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# Dynamic damage analysis of composites via a component-wise 1D approach

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*Abstract:* This paper proposes an innovative approach that is based on 1D (beam) advanced models for the damage analysis of composite structures. The present 1D formulation stems from the Carrera Unified Formulation (CUF) and it leads to a Component-Wise (CW) modelling. By means of the CUF, any-order 2D and 1D structural models can be developed in a unified and hierarchically manner and they provide extremely accurate results with very low computational costs. The computational cost reduction in terms of total amount of DOFs ranges from 10 to 100 times less then shell and solid models, respectively. The 1D CUF formulation, which is based on the use of Lagrange polynomials to describe the cross-section displacement field of the structure, is exploited in this paper. Such 1D models lead to the CW since each component of a complex structure can be modelled through a refined 1D model based on Lagrange expansions. The adoption of only 1D models to model complex structures improves the multi-dimension coupling capabilities and reduces the computational costs to a great extent. The CW can lead to a multi-scale approach for composites since each typical component of a composite structure can be modelled through the 1D CUF models and, moreover, different scale components can coexist in the same model with no need of further modelling tools. A detailed physical description of a real structure can be obtained since each component can be modelled with its own material characteristics, that is, no homogenization techniques are required. Furthermore, Although 1D models are exploited, the problem unknown variables can be placed on the physical surfaces of the real 3D model, that is, no artificial surfaces or lines have to be defined to build the structural model. In this paper, damaged composite structures are analysed by means of the CW approach. Static and dynamic responses are carried out and comparisons against classical approaches are provided to show the enhanced capabilities of the present approach in obtaining 3D-like accuracy with very low computational costs.

*Keywords:* Carrera unified formulation, beam, finite element, advanced models, damage analysis, composites.

## 1 Introduction

Computational models for the analysis of damaged composite structures should be able to provide very accurate displacement, strain and stress fields. The proper modelling of multiscale components – layers, fibres and matrix – enhances the accuracy of computational models to a great extent. Currently, most of the techniques that have been developed for these tasks are based on very cumbersome numerical models, such as the 3D solid finite elements. The accurate structural analysis of complex structures is almost impossible due to the enormous number of degrees of freedom that is required.

Beam models are widely adopted for many structural engineering applications because they are computationally cheaper and less cumbersome than plate, shell and solid finite elements. The classical beam theories are those by Euler-Bernoulli and Timoshenko [1-3]. None of these theories can detect non-classical effects such as warping, out- and in-plane deformations, torsion-bending coupling or localized boundary conditions (geometrical or mechanical). These effects are important when, for instance, small slenderness ratios, thin walls, the anisotropy of the materials and damages are considered.

Many methods have been proposed over the last decades to enhance classical theories and to extend the application of 1D models to any geometry or boundary condition. Among the others, some of the

most recent developments in 1D models have been obtained by means of the following approaches: the introduction of shear correction factors [1]; the use of warping functions based on the Saint-Venant solution [4, 5], asymptotic approaches [6]; generalized beam theories (GBT) [7]; higher-order beam models [8, 9].
This work exploits the Carrera Unified Formulation (CUF) for higher-order 1D models [2, 3, 10]. CUF was initially developed for plates and shells [10, 11], more recently for beams [2, 12, 13]. In CUF models, the displacement field above the cross-section is modelled through expansion functions whose order is a free parameter of the analysis. This means that any-order structural models can be implemented with no need of formal changes in the problem equations and matrices. CUF can therefore deal with arbitrary geometries, boundary conditions and material characteristics with no need of ad hoc formulations.
CUF 1D models have recently been applied to static [12-14] and free-vibration [15] analyses. The most recent extension of CUF models is the so-called Component-Wise approach (CW) that is based on the use of Lagrange polynomials for the cross-section displacement field description [16,17]. Multicomponent structures (e.g. aircraft wings or fibre reinforced composites) are modelled through a unique 1D formulation [18-21]. 1D CW leads to solid-like accuracies with far less computational costs than shell and solid FEs.
In this work, CUF 1D models are exploited to analyse damaged structures through the free vibration analysis. This paper outlines guidelines for the damage analysis of structures and it highlights the enhanced capabilities of the present formulation which can be easily adopted to detect the structural behaviour of damaged structures.

## 2 Carrera unified formulation

The beam cross-section displacement field is described by an expansion of generic functions ($F_\tau$)

$$\boldsymbol{u} = F_\tau \boldsymbol{u}_\tau, \qquad \tau = 1,2,\ldots.,M \tag{1}$$

Where $F_\tau$ are functions of the cross-section coordinates (x, z), $\boldsymbol{u}_\tau$ is the displacement vector and $M$ stands for the number of terms of the expansion. Taylor-like polynomial expansions of the displacement field above the cross-section of the structure can be used. The order of the expansion is arbitrary and is set as an input of the analysis. For example, the second-order model ($N = 2$) is based on the following displacement field:

$$\begin{aligned} u_x &= u_{x_1} + x\ u_{x_2} + z\ u_{x_3} + x^2\ u_{x_4} + xz\ u_{x_5} + z^2\ u_{x_6} \\ u_y &= u_{y_1} + x\ u_{y_2} + z\ u_{y_3} + x^2\ u_{y_4} + xz\ u_{y_5} + z^2\ u_{y_6} \\ u_z &= u_{z_1} + x\ u_{z_2} + z\ u_{z_3} + x^2\ u_{z_4} + xz\ u_{z_5} + z^2\ u_{z_6} \end{aligned} \tag{2}$$

The 1D model described by Eq. (2) has 18 generalized displacement variables: three constant, six linear, and nine parabolic terms. Lagrange polynomials can be used as well. For the sake of brevity, the Lagrange polynomial expressions are not reported here, they can be found in [3].
The governing equations were derived by means of the Principle of Virtual Displacements (PVD). A compact form of the virtual variation of the strain energy can be obtained as shown in [2, 3],

$$\delta L_{int} = \delta \boldsymbol{q}_{\tau i}^T \boldsymbol{K}^{ij\tau s} \boldsymbol{q}_{sj} \tag{3}$$

Where $\boldsymbol{K}^{ij\tau s}$ is the stiffness matrix written in the form of the fundamental nuclei. Superscripts indicate the four indexes exploited to assemble the matrix: $i$ and $j$ are related to the shape functions, $\tau$ and $s$ are related to the expansion functions. The fundamental nucleus is a $3 \times 3$ array which is formally independent of the order of the beam model. In a compact notation, the stiffness matrix for a given material property set can be written as:

$$\begin{aligned}\mathbf{K}^{ij\tau s} = \quad & I_l^{ij} \lhd (\mathbf{D}_{np}^T F_\tau \mathbf{I})[\,\tilde{\mathbf{C}}_{np}\,(\mathbf{D}_p F_s \mathbf{I}) + \tilde{\mathbf{C}}_{nn}\,(\mathbf{D}_{np} F_s \mathbf{I})] + \\ & (\mathbf{D}_p^T F_\tau \mathbf{I})[\,\tilde{\mathbf{C}}_{pp}\,(\mathbf{D}_p F_s \mathbf{I}) + \tilde{\mathbf{C}}_{pn}\,(\mathbf{D}_{np} F_s \mathbf{I})] \rhd_\Omega + \\ & I_l^{ij,y} \lhd [\,(\mathbf{D}_{np}^T F_\tau \mathbf{I})\tilde{\mathbf{C}}_{nn} + (\mathbf{D}_p^T F_\tau \mathbf{I})\tilde{\mathbf{C}}_{pn}]F_s \rhd_\Omega \mathbf{I}_{\Omega y} + \\ & I_l^{i,yj}\,\mathbf{I}_{\Omega y} \lhd F_\tau\,[\,\tilde{\mathbf{C}}_{np}\,(\mathbf{D}_p F_s \mathbf{I}) + \tilde{\mathbf{C}}_{nn}\,(\mathbf{D}_{np} F_s \mathbf{I})] \rhd_\Omega + \\ & I_l^{i,yj,y}\,\mathbf{I}_{\Omega y} \lhd F_\tau \tilde{\mathbf{C}}_{nn} F_s \rhd_\Omega \mathbf{I}_{\Omega y}\end{aligned} \qquad (4)$$

where

$$\mathbf{I}_{\Omega y} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \lhd \ldots \rhd_\Omega = \int_\Omega \ldots\, d\Omega \qquad (5)$$

$$(I_l^{ij},\, I_l^{ij,y},\, I_l^{i,yj},\, I_l^{i,yj,y}) = \int_l (N_i\, N_j, N_i\, N_{j,y}, N_{i,y}\, N_j, N_{i,y}\, N_{j,y})\; dy \qquad (6)$$

$\tilde{\mathbf{C}}$ is the material coefficient matrix and $\mathbf{D}$ is the differential operator matrix. It should be underlined that the formal expression of $\boldsymbol{K}^{ij\tau s}$

1. Does not depend on the expansion order.
2. Does not depend on the choice of the $F_\tau$ expansion polynomials.

These are the key-points of CUF which permits, with only nine FORTRAN statements, to implement any-order of multiple class theories.

The virtual variation of the work of the inertial loadings is

$$\delta L_{ine} = \int_V \rho \ddot{\boldsymbol{u}} \delta \boldsymbol{u}^T dV \qquad (7)$$

where $\rho$ stands for the density of the material, and $\ddot{\boldsymbol{u}}$ is the acceleration vector. Equation (7) can be rewritten in a compact manner as follows:

$$\delta L_{ine} = \delta \boldsymbol{q}_{\tau i}^T \boldsymbol{M}^{ij\tau s} \ddot{\boldsymbol{q}}_{sj} \qquad (8)$$

Where $\boldsymbol{M}^{ij\tau s}$ is the mass matrix in the form of the fundamental nucleus whose components can be found in [2, 3].

## 3 Damage modelling

A basic damage modelling approach was adopted in this work. Fig. 1 shows an example of locally damaged structure. In the damaged zone, the material characteristics were modified according to the following formulas:

$$E_d = d \times E \; with \; 0 \le d \le 1,\; E_0 = E;\; E_{0.9} = 0.9 \times E\,; \ldots.;\; D_{0.1} = 0.1 \times E \qquad (9)$$

Damages were introduced in different portions of the structure as will be shown in the result section of this paper.

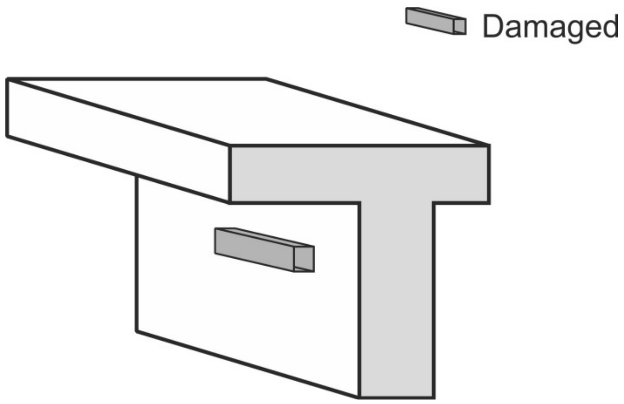


**Figure 1**: A locally damaged structure

## 4 Results and discussion

### 4.1 Isotropic beam

A square cantilever beam was considered, aluminium was used (E = 75 GPa, ν = 0.33, ρ = 2700 Kg/m$^3$). The length of the beam (L) was set equal to 2 m and the height (h) equal to 0.2 m. Damage was introduced at the root of the beam, see Fig. 2.

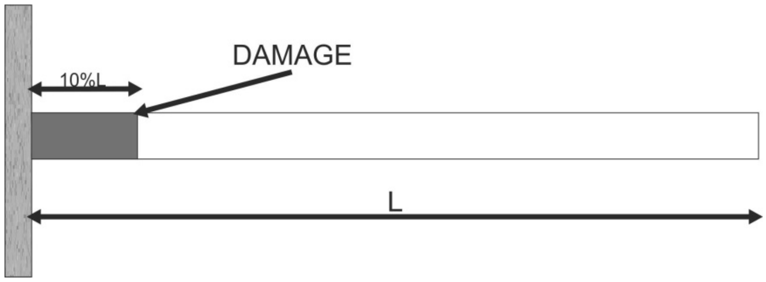


**Figure 2**: Damage at the root of a cantilever

Table 1 shows the first bending and torsional frequencies of the beam via different 1D models. A solid FE model was built in Abaqus for comparison purposes. The total amount of degrees of freedom (DOFs) of each model is shown to compare computational costs.

**Table 1:** First bending and torsional frequencies (Hz) of the damaged isotropic beam ($E_{0.1}$), 1D CUF models vs. Abaqus Solid model

| Model | EBBT | TBT | N = 1 | N = 2 | N = 3 | N = 4 | Solid |
|---|---|---|---|---|---|---|---|
| DOFs | 183 | 305 | 549 | 1098 | 1647 | 2196 | 139587 |
| Bending | 20.8 | 20.7 | 20.7 | 21.3 | 21.3 | 21.2 | 21.1 |
| Torsional | - | - | 233.9 | 233.9 | 233.9 | 218.3 | 216.0 |


Table 2 shows the effect of the damage intensity on the first bending and torsional frequencies. A fourth-order 1D CUF model was compared with a solid model.

**Table 2:** First bending and torsional frequencies (Hz) of the damaged isotropic beam, CUF N = 4 model (2196 DOFs) and Abaqus Solid model (139587 DOFs)

| **Model** | $E_1$ | $E_{0.9}$ | $E_{0.8}$ | $E_{0.7}$ | $E_{0.6}$ | $E_{0.5}$ | $E_{0.4}$ | $E_{0.3}$ | $E_{0.2}$ | $E_{0.1}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **CUF** | $42.5^b$ | $41.7^b$ | $40.8^b$ | $39.7^b$ | $38.4^b$ | $36.8^b$ | $34.7^b$ | $31.9^b$ | $27.8^b$ | $21.2^b$ |
| **Abaqus** | $42.4^b$ | $41.6^b$ | $40.7^b$ | $39.6^b$ | $38.3^b$ | $36.7^b$ | $34.5^b$ | $31.7^b$ | $27.7^b$ | $21.1^b$ |
| **CUF** | $372.1^t$ | $368.1^t$ | $363.3^t$ | $357.4^t$ | $349.8^t$ | $339.9^t$ | $326.3^t$ | $306.6^t$ | $275.4^t$ | $218.3^t$ |
| **Abaqus** | $369.7^t$ | $365.8^t$ | $360.9^t$ | $355.0^t$ | $347.4^t$ | $337.5^t$ | $323.8^t$ | $304.1^t$ | $272.9^t$ | $216.0^t$ |

The following comments hold:

- Refined beam models are mandatory to detect the damage effects.
- The detection of torsional models can be carried out by means of refined models only. As it is well knows, classical beam models cannot deal with torsion.
- Results from 1D CIF models perfectly match those from solid models.
- The computational costs of 1D CUF models are extremely lower than solid ones.

### 4.2 Orthotropic beam

A rectangular cross-section beam was considered. The length of the beam (L) equal to 2 m, the width (b) equal to 0.1 m and the thickness (h) equal to 1 mm. An orthotropic material was considered, with $E_L$ = 40 GPa, $E_T = E_Z$ = 4 GPa, G = 4 GPa, ν = 0.25 and ρ = 1600 Kg/m$^3$. Figure 3 shows the damage distribution along the thickness, the entire beam span was damaged.

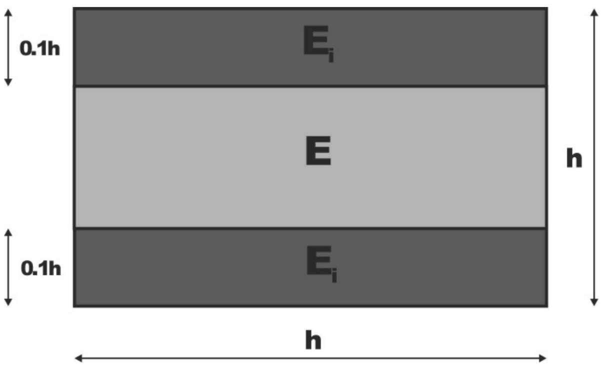


**Figure 3**: Damage distribution above the cross-section of the orthotropic beam

Table 2 presents the first bending and torsional frequencies for different damage levels, results were obtained through a fourth-order model (N = 4). Figure 4 shows the types of modal shapes for different beam models. It can be stated that

- Damage influences the natural frequencies of the beam.
- Refined models are needed to detect torsional modes.

**Table 2:** First bending and torsional frequencies (Hz) of the damaged orthotropic beam, N = 4 model

| $E_1$ | $E_{0.9}$ | $E_{0.8}$ | $E_{0.7}$ | $E_{0.6}$ | $E_{0.5}$ | $E_{0.4}$ | $E_{0.3}$ | $E_{0.2}$ | $E_{0.1}$ |
|---|---|---|---|---|---|---|---|---|---|
| $0.60^b$ | $0.59^b$ | $0.58^b$ | $0.57^b$ | $0.56^b$ | $0.54^b$ | $0.51^b$ | $0.48^b$ | $0.43^b$ | $0.35^b$ |
| $6.21^t$ | $6.15^t$ | $6.09^t$ | $6.00^t$ | $5.88^t$ | $5.75^t$ | $5.57^t$ | $5.33^t$ | $5.02^t$ | $4.62^t$ |

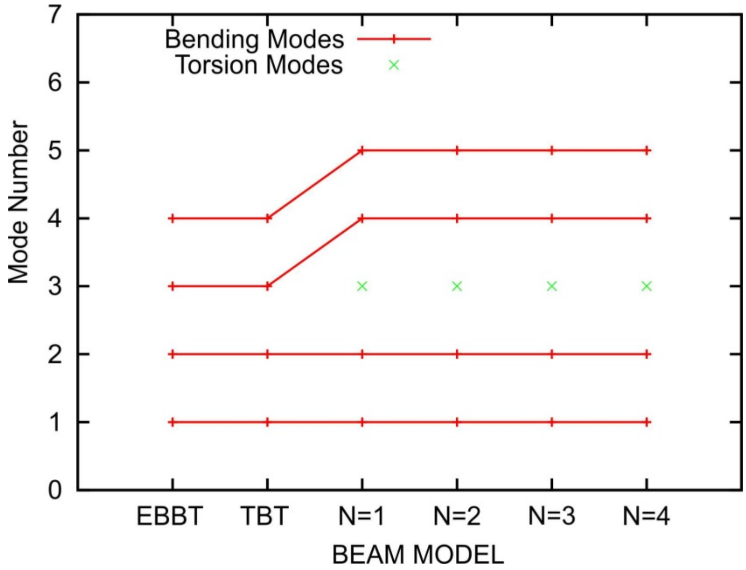


**Figure 4**: Modal shape types for different beam models

## 5 Conclusions

This paper has presented a preliminary study to analyse damaged structures via refined 1D models. These models were built through the Carrera Unified Formulation (CUF). The CUF has hierarchical capabilities that allow us to deal with any-order models with no need of ad hoc formulations. Different structures have been analysed and the results suggest the following:

- 1D CUF structural models are powerful and computationally cheap tools to analyze structures for different applications.
- CUF models provide 3D-like accuracies with low computational costs.
- 1D CUF models can deal with damaged structures, in particular, they can detect torsional modes.

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