Efficient Component-Wise Finite Elements for the Dynamic Response Analysis of Metallic and Composite Structures

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Abstract

This paper presents a novel component-wise (CW) approach for the finite element, dynamic response analysis of composite structures. In a CW model, each component of a complex structure can be modelled through a refined 1D model based on the Carrera Unified Formulation (CUF). The CUF allows the use of any order 1D structural models in a unified manner. Finite element matrices are obtained using a set of a few fundamental nuclei that are independent of the structural model order. The adoption of only 1D finite elements to model complex structures improves the multi-dimension coupling capabilities and reduces the computational costs to a great extent. The CW leads to finite element models in which fibres, matrix, and plies can be modelled through the 1D CUF models. Furthermore, the CW can be seen as a physics-based approach. In fact, a detailed physical description of a real structure can be obtained placing the problem unknown variables on the physical surfaces of the real 3D model. No artificial surfaces or lines are needed, and the CAD-FEM coupling is facilitated. Each component can be modelled with its material characteristics; that is, no homogenization techniques are required. In this paper, the CW is exploited for the dynamic response analysis of structures is presented. The CW capabilities are compared to those of classical techniques and commercial codes to prove the CW high accuracy and low computational cost.

Keywords: FEM, Beam, Composites, CUF, Component-Wise

Introduction

The dynamic analysis of structures is of primary importance in many applications, such as impact problems, damage detection, and health monitoring. Such analyzes can be challenging tasks. Computational models have to detect very accurate displacement, strain and stress fields. Moreover, local effects, multi-scale and multi-dimension structural components (e.g. layers and fibers, panels and stringers), and anisotropy have to be considered to obtain reliable results. Currently, most of the techniques that have been developed for these tasks are based on very cumbersome numerical models, such as the 3D solid finite elements. The accurate structural analysis of complex structures is almost impossible due to the enormous number of degrees of freedom that is required.

This paper presents the dynamic analysis of metallic and composite structures via refined structural models and the finite element modeling. In particular, 1D advanced structural models were used. Classical 1D models, or beams, were provided by Euler-Bernoulli [1, 2], and Timoshenko [3], hereafter referred to EBBT and TBT, respectively. These models are computationally cheap and, to some extent, reliable for many structural mechanics problems. However, EBBT and TBT cannot detect many mechanical behaviors; such as out-of-plane warping, in-plane distortions, torsion, coupling effects, or local effects. These effects are usually due to small slenderness ratios, thin walls, geometrical and mechanical asymmetries, and the anisotropy of the material [4]. Due to their computational efficiency, 1D models with advanced capabilities have been developed over the last decades. Some of the most relevant are based on the use of higher-order displacement fields [5, 6], the Variational Asymptotic

Method [7-9], the Generalized Beam Theory [10], and the Carrera Unified Formulation (CUF) [11]. Some works focused on the development of advanced 1D models for the dynamic response analysis [12-15].

This paper makes use of the advanced 1D models based on the CUF. The CUF [16, 17] provides refined 1D and 2D structural theories that are extremely accurate and computationally cheap. Recently, the 1D CUF has led to the development of the component-wise approach (CW) [18]. Figure 1 shows an example of CW modeling for a layered composite plate. The CW can model the macro (layers) and microscale components (fibers and matrix) using 1D models only. All these components can be coupled straightforwardly by imposing the displacement continuity at the interfaces. A detailed, physical description of composites can be obtained since the problem unknowns can be placed on the physical surfaces of the real 3D model. Moreover, each component is modeled using its material characteristics, that is, no homogenization techniques are required.

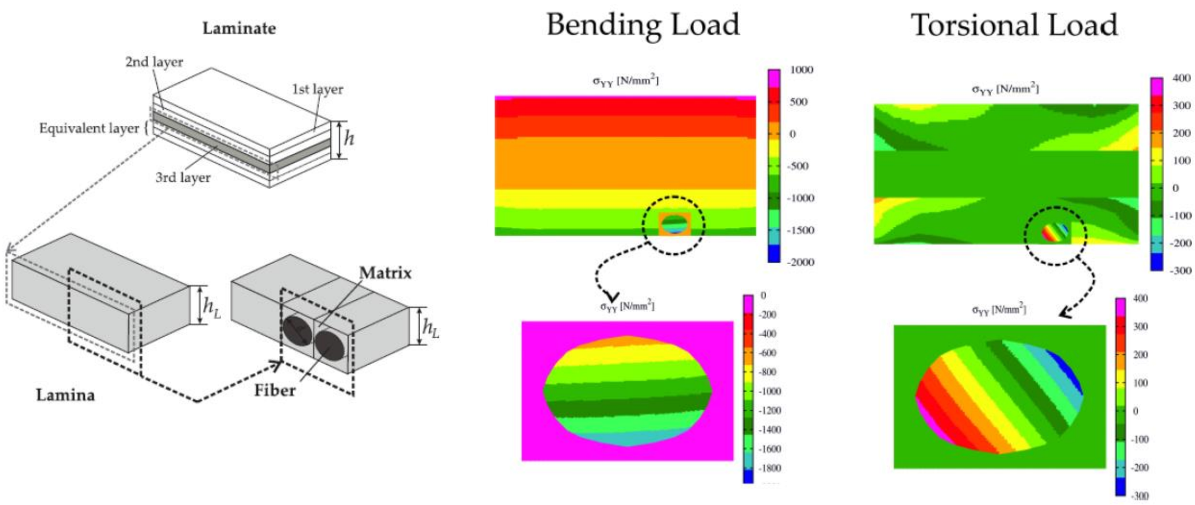

Figure 1: CW modeling of composites

The CW can be exploited to model other types of complex structures, such as aircraft wings [19, 20]. Recently the authors have extended the CW to the analysis of damaged isotropic, thin-walled structures [21].

This paper evaluates the enhanced capabilities of the CW for the dynamic response analysis of metallic and composite structures. Free vibration and dynamic response analyses are carried out. The results are compared with those of 3D solid finite elements.

1. Carrera Unified Formulation

The cross-section of the beam lies on the *xz*-plane, and it is denoted by Ω, whereas the boundaries over y are 0≤y≤L. Within the framework of the CUF, the 3D displacement field is expressed as

$$\boldsymbol{u}(x,y,z;t) = F_\tau(x,z)\boldsymbol{u}_\tau(y;t),\ \tau = 1,2,\dots,M \tag{1}$$

Where $\boldsymbol{F_\tau}$ are the functions of the coordinates x and z on the cross-section. $\boldsymbol{u_\tau}$ is the vector of the generalized displacements. $\boldsymbol{M}$ stands for the number of the terms used in the expansion, and the repeated subscript, τ indicates summation. LE (Lagrange Expansion) 1D CUF models exploit 2D Lagrange polynomials to model the displacement field of the structure above the cross-section. For instance, the displacement field of an L9 LE model can be expressed as

$$u_x = L_1 u_{x_1} + L_2 u_{x_2} + \cdots + L_9 u_{x_9} \tag{2}$$

For the sake of brevity, only the x-component of the displacement field is reported. The L9 model has 27 displacement variables that coincide with the three displacement components of the 9 Lagrange nodes. Two or more Lagrange elements can be conveniently assembled to discretized cross-sections, and improve the accuracy of the model. Figure 2 shows a typical cross-section modelling in which a finer modelling is used in the proximity of the applied load.

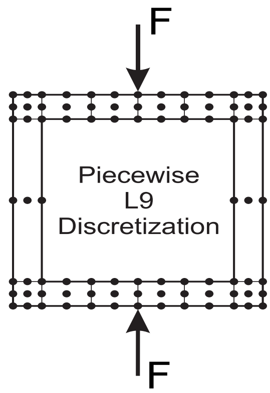

Figure 2: Multiple L9 discretization

A compact form of the virtual variation of the strain energy can be obtained as shown in [17],

$$\delta L_{int} = \delta \boldsymbol{q}_{sj}^T \boldsymbol{K}^{ij\tau s} \boldsymbol{q}_{\tau i} \tag{3}$$

Where $\boldsymbol{K}^{ij\tau s}$ is the stiffness matrix written in the form of the fundamental nuclei whose components can be found in [17]. δindicates the virtual variation. $\boldsymbol{q}_{\tau i}$ is the nodal vector. Superscripts indicate the four indexes exploited to assemble the matrix: i and j are related to the shape functions, τ and s are related to the expansion functions. The fundamental nucleus is a 3×3 array that is formally independent of the order of the beam model. It should be underlined that the formal expression of $\boldsymbol{K}^{ij\tau s}$ does not depend on the expansion order and on the choice of the F_τ expansion polynomials. All the other FEM matrices can be obtained in a similar manner.

The advanced capabilities of CUF 1D models can be particularly convenient in the case of multicomponent structures (MCS). The Component-Wise approach exploits LE 1D elements to model each component of a structure separately and independently of their geometrical and material characteristics. In other words, each 1D, 2D, 3D or micro and macro component can be modeled via LE 1D models with no need for ad hoc coupling and interface techniques. Figure 1 shows a typical CW strategy for a composite plate; 1D LE models can be simultaneously adopted to model layers (macroscale), matrix and fibers (microscale). This methodology can be very powerful when, for instance, detailed stress fields are required in a specific portion of the structures. Similar strategies can be used for aircraft structures as in [20].

2. Numerical Example

A clamped-clamped, thin-walled, isotropic cylinder was considered to highlight the enhanced capabilities of the present formulation. The outer diameter d is equal to 0.1 (m), the thickness is equal to 0.001 (m), and the span-to-diameter ratio (L=d) is equal to 10. The material is aluminum (E = 69 GPa, ν = 0:33, ρ = 2700 kg/m^3). Four points were considered over the mid-span cross-section as shown in Fig. 3. Four concentrated forces were applied as time-dependent sinusoids with amplitude P_{z0} = 10000 (N) and a phase shift,

$$P_{zA}(t) = P_{z0}\sin(\omega t + \varphi_A), P_{zB}(t) = P_{z0}\sin(\omega t + \varphi_B),$$

$$P_{zC}(t) = P_{z0}\sin(\omega t + \varphi_C), P_{zD}(t) = P_{z0}\sin(\omega t + \varphi_D),$$

$$\varphi_A = 0, \varphi_B = 30, \varphi_C = 60, \varphi_D = 90$$

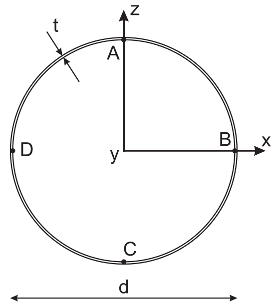

Figure 3: Thin walled cross-section

Where the angular frequency is ω = 100 rad/s. The dynamic response of the structure was evaluated over the time interval [0; 0.025] s. The Newmark integration scheme was exploited. Table 1 shows the transverse displacements of point A at t = 0 s. A 44 L9 model was used in this paper. The configuration at the final time instant t = 0.025 s is shown in Fig. 4 (mid-span cross-section). The results show how the present 1D formulation can detect severe cross-section deformations and match 3D solid results with much lower computational costs.

Table 1: Transverse displacement of point A

	CW (44 L9)	Solid
DOFs	24552	268440
u_{zA}	-9.5388	-9.8840

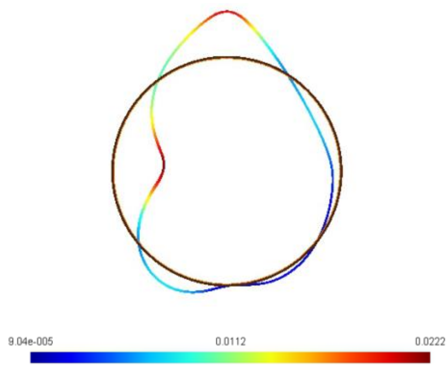

Figure 4: Deformation of the mid-span circular cross-section, t = 0:025s, 44 L9

3. Conclusions

This paper has presented a brief overview of the Component-Wise approach (CW) for the high-fidelity dynamic analysis of metallic and composite structures. The CW is based on the 1D CUF models. Such structural models are computationally efficient and accurate. 1D CUF, in fact, can provide 3D-like accuracy with 10-100 times fewer degrees of freedom. A numerical example has been carried out on a thin-walled structure undergoing a dynamic load. The present approach could be useful for impact problems in which high accuracy is needed, and a low computational cost is desirable.

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