V ECCOMAS Thematic Conference on the Mechanical Response of Composites
COMPOSITES 2015
S.R. Hallett and J.J.C. Remmers (Editors)

# COMPONENT-WISE 1D MODELS FOR DAMAGED LAMINATED, FIBER-REINFORCED COMPOSITES

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**Keywords:** CUF, beam, damage, component-wise, finite elements.

**Summary:** *The structural analysis of damaged composite structures requires high fidelity models to detect very accurate displacement, strain and stress fields. In particular, local effects and 3D stress fields have to be dealt with. The proper modeling of multiscale components – layers, fibers and matrix – enhances the accuracy of computational models to a great extent. To date, 3D solid finite elements represent the most reliable tool for this kind of analyses. However, such finite elements can lead to very cumbersome numerical models. In other words, the accurate structural analysis of complex structures is quite impossible due to the very high number of degrees of freedom that is necessary. This paper presents free vibration analyses of damaged composite structures via an innovative approach that is based on 1D (beam) advanced models. The present 1D FEs stems from the Carrera Unified Formulation (CUF) and provide a Component-Wise (CW) modeling. In a CW model, each component of a complex structure is modeled through the refined 1D CUF models. A detailed physical description of the real structure is achieved because each component can be modeled with its material characteristics, and no homogenization techniques are required. Furthermore, although 1D models are exploited, the problem unknown variables are located on the physical surfaces of the real 3D model, and no artificial surfaces or lines have to be defined to build the structural model. The CW can lead to a multiscale approach for composites since each typical component of a composite structure - fibers, matrix, plies - can be modeled through the 1D CUF models. Different scale components can be assembled straightforwardly without ad hoc coupling techniques. The adoption of 1D models enhances the multi-dimension coupling capabilities and reduces the computational costs to a great extent. The computational cost reduction in terms of total amount of DOFs ranges from 10 to 100 times less than shell and solid models, respectively. In this paper, damaged composite structures are analyzed using the CW approach. Free vibration analyses are carried out, and comparisons against classical approaches are provided to show the enhanced capabilities of the present approach to providing 3D-like accuracy with very low computational costs.*

## 1. INTRODUCTION

The inclusion of the multi-scale characteristics of composites is often mandatory to enhance the quality of the structural analysis. To date, most of the techniques that have been developed

for this task are based on very cumbersome numerical models, such as the 3D solid finite elements. This means that the accurate structural analysis of complex composite structures is almost impossible due to the enormous number of degrees of freedom that is required. Examples of multiscale techniques can be found in [1, 2, 3, 4].
This paper deals with damaged composite structures, and, in particular, free vibration analyses were carried out. Many damage detection techniques exploit the changes in frequencies and modal shapes due to the presence of damage [5, 6, 7, 8]. The Modal Assurance Criterion (MAC) [9] is one of the most common tools to evaluate damage through the modal shapes of a structure [10, 11, 12]. The MAC is defined as a scalar representing the degree of consistency (linearity) between one modal and another reference modal vector.
The Carrera Unified Formulation (CUF) [13, 14] is a modeling technique to build 1D and 2D structural theories that are extremely accurate and computationally cheap. The 1D CUF, in particular, has been recently exploited to develop the component-wise approach (CW)[15]. Figure

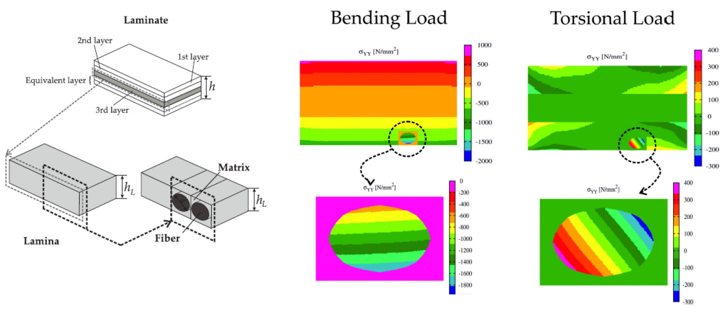



Figure 1. Component-Wise model of a laminated plate.

1 shows an example in which the CW was applied to model a layered composite plate. The CW can model the macro (layers) and microscale components (fibers and matrix) using 1D models only. All these components can be coupled straightforwardly by imposing the displacement continuity at the interfaces. A detailed, physical description of composites can be obtained since the problem unknowns can be placed on the physical surfaces of the real 3D model. Moreover, each component is modeled using its material characteristics, that is, no homogenization techniques are required. The CW can be exploited to model other types of complex structures, such as aircraft wings [16, 17]. In [18], the authors exploited the CW for the analysis of damaged isotropic, thin-walled structures.
This paper evaluates the effects of damages on composite structures via the CW. In particular, the effects on the natural frequencies and the MAC matrix are investigated.

## 2. CARRERA UNIFIED FORMULATION

The CUF is a hierarchical methodology to reduce 3D problems to 2D or 1D formulations in a unified manner. CUF exploits arbitrary rich expansions of the unknown variables. In the structural mechanics scenario, and in a displacement-based formulation, the CUF defines the displacement field of a structural model as the expansion of generic functions $F_\tau$,

$$\mathbf{u} = F_\tau \mathbf{u}_\tau, \qquad \tau = 1, 2, ...., M \tag{1}$$

Where $\mathbf{u}$ is the displacement vector, $\mathbf{u}_\tau$ is the generalized displacements unknown array and $M$ stands for the number of terms of the expansion. According to the Einstein notation, the repeated subscript, $\tau$, indicates summation.
The 1D Taylor Expansion (TE) models uses 2D polynomials $x^i\, z^j$ as base functions above the cross-section ($y$ is the axial coordinate). $i$ and $j$ are positive integers. For instance, the third-order ($N = 3$) TE model has the following displacement field:

$$\begin{aligned} u_x &= u_{x_1} + x\, u_{x_2} + z\, u_{x_3} + x^2\, u_{x_4} + xz\, u_{x_5} + z^2\, u_{x_6} + x^3\, u_{x_7} + x^2 z\, u_{x_8} + xz^2\, u_{x_9} + z^3\, u_{x_{10}} \\ u_y &= u_{y_1} + x\, u_{y_2} + z\, u_{y_3} + x^2\, u_{y_4} + xz\, u_{y_5} + z^2\, u_{y_6} + x^3\, u_{y_7} + x^2 z\, u_{y_8} + xz^2\, u_{y_9} + z^3\, u_{y_{10}} \\ u_z &= u_{z_1} + x\, u_{z_2} + z\, u_{z_3} + x^2\, u_{z_4} + xz\, u_{z_5} + z^2\, u_{z_6} + x^3\, u_{z_7} + x^2 z\, u_{z_8} + xz^2\, u_{z_9} + z^3\, u_{z_{10}} \end{aligned} \tag{2}$$

TE model unknowns are displacements and $N$-order derivatives of the displacement field. These variables are usually defined along the axis of the beam. The unknown variables become pure displacements if Lagrange polynomials are adopted as expansion functions. The resulting models are referred to as LE (Lagrange-Expansion) models in the framework of the CUF. Three- (L3), four- (L4) and nine-point (L9) polynomials were formulated which lead to linear, quasi-linear (bilinear), and quadratic kinematics, respectively. For instance, the interpolation functions in the case of an L4 element are the following ones:

$$F_\tau = \frac{1}{4}(1 + r\, r_\tau)(1 + s\, s_\tau) \qquad \tau = 1, 2, 3, 4 \tag{3}$$

where $r$ and $s$ vary from $-1$ to $+1$, and $r_\tau$ and $s_\tau$ are the coordinates in the natural plane of the four Lagrange nodes. According to the CUF, the displacement field given by an L4 element is

$$\begin{aligned} u_x &= F_1\, u_{x_1} + F_2\, u_{x_2} + F_3\, u_{x_3} + F_4\, u_{x_4} \\ u_y &= F_1\, u_{y_1} + F_2\, u_{y_2} + F_3\, u_{y_3} + F_4\, u_{y_4} \\ u_z &= F_1\, u_{z_1} + F_2\, u_{z_2} + F_3\, u_{z_3} + F_4\, u_{z_4} \end{aligned} \tag{4}$$

Where $u_{x_1}, ..., u_{z_4}$ are the displacement variables of the problem and represent the translational displacement components of each of the four points of the L4 element. For further refinements, the cross-section can be discretized by using several L-elements.
LE leads to FE mathematical models built by using only physical boundaries; artificial lines (beam axes) and surfaces (plate/shell reference surfaces) are no longer necessary. Figure 2 shows the physical volume/surface approach of the present modelling technique. A 3D geometry can be accurately modeled via LE since the problem unknowns can be spread over the physical surfaces of the structure. This capability can be extremely powerful in a CAD-FEM coupling scenario, for instance in an optimization problem, since the 3D CAD geometry can be straightforwardly exploited to build the FE model. FE matrices can be obtained upon the

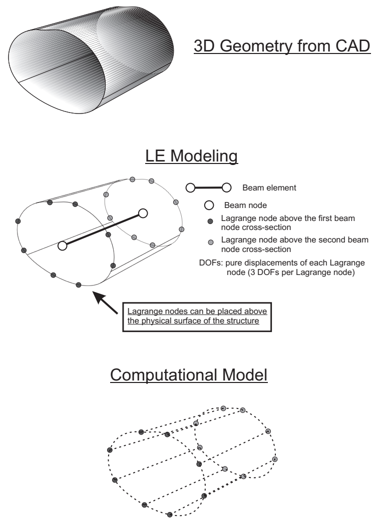



Figure 2. The physical volume/surface approach of LE.

introduction of the 3D material constitutive relations and the differential geometrical relations. For example, the virtual variation of the internal work can be written as follows:

$$\delta L_{int} = \delta \mathbf{u}_{sj}^T \mathbf{K}^{\tau sij} \mathbf{u}_{\tau i} \tag{5}$$

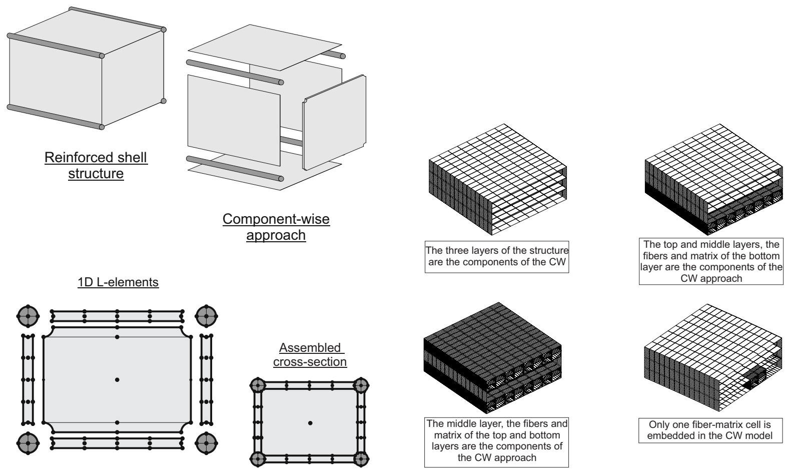
Where $\mathbf{K}^{\tau sij}$ is the stiffness matrix written in the form of the fundamental nuclei, **u** is the nodal displacement vector and $\delta$ is the virtual variation. The superscripts indicate the four indexes exploited to assemble the matrix. $i$ and $j$ are related to the shape functions along the beam axis, while $\tau$ and $s$ are related to the expansion functions over the cross-section. The fundamental nucleus is a $3 \times 3$ array that is formally independent of the order of the structural model. More details about CUF models can be found in authors' books [13, 14].

## 3. COMPONENT-WISE APPROACH AND DAMAGE MODELING

The component-wise (CW) approach has been introduced in the framework of the CUF. The CW approach allows each typical component of a structure to be modeled through the 1D CUF formulation. In an FE framework, this means that different components are modelled using the same 1D FE, i.e. the same stiffness matrix is used for each component. Figure 3 shows two examples of CW applications to aerospace and composite structures, respectively. The CW methodology enables an optimized finite element modeling by

- choosing the component that requires a more detailed modeling;
- setting the order of the structural model to be used.

The CW can be exploited for the analysis of composite structures and can be seen as a computationally cheap multiscale approach. In fact,




(a) A reinforced shell structure for aerospace applications

(b) A fiber reinforced composite structure

Figure 3. CW modeling of multi-component structures.

- Macroscale components, e.g. layers, and microscale components, e.g. fibers, can be simultaneously modeled through the same 1D formulation. Coupling techniques to deal with different scales are not required.
- Each component can be modeled with its material characteristics, in other words, no homogenization techniques are necessary.
- The adoption of the same type of 1D FEs allow highly accurate modelings to be used only where needed, e.g. in the proximity of failure zones, whereas lower fidelity modelings can be used elsewhere.
- The adoption of 1D FEs makes the computational costs of the CW approach 10-100 times lower than solid elements.

A basic damage modelling approach was adopted in this work. Figure 4 shows an example of

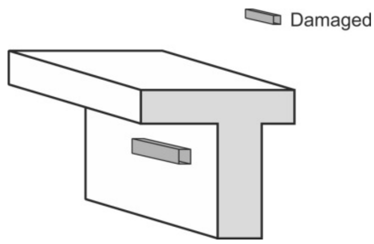


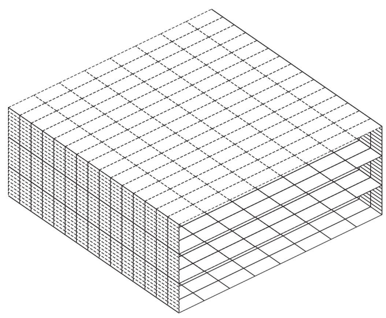

Figure 4. Local damage in a beam.

locally damaged structure. In the damaged zone, the Young and shear moduli were modified according to

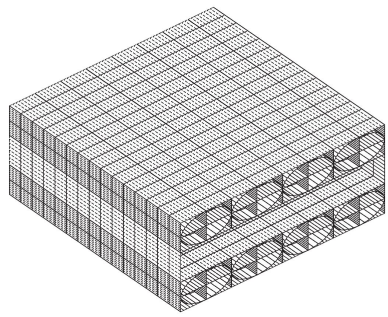
$$E_d = d \times E, \quad 0 < d < 1 \tag{6}$$

## 4. RESULTS

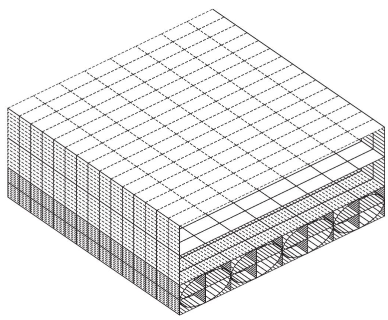
A simple cell of a laminated structure is considered as a numerical example. The cell length is $40$ mm, the height is $0.6$ mm and the width is $0.8$ mm. One end of the structure is clamped. A $0/90/0$ lamination was considered. Figure 5 shows the four modeling approaches adopted. Fibers were modeled with a circular cross-section, where the diameter is equal to $0.2$ mm. Up



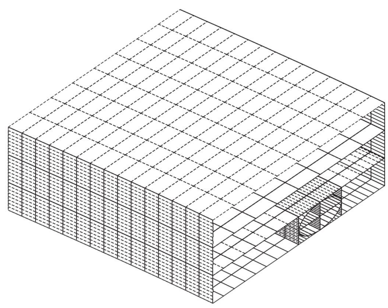
(a) Model 1: the three layers of the structure are the components of the CW approach



(b) Model 2: the top and middle layers and the fibers and matrices of the bottom layer are the components of the CW approach



(c) Model 3: the middle layer and the fibers and matrices of the top and bottom layers are the components of the CW approach

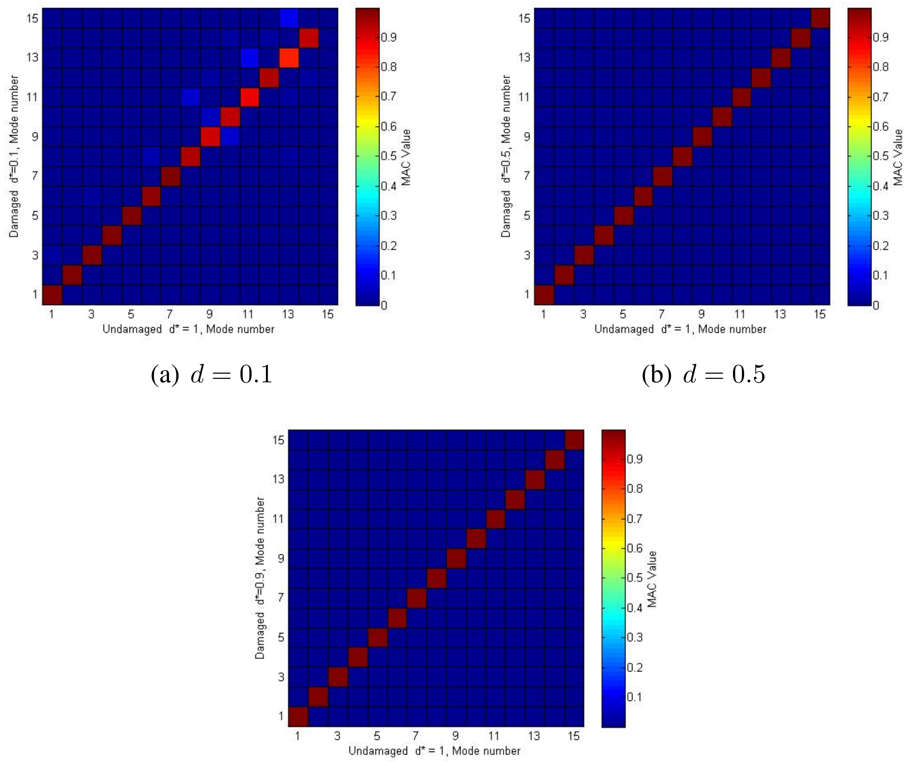


(d) Model 4: only one fiber-matrix cell is inserted in CW model

Figure 5. Various modeling approaches for the laminate.

to four fibers per layer were considered. Fibers are orthotropic, with $E_L$ = $202.038$ GPa, $E_T$ = $E_Z$ = $12.134$ GPa, $G_{LT}$ = $G_{LZ}$ = $8.358$ GPa, $G_{TZ}$ = $47.756$ GPa, $\nu_{LT}$ = $\nu_{LZ}$ = $0.2128$ und $\nu_{TZ}$ = $0.2704$. The matrix is made of an isotropic material, with $E$ = $3.252$ GPa and $\nu$ = $0.355$. Layer properties are orthotropic and are as the following: $E_L$ = $159.380$ GPa, $E_T$ = $E_Z$ = $14.311$ GPa, $G_{LT}$ = $G_{LZ}$ = $3.711$ GPa, $G_{TZ}$ = $5.209$ GPa, $\nu_{LT}$ = $\nu_{LZ}$ = $0.2433$ und $\nu_{TZ}$ = $0.2886$. The density, $\rho$, is $1300$ $kg/m^3$, $1500$ $kg/m^3$ and $1555$ $kg/m^3$ are adopted for the matrix, layer and fiber, respectively. Mixture rules were used. The bottom layer was damaged along the first $10\%$ of the span.

Table 1 shows the first five natural frequencies of the structure for various damage levels. CW




(a) $d = 0.1$

(b) $d = 0.5$

(c) $d = 0.9$

Figure 6. MAC matrices, Model 2.

models obtained through LEs were compared to solid FE models (Abaqus). Bending and torsional modes were considered. The total amount of degrees of freedom (DOFs) for each model is given in the last row. A good match between the two models was found. Figure 6 shows the MAC matrices relating the undamaged and damaged models. As it can be seen, such matrices are slightly significative only when severe damages are considered.

Table 1. First five natural frequencies (Hz).

| | $d = 0.1$ | | $d = 0.5$ | | $d = 0.9$ | | $d = 1.0$ | |
|---|---|---|---|---|---|---|---|---|
| | CW | Solid | CW | Solid | CW | Solid | CW | Solid |
| **Model 1** | | | | | | | | |
| Mode 1 | 420.49$^b$ | 412.45$^b$ | 568.05$^b$ | 562.66$^b$ | 607.25$^b$ | 601.84$^b$ | 613.01$^b$ | 607.56$^b$ |
| Mode 2 | 609.17$^b$ | 603.90$^b$ | 657.13$^b$ | 651.82$^b$ | 687.32$^b$ | 681.81$^b$ | 693.29$^b$ | 687.75$^b$ |
| Mode 3 | 3248.2$^b$ | 3211.4$^b$ | 3629.3$^b$ | 3597.1$^b$ | 3776.3$^b$ | 3744.1$^b$ | 3800.2$^b$ | 3767.8$^b$ |
| Mode 4 | 3985.1$^b$ | 3954.2$^b$ | 4132.6$^b$ | 4101.0$^b$ | 4246.4$^b$ | 4214.0$^b$ | 4271.0$^b$ | 4238.5$^b$ |
| Mode 5 | 8169.3$^t$ | 7903.9$^t$ | 8743.2$^t$ | 8503.6$^t$ | 8964.4$^t$ | 8724.0$^t$ | 9000.8$^t$ | 8759.7$^t$ |
| DOFs | 5859 | 75789 | 5859 | 75789 | 5859 | 75789 | 5859 | 75789 |
| **Model 2** | | | | | | | | |
| Mode 1 | 409.74$^b$ | 458.34$^b$ | 557.89$^b$ | 589.92$^b$ | 596.51$^b$ | 628.08$^b$ | 602.15$^b$ | 606.92$^b$ |
| Mode 2 | 599.46$^b$ | 624.55$^b$ | 648.81$^b$ | 671.44$^b$ | 678.95$^b$ | 701.59$^b$ | 684.86$^b$ | 687.77$^b$ |
| Mode 3 | 3144.1$^b$ | 3412.1$^b$ | 3512.2$^b$ | 3764.6$^b$ | 3653.3$^b$ | 3908.4$^b$ | 3676.1$^b$ | 3766.5$^b$ |
| Mode 4 | 3941.2$^b$ | 4086.9$^b$ | 4094.4$^b$ | 4235.6$^b$ | 4209.3$^b$ | 4350.8$^b$ | 4233.8$^b$ | 4254.5$^b$ |
| Mode 5 | 7599.4$^t$ | 9046.3$^t$ | 8084.4$^t$ | 9738.6$^t$ | 8290.0$^t$ | 9984.4$^t$ | 8324.7$^t$ | 10187$^t$ |
| DOFs | 16275 | 92346 | 16275 | 92346 | 16275 | 92346 | 16275 | 92346 |
| **Model 3** | | | | | | | | |
| Mode 1 | 417.06$^b$ | 420.13$^b$ | 563.22$^b$ | 566.17$^b$ | 601.91$^b$ | 604.98$^b$ | 607.58$^b$ | 608.44$^b$ |
| Mode 2 | 605.38$^b$ | 608.16$^b$ | 653.52$^b$ | 654.90$^b$ | 683.2$^b$ | 684.66$^b$ | 689.10$^b$ | 687.67$^b$ |
| Mode 3 | 3195.7$^b$ | 3238.8$^b$ | 3568.7$^b$ | 3618.1$^b$ | 3713.89$^b$ | 3764.0$^b$ | 3737.5$^b$ | 3773.7$^b$ |
| Mode 4 | 3968.3$^b$ | 3976.9$^b$ | 4115.6$^b$ | 4124.3$^b$ | 4227.91$^b$ | 4238.2$^b$ | 4252.1$^b$ | 4244.0$^b$ |
| Mode 5 | 7903.3$^t$ | 8612.5$^t$ | 8403.6$^t$ | 9303.5$^t$ | 8619.94$^t$ | 9536.0$^t$ | 8656.7$^t$ | 9497.5$^t$ |
| DOFs | 9765 | 268440 | 9765 | 268440 | 9765 | 268440 | 9765 | 268440 |
| **Model 4** | | | | | | | | |
| Mode 1 | 420.11$^b$ | 436.18$^b$ | 567.41$^b$ | 577.96$^b$ | 606.50$^b$ | 616.97$^b$ | 612.24$^b$ | 612.01$^b$ |
| Mode 2 | 608.85$^b$ | 616.76$^b$ | 656.61$^b$ | 664.37$^b$ | 686.70$^b$ | 694.57$^b$ | 692.66$^b$ | 692.12$^b$ |
| Mode 3 | 3244.21$^b$ | 3318.8$^b$ | 3624.74$^b$ | 3689.9$^b$ | 3771.46$^b$ | 3836.4$^b$ | 3795.3$^b$ | 3794.7$^b$ |
| Mode 4 | 3982.42$^b$ | 4030.7$^b$ | 4129.66$^b$ | 4178.7$^b$ | 4243.23$^b$ | 4293.1$^b$ | 4267.8$^b$ | 4266.6$^b$ |
| Mode 5 | 8171.29$^t$ | 8238.5$^t$ | 8745.96$^t$ | 8837.0$^t$ | 8966.89$^t$ | 9061.1$^t$ | 9003.2$^t$ | 9106.2$^t$ |
| DOFs | 11811 | 60282 | 11811 | 60282 | 11811 | 60282 | 11811 | 60282 |

(*): $b$, $t$ refer to bending and torsional mode, respectively.

## 5. CONCLUSIONS

This paper has presented free vibration analyses of damaged composite structures. A simple cell model was considered in which fibers were embedded. Component-Wise (CW) models based on 1D advanced finite elements were used. The 1D elements were obtained using Lagrange expansions of the cross-section displacement field. 1D models were built through the Carrera Unified Formulation (CUF). The CUF has hierarchical capabilities that allow us to deal with any order models with no need for ad hoc formulations. The use of CW features leads to models that provide high-fidelity geometrical and material descriptions of the structure. No reference axes or surfaces are, in fact, needed to define an LE model. Furthermore, the use of homogenised material characteristics can be avoided. The damage has been introduced using reduced stiffness areas. Local damage was considered. Various modeling approaches have been considered in which multiscale components have been modeled in different ways. Comparisons with 3D finite elements have been carried out. The results suggest that

- A good match was found between the 1D models and the 3D ones.
- The number of degrees of freedom of 1D models is at least ten times lower than 3D models.
- The CW was proved to be reliable in dealing with multiscale components and various modeling strategies can be considered.

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