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Vibration analysis of metallic and composite damaged structures by Component-Wise models

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Abstract

This work proposes an innovative approach that is based on higher-order beam models for the damage analysis of metallic and composite structures. The present 1D formulation stems from the Carrera Unified Formulation (CUF) and it leads to a Component-Wise (CW) modelling.

According to CUF, the accuracy of the analysis is a parameter of the formulation. The displacement field is, in fact, expressed as an arbitrary expansion of the generalized unknowns via user-defined cross-sectional functions, F_τ. Formally,

$$\mathbf{u}(x,y,z) = F_\tau(x,z)\mathbf{u}_\tau(y), \qquad \tau = 1,2,....,M \tag{1}$$

where $\mathbf{u}_\tau$ is the vector of the *generalized displacements* and M stands for the number of terms used in the expansion. Taylor-like polynomials, i.e. power series of the coordinates x and z of generic order N, can be used as basis functions (F_τ) to enrich the beam kinematics. Taylor Expansion (TE) models are not described in this paper but they can be found in [1]. Here, the attention is focused on CW CUF models, which make use of Lagrange polynomials as F_τ expanding functions. Lagrange polynomials are reported in many reference books, see for example [2]. In this paper, nine-point cubic (L9) Lagrange polynomials are employed. Nevertheless, thanks to the hierarchical capability of CUF, the order and the number of the Lagrange elements used to discretize the beam cross-section can be arbitrarily varied without changing the formal expression of the problem. In the simple case of single-L9 beam model, the displacement field is

$$\begin{aligned}
u_x &= F_1\, u_{x_1} + F_2\, u_{x_2} + F_3\, u_{x_3} + F_4\, u_{x_4} + F_5\, u_{x_5} + F_6\, u_{x_6} + F_7\, u_{x_7} + F_8\, u_{x_8} + F_9\, u_{x_9}\\
u_y &= F_1\, u_{y_1} + F_2\, u_{y_2} + F_3\, u_{y_3} + F_4\, u_{y_4} + F_5\, u_{y_5} + F_6\, u_{y_6} + F_7\, u_{y_7} + F_8\, u_{y_8} + F_9\, u_{y_9}\\
u_z &= F_1\, u_{z_1} + F_2\, u_{z_2} + F_3\, u_{z_3} + F_4\, u_{z_4} + F_5\, u_{z_5} + F_6\, u_{z_6} + F_7\, u_{z_7} + F_8\, u_{z_8} + F_9\, u_{z_9}
\end{aligned} \tag{2}$$

where $u_{x_1}, ..., u_{z_9}$ are the displacement variables of the problem and represent the translational displacement components of each of the nine points of the L9 element; $F_1, ..., F_9$ are the Lagrange polynomials. Further details about CUF models based on Lagrange polynomials expansions can be found in [3].

In this work, a finite element approximation is adopted; the generalized displacements are thus expressed as a linear combinations of the nodal unknowns, $\mathbf{q}_{\tau i}$, through classical shape functions N_i. The governing equations are derived by means of the principle of virtual displacements. A compact form of the virtual variation of the strain energy can be obtained as shown in [1].

$$\delta L_{\text{int}} = \delta \mathbf{q}_{\tau i}^T \mathbf{K}^{ij\tau s} \mathbf{q}_{sj} \tag{3}$$

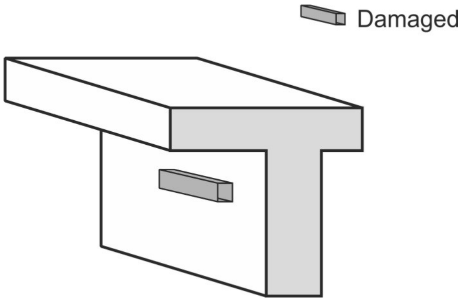

Figure 1: Locally damaged structure.

where $\mathbf{K}^{ij\tau s}$ is the stiffness matrix in the form of the fundamental nucleus. Superscripts indicate the four indexes exploited to assemble the matrix: i and j are related to the shape functions, τ and s are related to the theory expansion functions. The fundamental nucleus is a 3×3 array whose components can be found in [1, 3]. Matrix $\mathbf{K}^{ij\tau s}$ has to be expanded versus the four indexes to obtain any desired class of refined beam finite elements. Similarly, the fundamental nucleus of the mass matrix, $\mathbf{M}^{ij\tau s}$, can be easily obtained from the virtual variation of the work of inertial loadings, see [4].

A basic damage modelling approach is adopted in this work. Figure 1 shows an example of locally damaged structure. In the damaged zone, the material characteristics were modified according to the following formula:

$$E_d = d \times E \text{ with } 0 \leq d \leq 1 \tag{4}$$

i.e.;

$$E_0 = E; E_{0.9} = 0.9 \times E; \ldots; E_{0.1} = 0.1 \times E \tag{5}$$

A cantilever I shaped cross-section beam is discussed as a numerical example and it is shown in Figure 2. The main dimensions of the structure were: height, $h = 0.1$ m; width, $w = 0.1$ m; thickness of the flanges and the web, $t = 2$ mm; length, $L = 1$ m. The whole beam was made of an aluminium alloy ($E = 75$ GPa, $\nu = 0.33$, $\rho = 2700$ Kg m^{-3}). Damage was introduced in the whole top flange. Table 1 shows the first five natural frequencies of the structure subjected to different damage intensities. A CUF CW model built with 8L9 elements on the cross-section is compared to classical beam theories (Euler-Bernoulli, EBBM, and Timoshenko beam models, TBM) and to a 2D finite element plate model obtained with the commercial code Abaqus. Figure 3 shows the mode shapes for the un-damaged structure by the proposed CW CUF model. Also,

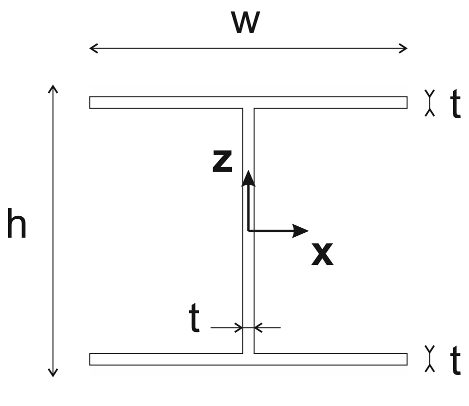

Figure 2: Cross-section of the I shaped beam.

Table 1: First natural frequencies (Hz) for different models and damage intensities, I-section beam.

Models	EBBM (193)*			TBM (305)			8L9 CUF (4743)			2D Abaqus (27972)		
$(1-d)$	0	0.5	0.9	0	0.5	0.9	0	0.5	0.9	0	0.5	0.9
f_1	69.9	60.5	51.8	69.7	60.3	51.7	69.7	51.7	26.4	69.2	51.1	26.2
f_2	127.0	109.2	84.1	125.6	108.1	83.5	73.8	71.6	70.9	72.8	70.7	70.0
f_3	434.7	376.4	322.4	424.7	368.6	346.5	122.7	106.4	82.4	119.8	103.9	80.3
f_4	776.3	666.2	510.6	723.9	626.0	488.5	239.1	234.2	136.9	232.3	226.3	134.4
f_5	1202.2	1202.3	891.7	1142.0	993.8	855.8	252.8	243.3	199.7	246.3	236.0	183.4

*The number of degrees of freedom are given in brackets

the same figure graphically shows the effects of damages in the top flange on the first five natural frequencies. The analysis clearly demonstrates that refined beam models are mandatory to detect the damage effects. Moreover, it is clear that the results from 1D CUF models perfectly match those from a 2D FEM model with very low computational costs. The enhanced capabilities of refined CW models in dealing with damaged complex structures [5] and composite materials [6] will be further discussed during the 10th International Symposium on Vibrations of Continuous Systems.

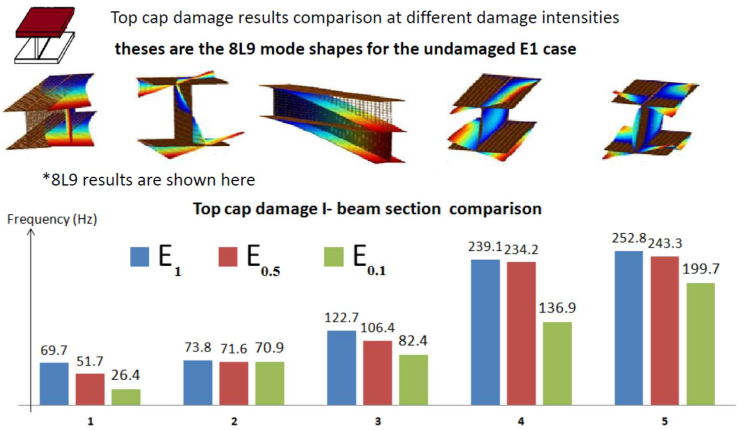

Figure 3: Mode shapes and damage effects on the natural frequencies by the 8L9 CUF model

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