

A FINITE ELEMENT WITH CONTINUOUS TRANSVERSE ELECTRIC DISPLACEMENT FOR STATIC AND FREE-VIBRATION ANALYSIS OF PIEZOELECTRIC SHELLS

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Abstract

The present work deals with the static and free-vibration analysis of multilayered plates and shells embedding piezoelectric layers as actuators and sensors, see Fig. 1. Finite elements with layer-wise capabilities are employed to ensure an accurate description of the mechanical and electric fields in the layers. It is essential to take into account the discontinuity of the mechanical and electrical properties at the layer interfaces. For these reasons, the use of classical plate theories based on Kirchhoff and Reissner-Mindlin hypotheses can lead to inaccurate results. Even if analytical models are available, the solution of practical problems often demand the use of computational methods such as the finite element method. To this aim, the refined shell elements, recently formulated by the authors on the basis of Carrera Unified Formulation (CUF), have been extended to the electro-mechanical problem. The governing equations are derived using the Reissner's Mixed Variational Theorem (RMVT) extended to the electro-mechanical case. The original RMVT has been modified to model a priori interlaminar continuous D_3 . Continuity of mechanical variables, such as transverse shear and normal stress components, has been discarded to provide a simple "electrical" modified RMVT, here called (*RMVT* - D_3).

$$\int_V \left(\delta \epsilon_{pG}^T \sigma_{pC} + \delta \epsilon_{nG}^T \sigma_{nC} - \delta \mathcal{E}_{pG}^T \mathcal{D}_{pC} - \delta \mathcal{E}_{nG}^T \mathcal{D}_{nM} - \delta \mathcal{D}_{nM}^T (\mathcal{E}_{nG} - \mathcal{E}_{nC}) \right) dV = \delta L_e - \delta L_{in}$$

This model has already shown good results in the electro-mechanical analysis of multilayered plates and shells with analytical code, and only plate with FEM code [1], [2]. The Mixed Interpolated Tensorial Components (MITC) method is employed to contrast the membrane-shear locking phenomenon that usually affects shell finite elements. This formulation has already shown all its potentiality as a base for finite elements in the mechanical analysis of multilayered shells [3]. Moreover, plate finite elements based on CUF for the analysis of electro-mechanical problem have been already presented

in [4]. One of the most interesting features of the unified formulation consists in the possibility to keep the order of the expansion of the state variables along the thickness of the plate as a parameter of the model. The electrical potential assumption for the layered actuators and sensors has been limited to a layer-wise (LW) description, in the same way the displacements assumptions on the composite layers are described by a LW model. Some results from the static and dynamic analysis of plates and shells under electro-mechanical loads will be provided, in order to show the efficiency of models presented.

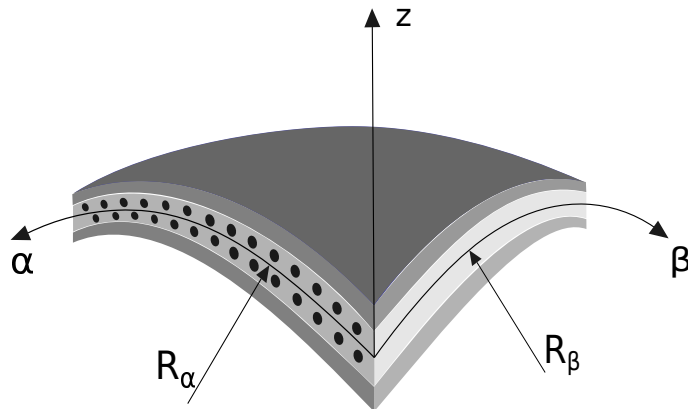


Figure 1: Double curvature shell with piezoelectric layers.

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