Application of the Unified Formulation to bending analysis of magnetostrictive unimorph microbeams

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Magnetostrictive-based sensors and actuators are widely used in high-tech MEMS, recently. In such smart devices, thin films made of special magnetostrictive materials are bonded to a non-magnetic substrate to produce bending when subjected to magnetic fields. Since special magnetostrictive materials like Terfenol-D ($Tb_xDy_{1-x}Fe_2$) are able to produce high magnetic strains, they can be used in such sensitive devices. Consider a micro-bilayer consisting of a Terfenol-D thin film and a silicon substrate perfectly bonded together, as shown in Figure 1. The cantilever beam is subjected to a longitudinal magnetic field in actuator case as well as a tip deflection in sensor case.

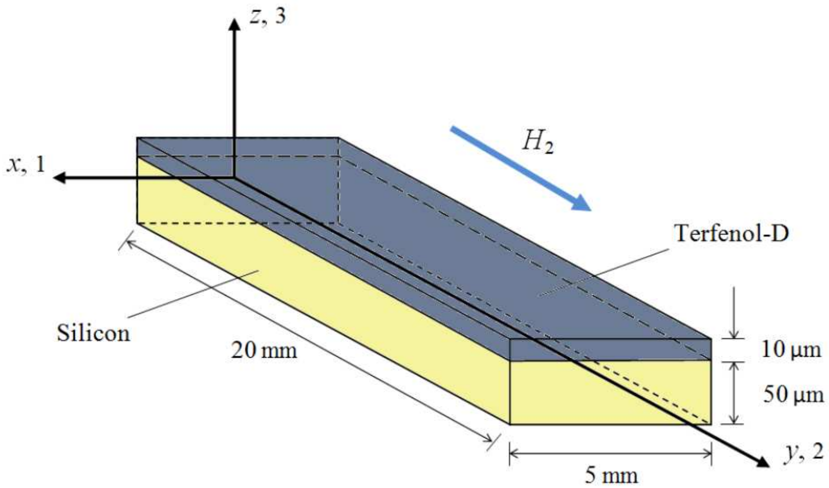

Figure 1: Example figure.

For the designers of MEMS, the determination of actuator deflection is very important. Such devices may be constructed in the form of sandwich beams, fully or partially -coated substrates. Thus, it is necessary to use suitable theoretical equations capable of dealing with different geometry and load conditions. In this paper, a new refined Finite Element (FE) model based on Carrera Unified Formulation (CUF) is used to predict the deflection and magnetic potential in the microbeams. The advantages of the CUF over conventional FE are widely discussed in the literature. The generalized displacement components for magnetomechanical analysis are

$$\boldsymbol{u}^T = \left\{ \begin{array}{cccc} u_x & u_y & u_z & \phi \end{array} \right\} \tag{1}$$

where u_1, u_2 and u_3 are the displacement components in the x, y and z directions, respectively, and ϕ is the magnetic potential. The generalized strain and stress vectors can be written as

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$$
\begin{aligned}
\boldsymbol{X}^T &= \left\{ \begin{array}{ccccccccc} \epsilon_{11} & \epsilon_{22} & \epsilon_{33} & \epsilon_{23} & \epsilon_{1} & \epsilon_{12} & H_1 & H_2 & H_3 \end{array} \right\}^T \\
\boldsymbol{Y}^T &= \left\{ \begin{array}{ccccccccc} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{23} & \sigma_{1} & \sigma_{12} & B_1 & B_2 & B_3 \end{array} \right\}^T
\end{aligned} \tag{2}
$$

where σ, ϵ, B and H stand for stress, strain, magnetic flux density and magnetic field intensity, respectively. In general, Terfenol-D has a non-linear behavior, however, in low magnetic field intensities, the following linear piezomagnetic constitutive relations can be used to present a reasonably accurate model of the material:

$$
\begin{aligned}
\sigma_{ij} &= C_{ijkl}\epsilon_{kl} - e_{kij}H_k \\
B_i &= e_{ikl}\epsilon_{kl} + \mu_{ik}H_k
\end{aligned} \tag{3}
$$

where C, e and μ are magnetomechanical constants. Eqs. 3 can be written in a compact form as

$$
\boldsymbol{Y} = \boldsymbol{R}\boldsymbol{X} \tag{4}
$$

where R is the matrix of magnetomechanical constants. The principle of virtual work in the absence of inertia indicates that

$$
\delta L_{int} = \int_V \delta \boldsymbol{X}^T \boldsymbol{Y} dV = \delta L_{ext} \tag{5}
$$

where L_{int} and L_{ext} stand for internal and external works, respectively, and δ is the variational operator. For a beam with a y-axis coinciding longitudinal axis, the 1D refined FE model can be used to discretize the domain. In this way, the generalized displacement vector and its variation can be written as [1]:

$$
\begin{aligned}
\mathbf{u}(x,y,z) &= N_i(y)F_\tau(x,z)\mathbf{U}_{\tau i} \\
\delta\mathbf{u}(x,y,z) &= N_j(y)F_s(x,z)\delta\mathbf{U}_{sj}
\end{aligned} \tag{6}
$$

where N_i and N_j denote the 1D shape function along the beam longitudinal axis, F_τ and F_s are the expansion functions of cross-sectional displacement components and $U_{\tau i}$ and U_{sj} are the vectors of nodal generalized displacement components. Substituting Eqs. and into , after some mathematical manipulations, on may get

$$
\delta L_{int} = \delta \boldsymbol{U}_{sj}^T \boldsymbol{k}^{\tau sij} \boldsymbol{U}_{\tau i}^T \tag{7}
$$

where the 4×4 matrix $\boldsymbol{k}^{\tau sij}$ is the fundamental nucleus of 1D piezomagnetic beam element. Assembling the fundamental nuclei matrices in a proper way leads to the following linear set of equations:

$$
[\boldsymbol{K}]\{\boldsymbol{U}\} = [\boldsymbol{Q}] \tag{8}
$$

where Q is the vector of nodal external forces. Solving Eq. 8 together with magnetomechanical boundary conditions yields the nodal values of generalized displacements. In order to validate the results of present study, the tip deflection of a bilayer microbeam is compared with the results of an experimental work made by [2] in Table 1. The material properties of silicon substrate and Terfenol-D film are (E_{Si}=169 GPa, ν_{Si}=0.067) [2] and (E_{Te}=50 GPa, ν_{Te}=0, d_{33}=10 nm/A, d_{15}=28 nm/A, μ_{33}=6.29 μH/m) [2, 3],

Table 1: Vertical displacement on the loading point for different LE structural model.

H_2 (Oe)	u_3 μm				
	5B2,4L9	5B3,4L4	5B3,2L9	5B3,4L9	Ref. [2]
26	27.18	26.38	27.3	27.25	24.59
56.4	59.66	57.91	59.93	59.81	65.57
86.8	91.48	88.79	91.89	91.72	94.26
117	123.3	119.67	123.85	123.62	117.1
148	156.45	151.85	157.14	156.85	135.83
178	187.61	182.09	188.44	188.1	155.73
213	224.73	218.12	225.72	225.31	168.03
243	256.55	248.97	257.69	257.21	176.81

respectively. A convergence analysis is also done and reported in the same table to find the proper set of beam and cross-sectional elements. In this table, 2 and 3-node beam elements (B2 and B3) are used for beam discretization while 4 and 9-node Lagrange elements (L4 and L9) are used as the expansion functions to model the cross-section. As it can be seen, in the region of low magnetic field intensities (< 120 Oe), the present results match well the experimental data, while in moderate and high field intensities, due to the nonlinear nature of Terfenol-D, the error grows gradually.

References

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