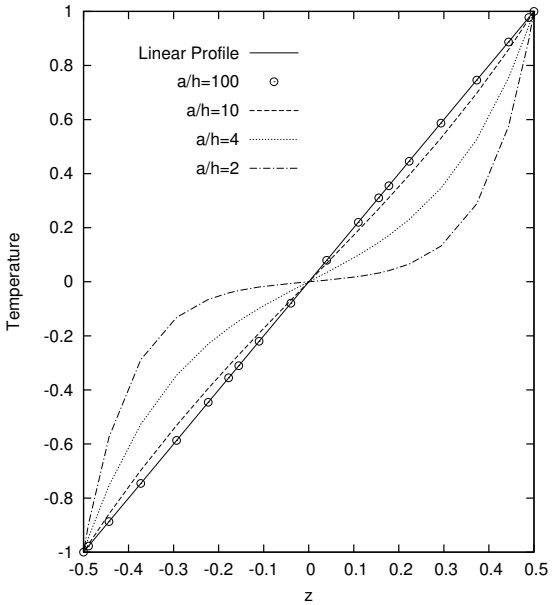
Cinefra M., Valvano S., and Carrera E.,
Thermal stress analysis of laminated structures by a variable kinematic MITC9 shell element,
Journal of Thermal Stresses, 39(2), 121-141, 2016.
http://dx.doi.org/10.1080/01495739.2015.1123591

Partially coupled thermo-mechanical problems

Static Analysis

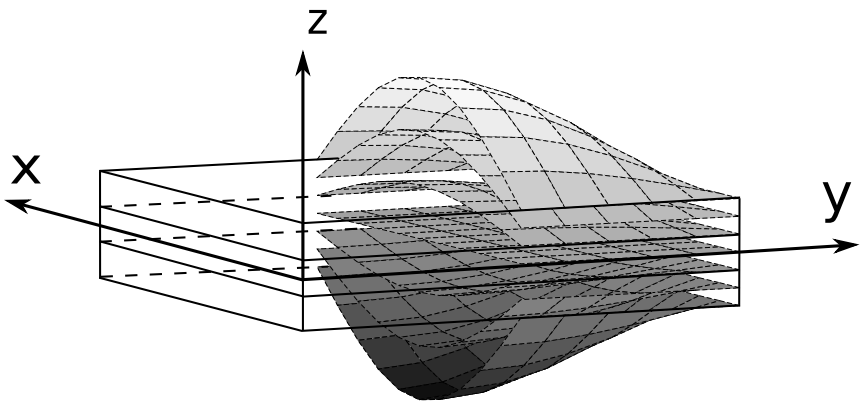
Principle of Virtual Displacements

$$\int_V \delta\boldsymbol{\epsilon}^{k^T} \boldsymbol{\sigma}_u^k dV = \int_V \delta\boldsymbol{\epsilon}^{k^T} \boldsymbol{\sigma}_\Theta^k dV$$



$$\Theta(x,y,z) = \Theta(z) sin\left(\frac{mx\pi}{a}\right) sin\left(\frac{ny\pi}{b}\right)$$

$\Theta(z)$ is calculated via the Fourier Heat conduction equations



Cinefra M., Valvano S., and Carrera E.,
Heat conduction and Thermal Stress Analysis of laminated composites by a variable kinematic MITC9 shell element,
Curved and Layered Structures, 2, 301-320, 2015.
http://dx.doi.org/10.1515/cls-2015-0017

Fully coupled thermo-mechanical problems

Static Analysis

Principle of Virtual Displacements

$$\int_V \left\{ \delta\boldsymbol{\epsilon}^{k\,T}\boldsymbol{\sigma}^k - \delta\Theta^k\boldsymbol{\eta}^k - \delta\boldsymbol{\vartheta}^{k\,T}\boldsymbol{h}^k \right\} dV = \delta L_e$$

$$\boldsymbol{\sigma}^k = \boldsymbol{C}^k\boldsymbol{\epsilon}^k - \boldsymbol{\lambda}^k\Theta^k$$

$$\boldsymbol{\eta}^k = \boldsymbol{\lambda}^k\boldsymbol{\epsilon}^k + \chi^k\Theta^k$$

$$\boldsymbol{h}^k = \boldsymbol{\kappa}^k\boldsymbol{\vartheta}^k \qquad \chi = \frac{\rho C_v}{\Theta_0}$$

$$\boldsymbol{\epsilon}_{mn} = \frac{\partial \boldsymbol{u}}{\partial_{mn}} \qquad \boldsymbol{\vartheta}_m = \frac{\partial\Theta}{\partial_m}$$

In compact form:

$$\begin{array}{l} \delta\boldsymbol{u}^k_{\tau i} : \\ \delta\Theta^k_{\tau i} : \end{array} \overset{(4\times 4)}{\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\Theta} \\ \boldsymbol{K}_{\Theta u} & K_{\Theta\Theta} \end{bmatrix}^{k\tau sij}} \begin{Bmatrix} \boldsymbol{u} \\ \Theta \end{Bmatrix}^{ksj} = \begin{bmatrix} \boldsymbol{P}_u \\ \boldsymbol{P}_\Theta \end{bmatrix}^{k\tau i}$$

$$\boldsymbol{K}^{k\tau sij} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} & K_{x\Theta} \\ K_{yx} & K_{yy} & K_{yz} & K_{y\Theta} \\ K_{zx} & K_{zy} & K_{zz} & K_{z\Theta} \\ K_{\Theta x} & K_{\Theta y} & K_{\Theta z} & K_{\Theta\Theta} \end{bmatrix}^{k\tau sij}$$

Fully coupled thermo-mechanical problems

Free-Vibrations Analysis

$$\int_V \left\{ \delta\boldsymbol{\epsilon}^{k\,T}\boldsymbol{\sigma}^k - \delta\Theta^k\boldsymbol{\eta}^k - \delta\boldsymbol{\vartheta}^{k\,T}\boldsymbol{h}^k \right\} dV = -\int_V \left\{ \rho\,\delta\boldsymbol{u}^T\ddot{\boldsymbol{u}} \right\} dV$$

In compact form:

$$\delta\boldsymbol{u}^k_{\tau i} \;:\; \boldsymbol{K}^{k\tau sij}\,\boldsymbol{u}^k_{sj} \;=\; -\boldsymbol{M}^{k\tau sij}\ddot{\boldsymbol{u}}^k_{sj}$$

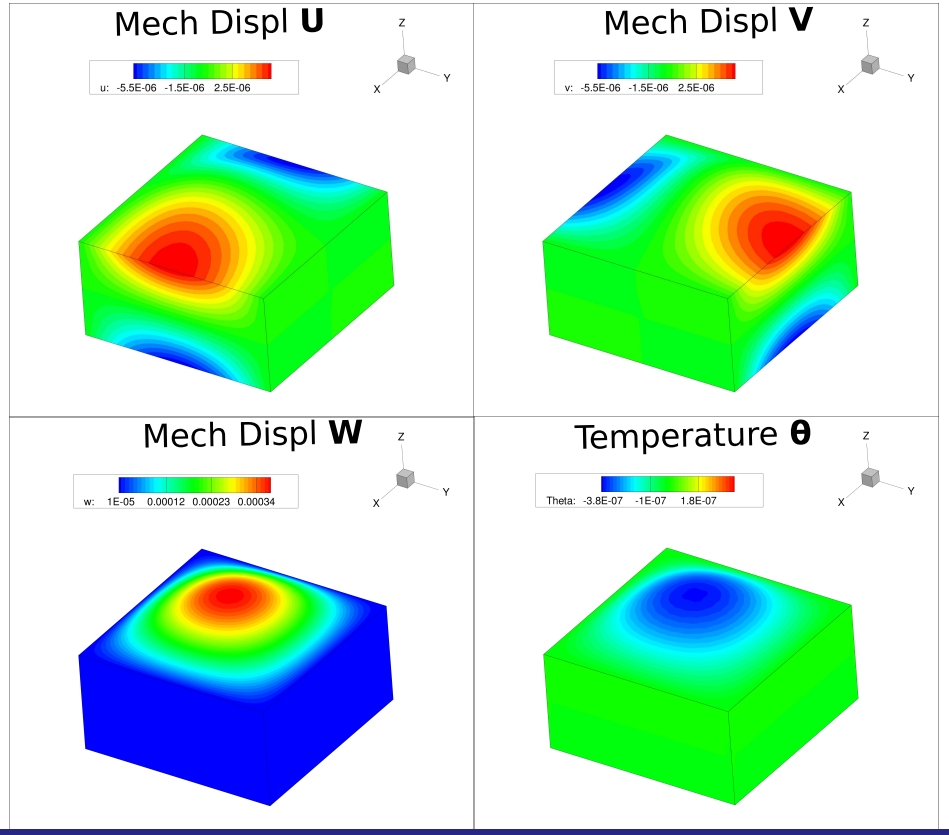
$$\boldsymbol{K}^{k\tau sij} = \overset{(4\times 4)}{\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\Theta} \\ \boldsymbol{K}_{\Theta u} & K_{\Theta\Theta} \end{bmatrix}^{k\tau sij}} \qquad \boldsymbol{M}^{k\tau sij} = \overset{(4\times 4)}{\begin{bmatrix} \boldsymbol{M}_{uu} & 0 \\ 0 & 0 \end{bmatrix}^{k\tau sij}}$$

$$\boldsymbol{M}^{k\tau sij}\ddot{\boldsymbol{u}}^k_{sj} + \boldsymbol{K}^{k\tau sij}\,\boldsymbol{u}^k_{sj} = 0 \quad \Rightarrow \text{Harmonic Solution}$$

$$\Rightarrow \quad \left(\boldsymbol{K}^{k\tau sij} - \omega_n^2\boldsymbol{M}^{k\tau sij}\right)\boldsymbol{u}^k_{sj} = 0$$

1 Layered Isotropic Simply-Supported Plate

MODE 1



Mixed ESL/LW

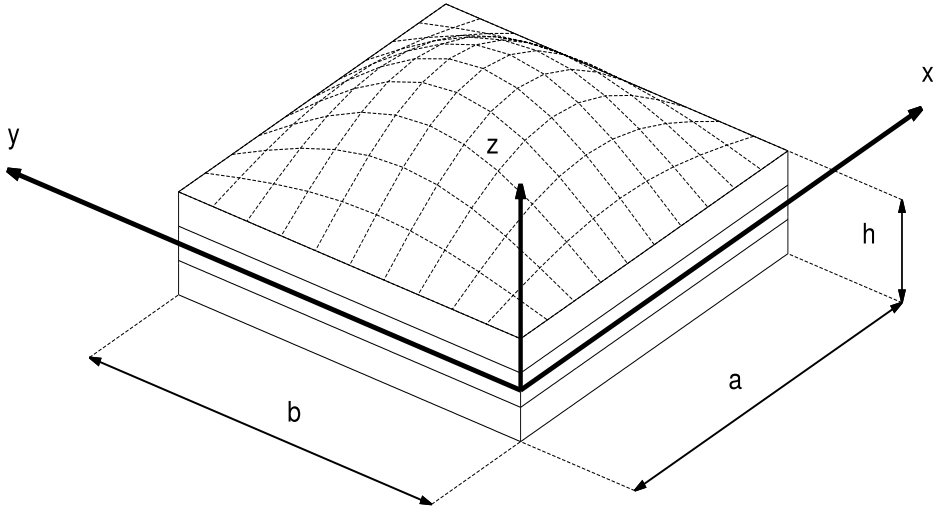
Preliminaries Static Analysis Results

Composite Square Plate [0°/90°/0°]

Mechanical Analysis

$$p(x, y, z_{top}) = \hat{p}_z \, sin\left(\frac{\pi x}{a}\right) sin\left(\frac{\pi y}{b}\right)$$

$$\hat{p}_z = 1, 0$$



B.C.= Simply-Supported

Material Properties:

$$E_L/E_T = 25$$

$$G_{LT}/E_T = 0,5$$

$$G_{TT}/E_T = 0,2$$

$$\nu_{LT} = \nu_{TT} = 0,25$$

Pagani A., Valvano S., and Carrera E.,
Analysis of laminated composites and sandwich structures by variable-kinematic MITC9 plate elements,
Journal of Sandwich Structures and Materials, 2016.
http://dx.doi.org/10.1177/1099636216650988

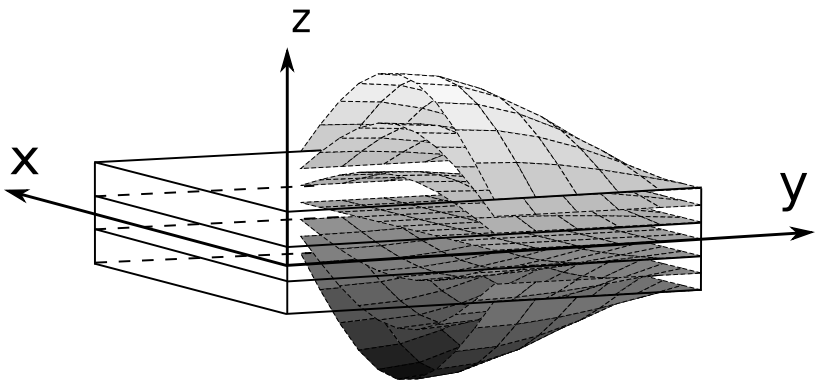
ESL Approach, Taylor vs Legendre Polynomials

	$a/h = 4$					$a/h = 100$					*DOFs*
	$\hat{w}$	$\hat{\sigma}_{xx}$		$\hat{\sigma}_{xz}$	$\hat{\sigma}_{yz}$	$\hat{w}$	$\hat{\sigma}_{xx}$		$\hat{\sigma}_{xz}$	$\hat{\sigma}_{yz}$	
		top	*bottom*				*top*	*bottom*			
3*D* [Pagano 1970]	-	0.801	-0.755	0.256	0.2172	-	0.539	-0.539	0.395	0.0828	
$LW4_a$ [Petrolo et al. 2015]	2.1216	0.801	-0.755	0.256	0.2180	0.4347	0.539	-0.539	0.395	0.0828	
$ET4_a$ [Petrolo et al. 2015]	2.0083	0.786	-0.740	0.205	0.1830	0.4342	0.539	-0.539	0.281	0.0734	
*LW*4	2.1216	0.807	-0.761	0.258	0.2197	0.4347	0.544	-0.544	0.398	0.0836	17199
*ET*4	2.0082	0.7926	-0.7461	0.2067	0.1845	0.4342	0.5435	-0.5436	0.2830	0.0742	6615
*ET*3	2.0069	0.7940	-0.7479	0.2068	0.1845	0.4342	0.5436	-0.5436	0.2830	0.0742	5292
*ET*2	1.6499	0.4714	-0.4252	0.1219	0.1258	0.4333	0.5428	-0.5428	0.1436	0.0603	3969
$ET1^*$	1.6574	0.4484	-0.4537	0.1234	0.1237	0.4333	0.5428	-0.5428	0.1428	0.0592	2646
$ET1^-$	1.6448	0.4465	-0.4517	0.1227	0.1258	0.4282	0.5404	-0.5404	0.1421	0.0614	2646
*EL*4	2.0082	0.7926	-0.7461	0.2067	0.1845	0.4342	0.5435	-0.5436	0.2830	0.0742	6615
*EL*3	2.0069	0.7940	-0.7479	0.2068	0.1845	0.4342	0.5436	-0.5436	0.2830	0.0742	5292
*EL*2	1.6499	0.4714	-0.4252	0.1219	0.1258	0.4333	0.5428	-0.5428	0.1436	0.0603	3969
*EL*1	1.6448	0.4465	-0.4517	0.1227	0.1258	0.4282	0.5404	-0.5404	0.1421	0.0614	2646

$*$ *thickness locking correction*
$-$ *no correction*

Thermal loads

$$T(x,y,z) = \hat{T}(z)\, sin\left(\frac{\pi x}{a}\right) sin\left(\frac{\pi y}{b}\right)$$



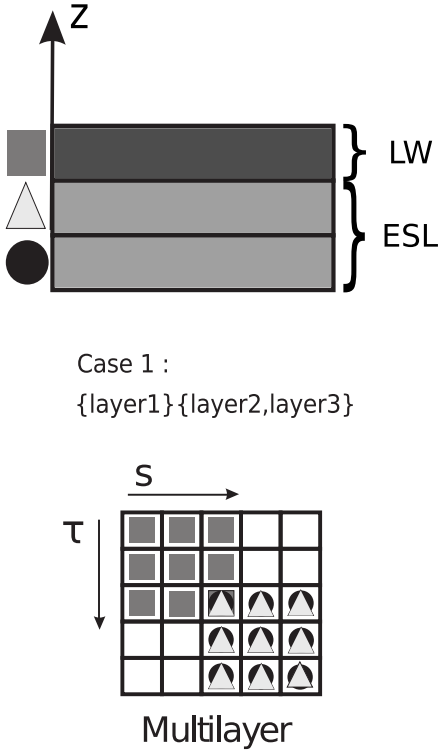
Carrera E., Valvano S.,
A variable kinematic shell formulation applied to thermal stress of laminated structures,
Journal of Thermal Stresses, 40(7): 803-827, 2017.
http://dx.doi.org/10.1080/01495739.2016.1253439

$\hat{T}(z = top) = +1.0$,
$\hat{T}(z = bottom) = -1.0$, $a = b = 1$,
$h = 0.1$

Mechanical properties:
$E_1/E_2 = 25$, $E_2 = E_3$
$G_{12}/E_2 = 0.5$, $G_{23}/E_2 = 0.2\, GPa$,
$G_{12} = G_{13}$
$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$

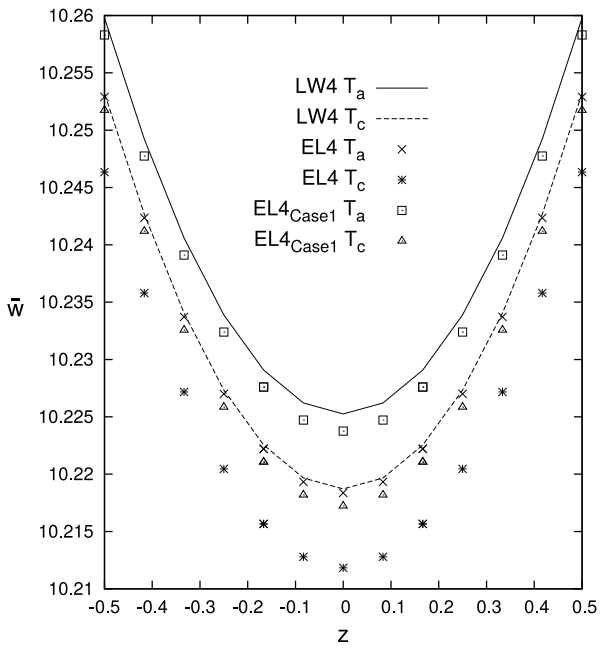
Thermal properties:
$\alpha_2/\alpha_1 = 3$, $\alpha_1 = \alpha_3$
$\mathcal{K}_1/\mathcal{K}_2 = 36.42/0.96$, $\mathcal{K}_2 = \mathcal{K}_3$

Mixed ESL-LW Variable-Kinematics

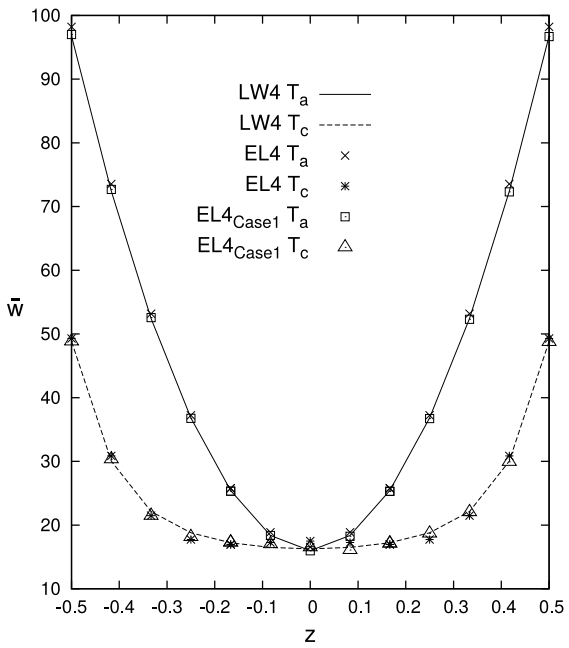


Transverse mechanical displacement **w**

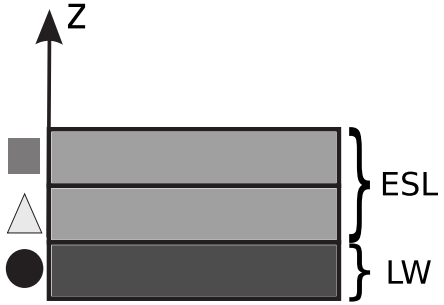
a/h=100



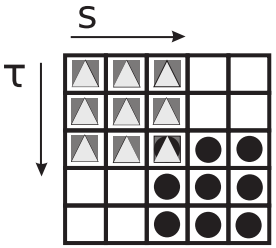
a/h=2



Mixed ESL-LW Variable-Kinematics



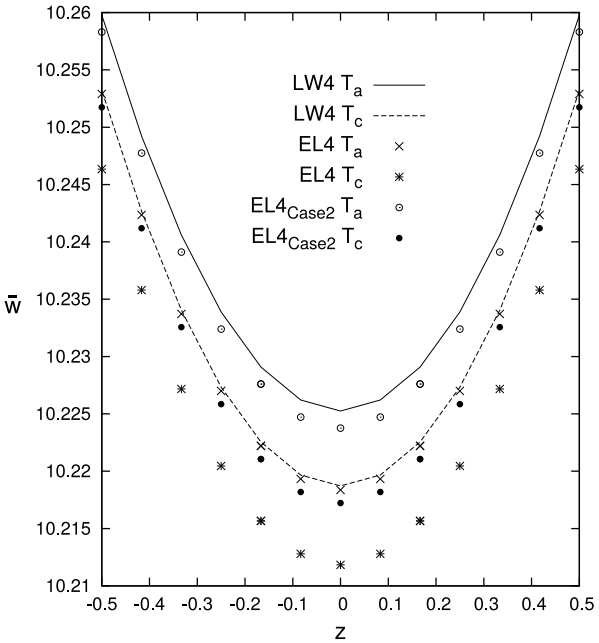
Case 2:
{layer1,layer2}{layer3}



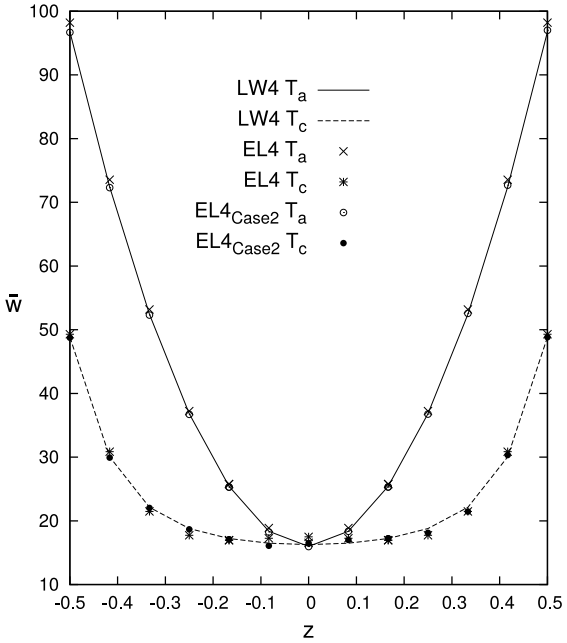
Multilayer

Transverse mechanical displacement **w**

a/h=100

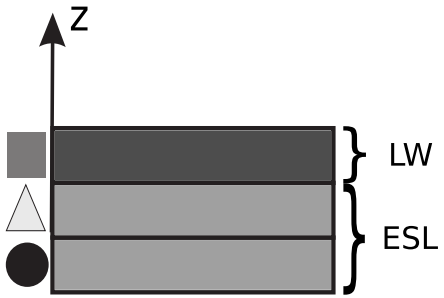


a/h=2

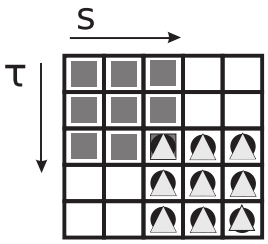


Mixed ESL-LW Variable-Kinematics

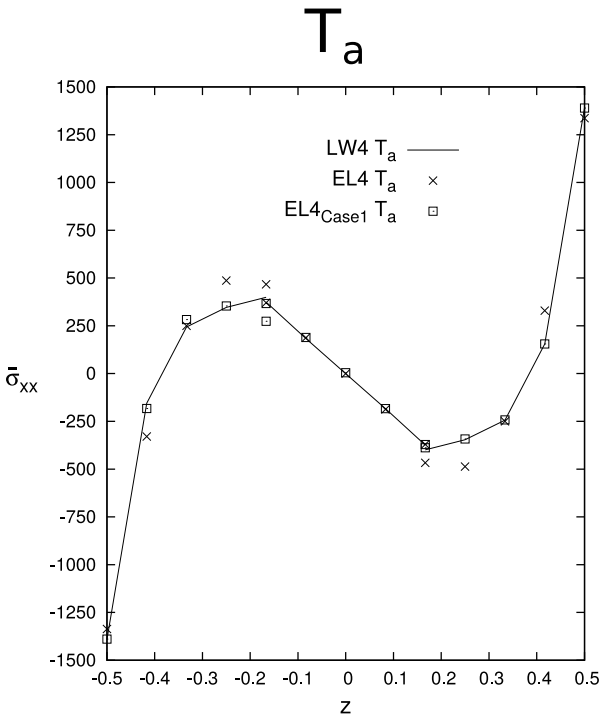
In-plane mechanical stress σ_{xx}, $(a/h = 2)$

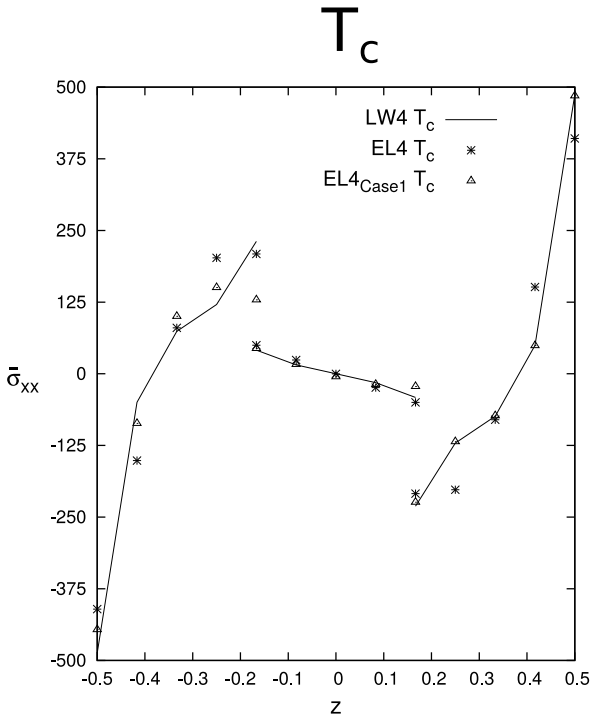


Case 1 :
{layer1}{layer2,layer3}

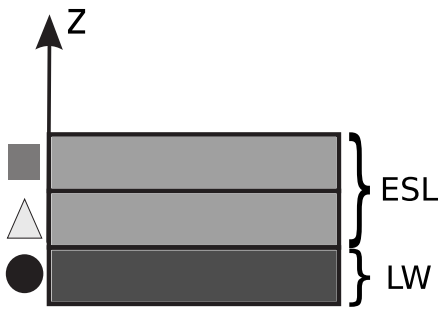


Multilayer

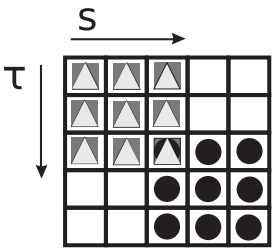




Mixed ESL-LW Variable-Kinematics

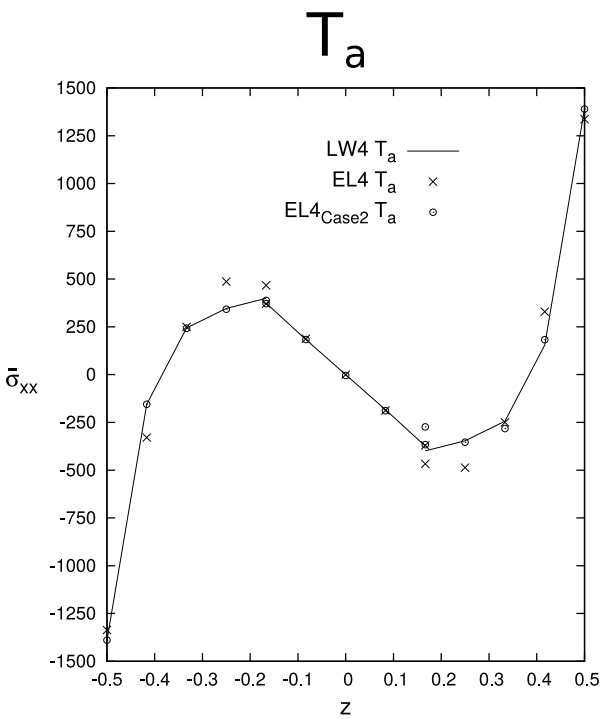


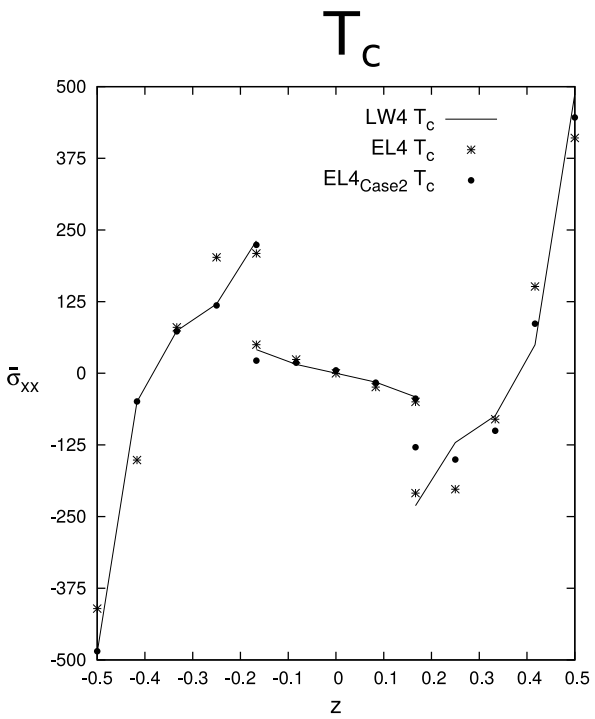
Case 2:
{layer1,layer2}{layer3}



Multilayer

In-plane mechanical stress σ_{xx}, $(a/h = 2)$



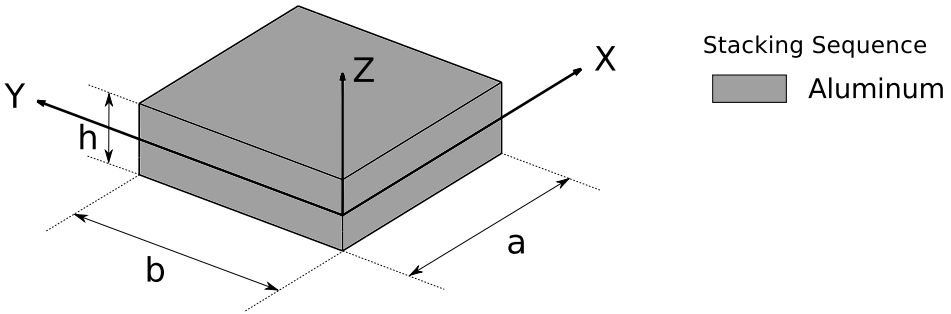


Mixed ESL/LW

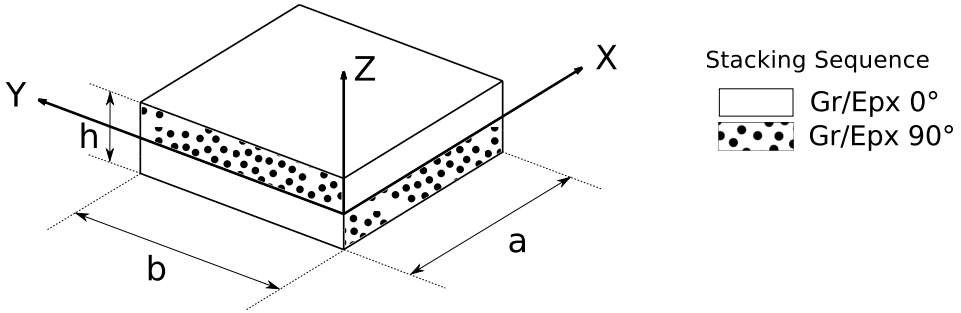
Free-Vibration Analysis Results

Isotropic and Composite Simply-Supported Square Plates

1 layered Isotropic



2 layered Composite



Aluminum Properties:

$$E = 73\,GPa,\ \nu = 0.3$$

$$\rho = 2800\,\frac{Kg}{m^3},\ c_V = 897\,\frac{J}{Kg\,K}$$

$$\alpha = 25\,E-6\,\frac{1}{K},\ \kappa = 130\,\frac{W}{m\,K}$$

Composite Properties:

$$E_L/E_T = 172.72/6.909\,(GPa)$$

$$G_{LT}/G_{TT} = 3.45/1.38\,(GPa)$$

$$\nu_{LT} = \nu_{TT} = 0,25$$

$$\rho = 1940\,\frac{Kg}{m^3},\ c_V = 846\,\frac{J}{Kg\,K}$$

$$\alpha_L/\alpha_T = 0.57\,E-6/35.6\,E-6\,(\frac{1}{K})$$

$$\kappa_L/\kappa_T = 36.42/0.96\,(\frac{W}{m\,K})$$

1 Layered Isotropic Plates

	$a/h = 2$				$a/h = 100$			
	$EL4_M$	$EL4_T$	$EL1_M$	$EL1_T$	$EL4_M$	$EL4_T$	$EL1_M$	$EL1_T$
Ref Analytical [1]	763.94	766.03			0.4852	0.4875		
Frequencies								
1	763.94	763.95	791.65	791.66	0.4856	0.4856	0.5374	0.5374
2	791.65	791.65	791.65	791.66	1.2165	1.2166	1.3458	1.3459
3	791.65	791.65	829.01	829.03	1.2165	1.2166	1.3458	1.3459
4	1119.6	1119.6	1119.6	1119.6	1.9440	1.9440	2.1507	2.1508
5	1414.8	1414.8	1522.2	1522.3	2.4457	2.4458	2.7033	2.7034
6	1414.8	1414.8	1522.2	1522.3	2.4457	2.4458	2.7033	2.7034
7	1583.3	1583.3	1583.3	1583.3	3.1673	3.1674	3.5023	3.5025
8	1583.3	1583.3	1583.3	1583.3	3.1673	3.1674	3.5023	3.5025
9	1770.2	1770.2	1770.2	1770.2	4.1896	4.1897	4.6253	4.6255
10	1770.2	1770.2	1770.2	1770.2	4.1896	4.1897	4.6253	4.6255
DOFs	16335	21780	6534	8712	16335	21780	6534	8712

[1] Brischetto S., Carrera E., "Coupled thermo-mechanical analysis of one-layered and multilayered plates", Composite Structures (2010) 92, 1793-1812

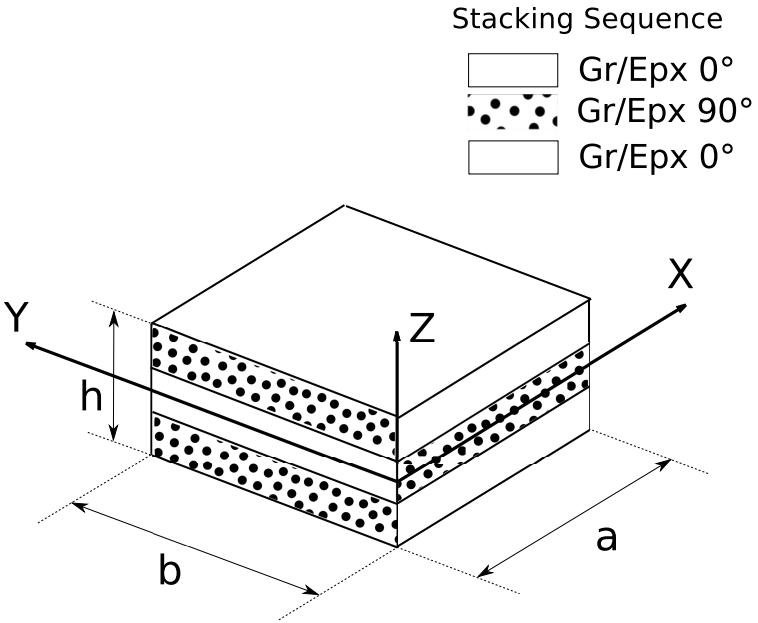
2 Layered Composite Plates

	$a/h = 2$								$a/h = 100$							
	$LW4_M$	$LW4_T$	$LW1_M$	$LW1_T$	$EL4_M$	$EL4_T$	$EL1_M$	$EL1_T$	$LW4_M$	$LW4_T$	$LW1_M$	$LW1_T$	$EL4_M$	$EL4_T$	$EL1_M$	$EL1_T$
Ref Analytical [1]	324.36	324.40			329.04	329.08			0.2909	0.2910			0.2909	0.2910		
Frequencies																
1	324.30	324.31	333.39	333.39	328.99	328.99	333.39	333.39	0.2908	0.2908	0.2919	0.2919	0.2908	0.2908	0.2945	0.2945
2	333.39	333.39	333.39	333.39	333.39	333.39	333.39	333.39	0.8040	0.8040	0.8069	0.8069	0.8042	0.8042	0.8128	0.8128
3	333.39	333.39	337.85	337.85	333.39	333.39	341.13	341.13	0.8040	0.8040	0.8069	0.8069	0.8042	0.8042	0.8128	0.8128
4	552.49	552.49	578.59	578.59	561.17	561.17	583.26	583.26	1.1616	1.1616	1.1665	1.1665	1.1618	1.1618	1.1769	1.1769
5	552.49	552.49	578.59	578.59	561.17	561.17	583.26	583.26	1.7176	1.7176	1.7240	1.7240	1.7181	1.7181	1.7353	1.7353
6	595.49	595.49	635.14	635.14	609.19	609.19	666.78	666.78	1.7176	1.7176	1.7240	1.7240	1.7181	1.7181	1.7353	1.7353
7	595.49	595.49	635.14	635.14	609.19	609.19	666.78	666.78	1.9709	1.9709	1.9800	1.9800	1.9716	1.9716	1.9958	1.9958
8	666.78	666.78	666.78	666.78	666.78	666.78	699.67	699.67	1.9709	1.9709	1.9800	1.9800	1.9716	1.9716	1.9958	1.9958
9	666.78	666.78	666.78	666.78	666.78	666.78	699.67	699.67	2.6078	2.6078	2.6204	2.6204	2.6088	2.6088	2.6436	2.6436
10	698.85	698.85	733.75	733.75	715.01	715.01	756.41	756.41	3.0059	3.0059	3.0191	3.0191	3.0076	3.0076	3.0382	3.0383
DOFs	29403	39204	9801	13068	16335	21780	6534	8712	29403	39204	9801	13068	16335	21780	6534	8712

[1] Brischetto S., Carrera E., "Coupled thermo-mechanical analysis of one-layered and multilayered plates", Composite Structures (2010) 92, 1793-1812

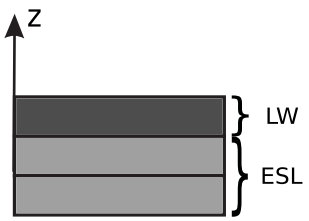
3 Layered Composite Plates

3 layered Composite

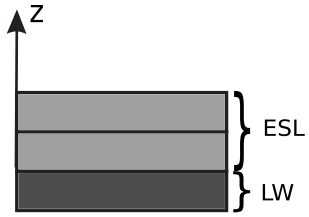


Mixed ESL/LW Variable Kinematic

Case 1



Case 2



3 Layered Composite Plates

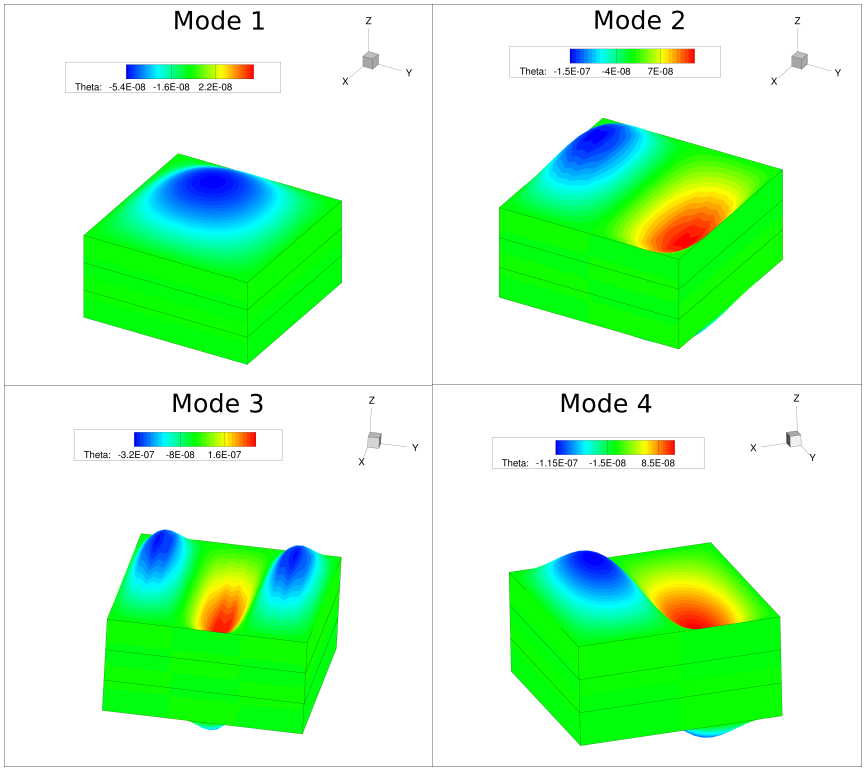
	$a/h = 100$							
	$LW4_M$	$LW4_T$	$EL4_M$	$EL4_T$	$EL4\ Case\ 1_M$	$EL4\ Case\ 1_T$	$EL4\ Case\ 2_M$	$EL4\ Case\ 2_T$
Frequencies								
1	0.4549	0.4549	0.4552	0.4552	0.4550	0.4550	0.4550	0.4550
2	0.6853	0.6853	0.6856	0.6856	0.6854	0.6854	0.6854	0.6854
3	1.2117	1.2117	1.2122	1.2122	1.2119	1.2119	1.2119	1.2119
4	1.6785	1.6785	1.6822	1.6822	1.6793	1.6793	1.6793	1.6793
5	1.7990	1.7990	1.8028	1.8028	1.7998	1.7998	1.7998	1.7998
6	2.0156	2.0156	2.0167	2.0167	2.0161	2.0161	2.0161	2.0161
7	2.1198	2.1198	2.1237	2.1237	2.1207	2.1207	2.1207	2.1207
8	2.7289	2.7289	2.7329	2.7329	2.7300	2.7300	2.7300	2.7300
9	3.0816	3.0816	3.0839	3.0839	3.0828	3.0828	3.0828	3.0828
10	3.6518	3.6518	3.6566	3.6566	3.6535	3.6535	3.6535	3.6535
DOFs	42471	56628	16335	21780	29403	39204	29403	39204

3 Layered Composite Plates

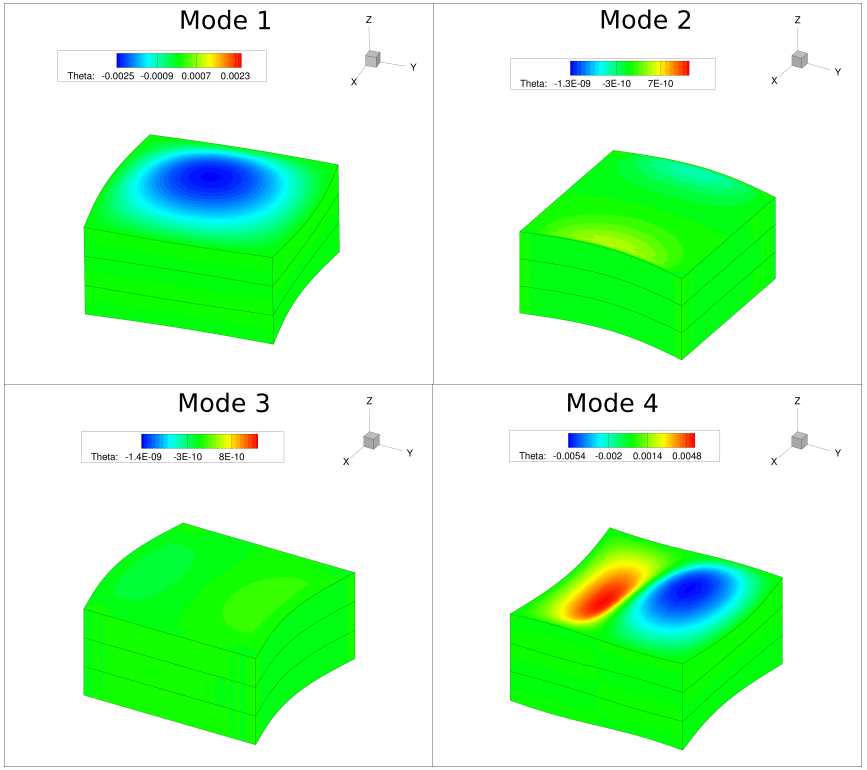
	$a/h = 2$							
	$LW4_M$	$LW4_T$	$EL4_M$	$EL4_T$	$EL4\ Case\ 1_M$	$EL4\ Case\ 1_T$	$EL4\ Case\ 2_M$	$EL4\ Case\ 2_T$
Frequencies								
1	316.93	316.93	323.17	323.17	319.92	319.92	319.92	319.92
2	333.39	333.39	333.39	333.39	333.39	333.39	333.39	333.39
3	333.39	333.39	333.39	333.39	333.39	333.39	333.39	333.39
4	512.55	512.55	520.41	520.42	517.22	517.22	517.22	517.22
5	583.46	583.46	598.31	598.31	589.72	589.72	589.72	589.72
6	585.39	585.39	624.88	624.88	590.96	590.96	590.96	590.96
7	621.14	621.14	655.86	655.86	628.20	628.20	628.20	628.20
8	666.78	666.78	666.78	666.78	666.78	666.78	666.78	666.78
9	666.78	666.78	666.78	666.78	666.78	666.78	666.78	666.78
10	712.56	712.56	726.41	726.41	719.40	719.40	719.40	719.40
DOFs	42471	56628	16335	21780	29403	39204	29403	39204

Three-dimensional view of the Temperature θ

Temperature for a/h=100 plates

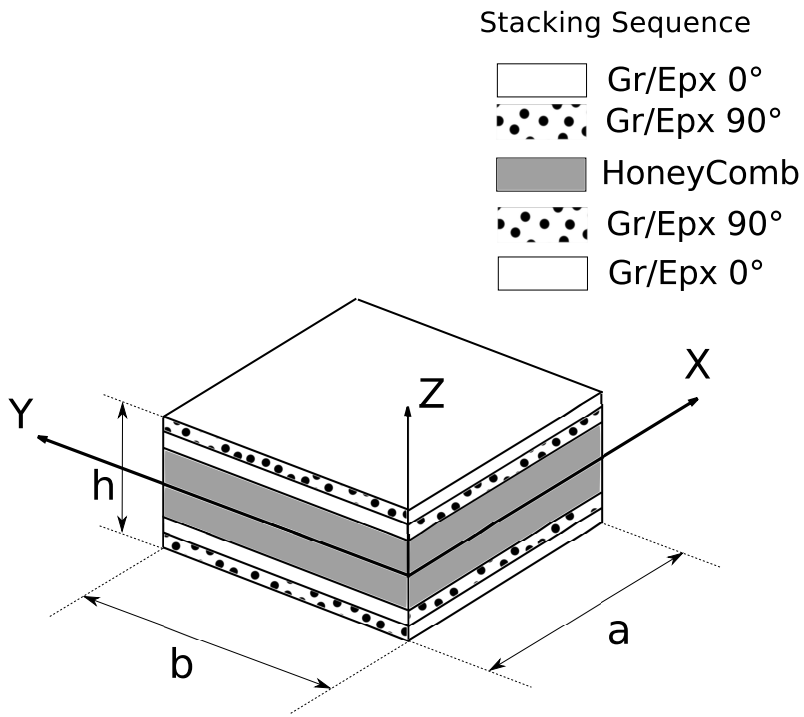


Temperature for a/h=2 plates



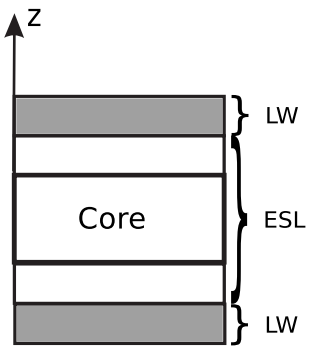
5 Layered Composite Sandwich Plates

5 layered Composite Sandwich

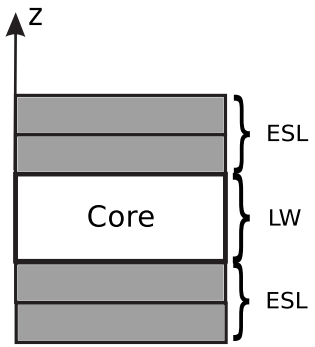


Mixed ESL/LW Variable Kinematic

Case 1



Case 2



5 Layered Composite Sandwich Plates

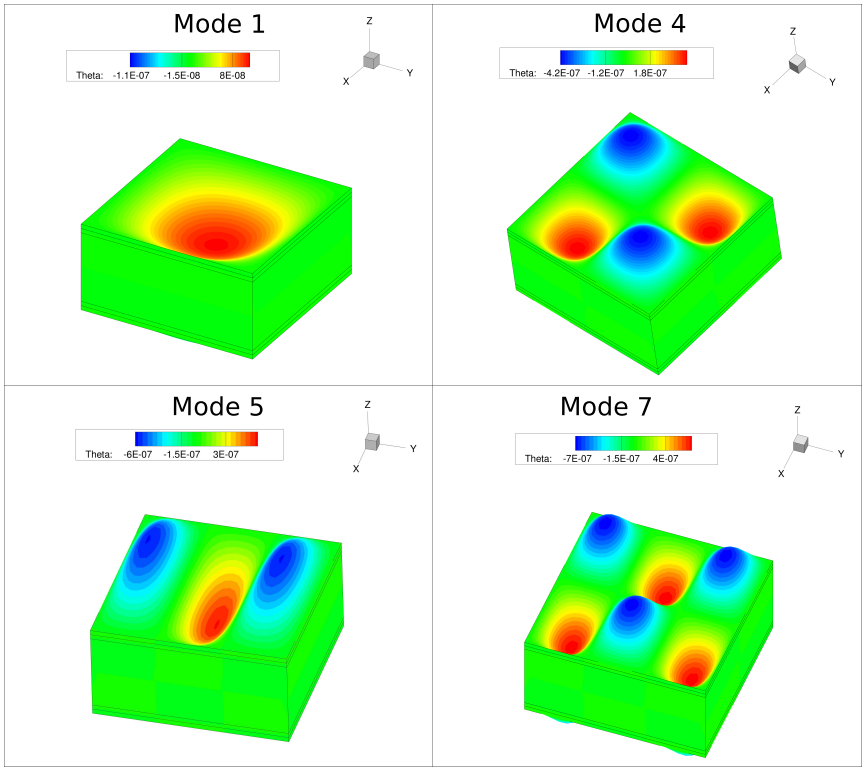
	$a/h = 100$							
	$LW4_M$	$LW4_T$	$EL4_M$	$EL4_T$	$EL4\ Case\ 1_M$	$EL4\ Case\ 1_T$	$EL4\ Case\ 2_M$	$EL4\ Case\ 2_T$
Frequencies								
1	0.6482	0.6482	0.6487	0.6487	0.6487	0.6487	0.6482	0.6482
2	1.7432	1.7432	1.7469	1.7469	1.7466	1.7466	1.7432	1.7432
3	1.8982	1.8982	1.9007	1.9007	1.9003	1.9003	1.8983	1.8983
4	2.5557	2.5557	2.5606	2.5606	2.5601	2.5601	2.5557	2.5557
5	3.6833	3.6833	3.6987	3.6987	3.6977	3.6977	3.6834	3.6834
6	4.0545	4.0545	4.0645	4.0645	4.0625	4.0625	4.0547	4.0547
7	4.1880	4.1880	4.2040	4.2040	4.2028	4.2028	4.1880	4.1880
8	4.4544	4.4544	4.4662	4.4662	4.4641	4.4641	4.4546	4.4546
9	5.6201	5.6201	5.6405	5.6405	5.6382	5.6382	5.6202	5.6202
10	6.3103	6.3103	6.3530	6.3530	6.3505	6.3505	6.3103	6.3103
DOFs	68607	91476	16335	21780	42471	56628	42471	56628
Δ *DOFs* %	0	0	76.2	76.2	38.1	38.1	38.1	38.1

5 Layered Composite Sandwich Plates

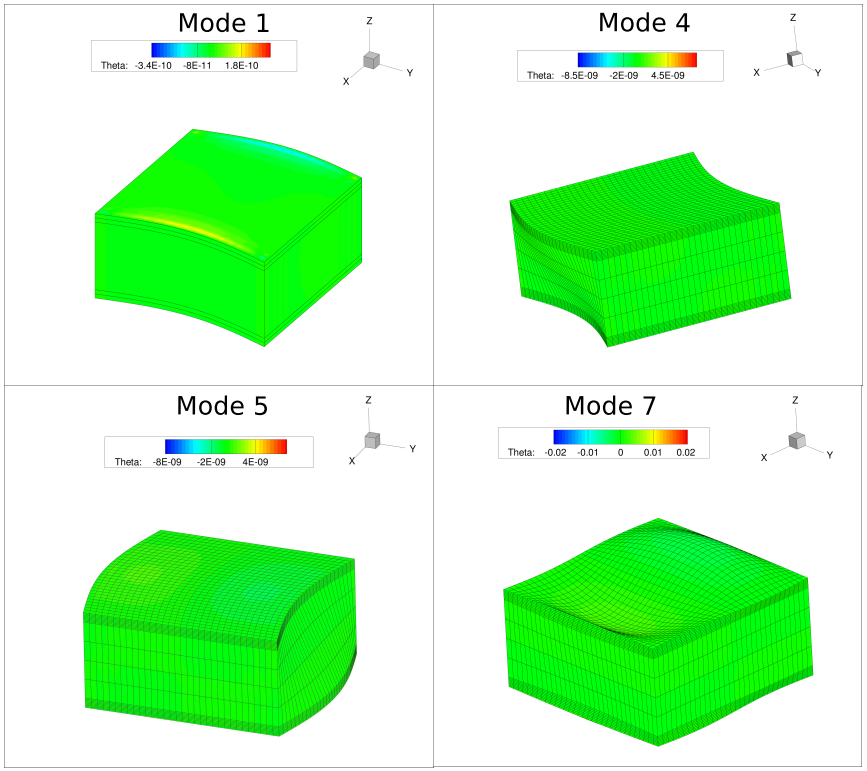
	$a/h = 2$							
	$LW4_M$	$LW4_T$	$EL4_M$	$EL4_T$	$EL4\ Case\ 1_M$	$EL4\ Case\ 1_T$	$EL4\ Case\ 2_M$	$EL4\ Case\ 2_T$
Frequencies								
1	322.15	322.15	322.17	322.17	322.17	322.17	322.15	322.15
2	322.16	322.16	322.20	322.20	322.19	322.19	322.16	322.16
3	324.31	324.31	336.31	336.31	334.98	334.98	324.44	324.44
4	480.74	480.74	487.59	487.59	486.63	486.63	480.78	480.78
5	481.92	481.92	493.18	493.18	492.16	492.16	481.95	481.95
6	514.01	514.01	533.00	533.00	531.82	531.82	514.22	514.22
7	536.55	536.55	552.02	552.02	549.78	549.78	537.00	537.00
8	643.14	643.14	643.33	643.33	643.30	643.30	643.14	643.14
9	643.19	643.19	643.54	643.54	643.45	643.45	643.19	643.19
10	663.70	663.70	684.52	684.52	682.33	682.33	664.20	664.20
DOFs	68607	91476	16335	21780	42471	56628	42471	56628
Δ *DOFs* %	0	0	76.2	76.2	38.1	38.1	38.1	38.1

Three-dimensional view of the Temperature θ

Temperature for a/h=100 plates



Temperature for a/h=2 plates



Conclusions

- Unified Formulation is the ideal tool for the implementation of variable kinematic theories. In fact, the theory approximation order and the modelling technique (ESL, LW) are free parameters of the FEM arrays, which are written in a compact and very general form.
- The present variable kinematic models, in general for static and free-vibration analysis, allow to locally improve the solution with a reduction of computational costs with respect to Layer-Wise solutions.
- The Mixed ESL/LW variable kinematic is effective for the free-vibration analysis of sandwich structures. Strong computational cost reductions can be obtained with high solution accuracy, respect to the full Layer-Wise model.
- The results show confidence for future extension of the present variable-kinematic methodology to thermography investigations analysis, and to the free-vibration analysis of multilayered piezoelectric components.

Thanks for the attention

Temperature for a/h=2 plates