Node-dependent kinematic shell elements for the analysis of smart structures

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- Results: Composite Shell with piezoelectric layers subjected to electrical load
- Results: Sandwich plate with piezoelectric patch subjected to electrical load
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Unified Formulation

2D approximation of **mechanical displacements** and **electric potential** using the **thickness functions**

\[
\begin{align*}
    u^k(\alpha, \beta, z) &= F_0(z) u_0^k(\alpha, \beta) + F_1(z) u_1^k(\alpha, \beta) + \ldots + F_N(z) u_N^k(\alpha, \beta) \\
    v^k(\alpha, \beta, z) &= F_0(z) v_0^k(\alpha, \beta) + F_1(z) v_1^k(\alpha, \beta) + \ldots + F_N(z) v_N^k(\alpha, \beta) \\
    w^k(\alpha, \beta, z) &= F_0(z) w_0^k(\alpha, \beta) + F_1(z) w_1^k(\alpha, \beta) + \ldots + F_N(z) w_N^k(\alpha, \beta) \\
    \Phi^k(\alpha, \beta, z) &= F_0(z) \Phi_0^k(\alpha, \beta) + F_1(z) \Phi_1^k(\alpha, \beta) + \ldots + F_N(z) \Phi_N^k(\alpha, \beta)
\end{align*}
\]

in compact form:

\[
\begin{align*}
    u^k(\alpha, \beta, z) &= F_\tau(z) u_\tau^k(\alpha, \beta) ; \quad \delta u^k(\alpha, \beta, z) = F_s(z) \delta u_s^k(\alpha, \beta) ; \quad \tau, s = 0, 1, \ldots, N \\
    \Phi^k(\alpha, \beta, z) &= F_\tau(z) \Phi_\tau^k(\alpha, \beta) ; \quad \delta \Phi^k(\alpha, \beta, z) = F_s(z) \delta \Phi_s^k(\alpha, \beta) ; \quad \tau, s = 0, 1, \ldots, N
\end{align*}
\]
Taylor Polynomials

\[ u^k = F_0 u_0^k + F_1 u_1^k + \ldots + F_N u_N^k = F_\tau u_\tau^k \]
\[ \Phi^k = F_0 \Phi_0^k + F_1 \Phi_1^k + \ldots + F_N \Phi_N^k = F_\tau \Phi_\tau^k \]
\[ \tau = 0, 1, \ldots, N \]
\[ F_0 = (z)^0 = 1 ; \quad F_1 = (z)^1 = z ; \ldots ; \quad F_N = (z)^N \]

Equivalent Single Layer Approach (ESL)

N = 1 \quad N = 2 \quad N = 3

Legendre Polynomials

\[ u^k = F_t u_t^k + F_b u_b^k + F_r u_r^k = F_\tau u_\tau^k \]
\[ \Phi^k = F_t \Phi_t^k + F_b \Phi_b^k + F_r \Phi_r^k = F_\tau \Phi_\tau^k \]
\[ \tau = t, b, r \quad ; \quad r = 2, \ldots, N \]
\[ F_t = \frac{P_0 + P_1}{2} \quad ; \quad F_b = \frac{P_0 - P_1}{2} \quad ; \quad F_r = P_r - P_{r-2} \]

Layer Wise Approach (LW)

N = 1 \quad N = 2

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Approximation of variables in the reference midplane surface using the Langrangian shape functions:

\[ \mathbf{u}_\tau = N_i(\xi, \eta) \mathbf{u}_{\tau i} \]

To overcome the problem of the membrane and shear locking, the strain components are calculated using a specific interpolation strategy:

For example:

\[ \epsilon_{\alpha\alpha} = N_{A1} \epsilon_{\alpha\alpha A1} + N_{B1} \epsilon_{\alpha\alpha B1} + N_{C1} \epsilon_{\alpha\alpha C1} + N_{D1} \epsilon_{\alpha\alpha D1} + N_{E1} \epsilon_{\alpha\alpha E1} + N_{F1} \epsilon_{\alpha\alpha F1} \]
PVD for electro-mechanical problems

Static Analysis
Principle of Virtual Displacements

\[ \int_V \left\{ \delta \epsilon^k \sigma^k - \delta \varepsilon^k \mathcal{D}^k \right\} dV = \delta L_e \]

\[ \sigma^k = C^k \epsilon^k - e^k \varepsilon^k \]

\[ \mathcal{D}^k = e^k \epsilon^k + e^k \varepsilon^k \]

In compact form:

\[ (4 \times 4) \]

\[ \begin{bmatrix} \delta u^k_{\tau i} : [K_{uu} & K_{u\Phi}]^{k\tau sij} \end{bmatrix} = \begin{bmatrix} \delta \Phi^k_{\tau i} : [K_{\Phi u} & K_{\Phi \Phi}] \end{bmatrix} = \begin{bmatrix} P_u \end{bmatrix}^{k\tau i} \]

\[ K^{k\tau sij} = \begin{bmatrix} K_{\alpha\alpha} & K_{\alpha\beta} & K_{\alpha z} & K_{\alpha \Phi} \\ K_{\beta\alpha} & K_{\beta\beta} & K_{\beta z} & K_{\beta \Phi} \\ K_{z\alpha} & K_{z\beta} & K_{zz} & K_{z \Phi} \\ K_{\Phi\alpha} & K_{\Phi\beta} & K_{\Phi z} & K_{\Phi \Phi} \end{bmatrix}^{k\tau sij} \]

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Node-dependent kinematic shell elements for the analysis of smart structures
In literature different kinematics can be coupled between elements with additional equations or mathematical artifices. The most common method to couple different kinematics are: the Arlequin Method, and the Lagrange Multipliers Method.
Node-Dependent Variable Kinematic Finite Element for Global/Local analysis

Different Kinematics are defined in the **Global Nodes**. **Shared nodes have the same kinematics.** The coupling of different kinematics is naturally obtained inside the finite element **without** any mathematical artifice or additional equations.
Node-Dependent Variable Kinematic Finite Element for Global/Local analysis

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Node-dependent kinematic shell elements for the analysis of smart structures
Finite Element assembling scheme

\[ u(\alpha, \beta, z) = F_5(z) N_j(\alpha, \beta) u_s \]
\[ \delta u(\alpha, \beta, z) = F_\tau(z) N_i(\alpha, \beta) \delta u_\tau \]
\[ \Phi(\alpha, \beta, z) = F_5(z) N_j(\alpha, \beta) \Phi_s \]
\[ \delta \Phi(\alpha, \beta, z) = F_\tau(z) N_i(\alpha, \beta) \delta \Phi_\tau \]

\[ S = 0, 1, \ldots, N \quad \tau = 0, 1, \ldots, N \quad i, j = 1, \ldots, 9 \]

ESL2 model for the entire finite element

\[ u(\alpha, \beta, z) = (u_{01} + z u_{11} + z^2 u_{21}) N_1(\alpha, \beta) + \ldots + (u_{03} + z u_{13} + z^2 u_{23}) N_3(\alpha, \beta) + \ldots + (u_{05} + z u_{15} + z^2 u_{25}) N_5(\alpha, \beta) + \ldots \]

\[ \Phi(\alpha, \beta, z) = (\Phi_{01} + z \Phi_{11} + z^2 \Phi_{21}) N_1(\alpha, \beta) + \ldots + (\Phi_{03} + z \Phi_{13} + z^2 \Phi_{23}) N_3(\alpha, \beta) + \ldots + (\Phi_{05} + z \Phi_{15} + z^2 \Phi_{25}) N_5(\alpha, \beta) + \ldots \]
Node-Dependent Variable assembling example

\[ u(\alpha, \gamma, z) = F_s(z) N_j(\alpha, \beta) u_s \]
\[ \delta u(\alpha, \gamma, z) = F^i(\gamma) N_l(\alpha, \beta) \delta u_l \]
\[ \Phi(\alpha, \gamma, z) = F_s(z) N_j(\alpha, \beta) \Phi_s \]
\[ \delta \Phi(\alpha, \gamma, z) = F^i(\gamma) N_l(\alpha, \beta) \delta \Phi_l \]

\[ s = 0, 1, \ldots, N_j \]
\[ \tau = 0, 1, \ldots, N_l \]
i, j = 1, \ldots, 9

Node 1 = LW1 \hspace{1cm} \text{node 3 = ESL2} \hspace{1cm} \text{node 5 = ESL3}

\[ N(\text{node } 1) = 1 \hspace{1cm} N(\text{node } 3) = 2 \hspace{1cm} N(\text{node } 5) = 3 \]

\[ u(\alpha, \gamma, z) = (1+\zeta_k/2)u_{01} + (1-\zeta_k/2)u_{11} N_1(\alpha, \beta) + \ldots + (u_{03} + z u_{13} + z^2 u_{23}) N_3(\alpha, \beta) + \ldots + (u_{05} + z u_{15} + z^2 u_{25} + z^3 u_{35}) N_5(\alpha, \beta) + \ldots \]

\[ \Phi(\alpha, \gamma, z) = (1+\zeta_k/2)\Phi_{01} + (1-\zeta_k/2)\Phi_{11} N_1(\alpha, \beta) + \ldots + (\Phi_{03} + z \Phi_{13} + z^2 \Phi_{23}) N_3(\alpha, \beta) + \ldots + (\Phi_{05} + z \Phi_{15} + z^2 \Phi_{25} + z^3 \Phi_{35}) N_5(\alpha, \beta) + \ldots \]
Composite Plate with Piezoelectric Skins

Sensor Case

\[ p(x, y, z_{\text{top}}) = \hat{p}_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \]

\[ \hat{\Phi}(z = \text{top}) = 0 \]

\[ \hat{\Phi}(z = \text{bottom}) = 0 \]

Actuator Case

\[ \Phi(x, y, z_{\text{top}}) = \hat{\Phi}_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \]

\[ \hat{\Phi}(z = \text{bottom}) = 0 \]

Stacking Sequence

- PZT-4
- Gr/EpX 0°
- Gr/EpX 90°
- PZT-4

Simply-Supported

Geometry:

\[ a = b = 4; \ h = 1 \]

\[ h_{\text{composite}} = 0.4 \times h_{\text{total}} \]

\[ h_{\text{skin}} = 0.1 \times h_{\text{total}} \]
Node-Dependent Variable Kinematics

Cases with Layer-Wise models

<table>
<thead>
<tr>
<th>Actuator Case</th>
<th>((x, y, z))</th>
<th>(\tilde{w})</th>
<th>(\tilde{\sigma}_{xx})</th>
<th>(\tilde{\sigma}_{xz})</th>
<th>(\tilde{\sigma}_{zz})</th>
<th>(\tilde{\Phi})</th>
<th>(\tilde{D}_z)</th>
<th>DOFs</th>
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<td>Reference solutions</td>
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<tr>
<td>LW4</td>
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<td>Arlequin solutions</td>
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<td>(LW1 - LWM3)A</td>
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<td>(LW2 - LWM3)C</td>
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<td>Present single- and multi-theory models</td>
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</tbody>
</table>

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Node-dependent kinematic shell elements for the analysis of smart structures
Node-Dependent Variable Kinematics

Cases with Layer-Wise models

Transverse displacement
\[ w(x, y) : (a/2, b/2) \]

Sensor

Actuator

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Node-dependent kinematic shell elements for the analysis of smart structures
Node-Dependent Variable Kinematics

Cases with Layer-Wise models

Transverse electric displacement and shear stress

\[ D_z(x, y) : \sigma_{xz}(x, y) : (a/2, b/2) \]

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Node-dependent kinematic shell elements for the analysis of smart structures
Node-Dependent Variable Kinematics

Cases with Layer-Wise models

Transverse electric displacement and in-plane stress along X-axis

\[ D_z(y, z) : \sigma_{xx}(y, z) : (b/2, +h/2) \]

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Node-dependent kinematic shell elements for the analysis of smart structures
Transverse Shear Stress $\sigma_{xz}$

Sensor

Case A

Actuator

Cut-view A-A

LW1 LW3 LW1

Case B

Node-dependent kinematic shell elements for the analysis of smart structures

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Cylindrical shell with Piezoelectric Skins

Actuator Case

Upper Skin
\[ \Phi_{\text{top}} = 50 \text{ V}, \Phi_{\text{bottom}} = 0 \text{ V} \]

Lower Skin
\[ \Phi_{\text{top}} = 0 \text{ V}, \Phi_{\text{bottom}} = 50 \text{ V} \]
Node-Dependent Variable Kinematics

Cases with Layer- Wise models

Transverse displacement

\( w(\alpha, \beta) : (a, 0) \)

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Node-dependent kinematic shell elements for the analysis of smart structures
Node-Dependent Variable Kinematics

Cases with Layer-Wise models

In-plane and transverse shear stresses

\[ \sigma_{\alpha\alpha}(\alpha, \beta) : \sigma_{\alpha z}(\alpha, \beta) : (0, 0) \]
Transverse displacement $w$

Present shell model **LW4**

Abaqus: 3D finite element C3D20RE

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Node-dependent kinematic shell elements for the analysis of smart structures
Transverse shear stress $\sigma_{xz}$

Present shell model LW4

Abaqus 3D finite element: C3D20RE

Case B

Case C

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Node-dependent kinematic shell elements for the analysis of smart structures
Sandwich plate with piezoelectric patch shear actuated mode-15

Piezoelectric patch used in **Actuator configuration**

\[ \Phi_{\text{patch top}} = -10 \ V \quad \Phi_{\text{patch bottom}} = +10 \ V \]
Model validation varying the patch position along the x-axis

Reference solutions:

C T Sun and X D Zhang.
Use of thickness-shear mode in adaptive sandwich structures.

A Benjeddou, M Trindade, and R Ohayon.
A unified beam finite element model for extension and shear piezoelectric actuation mechanisms.
Node-Dependent Variable Kinematics

Cases with Layer-Wise models

Transverse displacement and electric potential

\[ w(x, y) : (a, b/2) \quad \Phi(x, y) : (d = 85, b/2) \]
Node-Dependent Variable Kinematics

Cases with Layer-Wise models

In-plane and Transverse stresses

\[ \sigma_{xx}(x, y) : \sigma_{zz}(x, y) : (d = 85, b/2) \]

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Node-dependent kinematic shell elements for the analysis of smart structures
Transverse shear stress $\sigma_{xz}$

3D finite element Abaqus C3D20RE

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Node-dependent kinematic shell elements for the analysis of smart structures
Conclusions

Unified Formulation is the ideal tool for the implementation of **node-dependent** variable kinematic theories. In fact, the theory approximation order and the modelling technique (ESL, LW) are free parameters of the FEM arrays, which are written in a compact and very general form.

The present node-dependent variable kinematic model allows to locally improve the solution. Two main aspects can be highlighted: a reduction of computational costs with respect to Layer-Wise single-model solutions, and a simultaneous multi-models global-local analysis can be performed in one-single analysis step.

The **Node-Dependent Variable Kinematic** method permits to enrich locally the theory approximation accuracy by enforcing the same kinematics at the interface nodes between kinematically incompatible plate/shell elements. The resulting global/local approach is very efficient because it does not employ any mathematical artifice to enforce the displacement and stress continuity, such as those methods based on Lagrange multipliers or overlapping regions.
Conclusions

- An accurate representation of secondary variables (mechanical stresses and electric displacements) in localized zones is possible with DOFs reduction if an accurate distribution of the higher-order kinematic capabilities is performed. On the contrary, the accuracy of the solution in terms of primary variables (mechanical displacements and electric potential) values depends on the global approximation over the whole structure. The efficacy of the node-dependent variable kinematic and global/local models, thus, depends on the characteristics of the problem under consideration as well as on the required analysis type.

- The **Node-Dependent Variable Kinematic** method is very promising. Further investigations will be carried out on thermo-mechanical problems with localized loadings, plate/shell with non-uniform thickness problems etc. etc.
Thanks for the attention

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Node-dependent kinematic shell elements for the analysis of smart structures