COMPONENT-WISE APPROACH FOR EFFICIENT AND ACCURATE ANALYSIS OF STRUCTURES OF LAUNCHERS

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ABSTRACT

Reinforced structures are usually analyzed using one-dimensional (rod/beam) and two-dimensional (plate/shell) structural elements. The present work uses a one-dimensional (1D) structural model to analyze thin-walled structures with longitudinal and transversal stiffeners. The 1D model introduced in the present work is based on the Carrera Unified Formulation (CUF). The CUF allows any structural model to be derived in a compact and unified form. In the present work is derived a refined one-dimensional model which has only displacements as unknowns. Many assessment are proposed to show the performances of the model. Simple and reinforced panels are considered in the model assessment and reinforced cylindrical structures are then investigated. Finally a simplified launcher structural model, inspired to the Ariane 5, is analyzed. The results are compared with those from solid and two-dimensional/one-dimensional models.

Key words: CUF; Component-wise; One-dimensional models; Launcher design.

1. INTRODUCTION

The analysis of slender thin-walled structures, as a launchers, is a challenging problem, in fact, when these structures are discretized using solid FE models, the constraints on the element aspect-ratio force the model to have a large number of degrees of freedoms (DOFs) and therefore to be very computationally expensive.

The use of reduced model allows the computational cost of the analysis to be reduced but usually afflicts the accuracy of the results. Classical beam models, as Euler-Bernoulli [1] or Timoshenko [2] beam models, are suitable for the analysis of slender bodies typical of the launcher structure, but the kinematic assumptions do not allow these models to be used in the thin-walled structure analysis.

Different solutions have been proposed to overcome the limitations of the classical models in order to extend the use of 1D models to any beam geometries and with any boundary conditions, an example can be found in the book by Novozhilov [3]. Typical approaches consist to introduce corrector factors, as in the books by Timoshenko and Goodier [4] and by Sokolnikoff [5], which are based on shear corrector factors. Instead El Fatmi [6] [7] introduced a warping function ϕ to improve the description of normal and shear stress of the beam. Another approach based on asymptotic solution was used by Berdichevsky at al. [8], where a characteristic parameter (e.g., the cross-section thickness for a beam) is exploited to build an asymptotic series. This work has been the origin of an alternative approach to constructing refined beam theories. It is also appropriate to recall the works by Schardt [9, 10] which developed a generalized beam theories (GBT) widely used by Silvestre [11] for the thin-walled structures analysis.

The 1D model introduced in the present work is based on the Carrera Unified Formulation (CUF)Carrera [12]. The CUF allows any structural model to be derived in a compact and unified form. In the present work is used a refined one-dimensional model which has only displacements as unknowns, this model ws introduced in [13]. If the unknowns of the problem are expressed in terms of displacements only, a complex structure can be easily analyzed connecting simpler one-dimensional structures, this approach is called Component-Wise (CW)[14, 15]. The CW approach used in this work is used to analyze thin-walled structures. Many assessment are proposed to show the performances of the model. Simple and reinforced panels are considered in the model assessment and reinforced cylindrical structures are then investigated. Finally a simplified launcher structural model, inspired to the Ariane 5, is analyzed. The results are compared with those from solid and two-dimensional/one-dimensional models. Static and dynamic analyses are performed. The use of a refined one-dimensional theory allows the model to have e reduced number of degrees of freedom, typical of the beam theories, but, at the same time, allows the model to detect a quasi-3D solution. Refined models overcome the assumption of rigid cross-section and therefore are able to detect shell-like deformations, typical of the thin-walled structures.

The results of the dynamic analyses show the quasi-3D capabilities of the present model and the CW approach has been proved to be a competitor of the solid FE models. The natural frequencies and the modes computed using the CW approach can be compared whit those from the solid model but the present formulation has only 5% of DOF than solid model. The results have been compared also with a reduced model built using two- and one-dimensional classical elements (1D/2D). Also in this case, the CW approach requires a much lower number of DOF with respect to 1D/2D model and ensure the same level of accuracy.

2. REFINED ONE-DIMENSIONAL MODELS

2.1. Preliminaries

The coordinate frame adopted in shown in figure 1, where y-axis is the beam axis. The beam boundaries over y are $0 \leq y \leq L$, where L is the beam length. The displacement vector is

$$\boldsymbol{u}(x,y,z) = \left\{ \begin{array}{ccc} u_x & u_y & u_z \end{array} \right\}^T \tag{1}$$

The superscript "T'' denotes transposition. Stress, σ, and strain, ϵ, components are grouped as follows:

$$\boldsymbol{\sigma} = \left\{ \begin{array}{cccccc} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \sigma_{xy} & \sigma_{xz} & \sigma_{yz} \end{array} \right\}^T \tag{2}$$

$$\boldsymbol{\epsilon} = \left\{ \begin{array}{cccccc} \epsilon_{xx} & \epsilon_{yy} & \epsilon_{zz} & \epsilon_{xy} & \epsilon_{xz} & \epsilon_{yz} \end{array} \right\}^T \tag{3}$$

Linear strain-displacement relations are used,

$$\boldsymbol{\epsilon} = D\boldsymbol{u}$$

where D is a differential operator, the explicit formulation of D can be found in the book by Carrera *et al.*[16]. The materials is considered elastic and isotropic, therefore the stress can be derived using the hook's law:

$$\sigma = C\epsilon$$

The explicit form of $\mathbf{C}$ in the case of isotropic materials, can be found in the books by Tsai [17] or Reddy [18].

2.2. Displacement formulation

In the framework of the Carrera Unified Formulation [19, 20, 21, 16], the displacement field is assumed as an expansion in terms of generic functions, F_τ:

$$\mathbf{u} = F_\tau(x,z)\mathbf{u}_\tau(y), \qquad \tau = 1,2,....,T \tag{4}$$

where F_τ are defined above the cross-section. $\mathbf{u}_\tau$ is the displacement vector and T stands for the number of terms of the expansion.. In this work the Lagrange expansion, LE, is used to derive refined beam models.

LE models exploit Lagrange polynomials to build 1D higher-order theories. In this paper two type of cross-section polynomial sets are adopted: nine-point elements, L9, and four-point elements, L4. The isoparametric formulation is exploited to deal with arbitrary shaped geometries. The L9 interpolation functions are given by [22]:

$$F_\tau = \frac{1}{4}(r^2 + r\, r_\tau)(s^2 + s\, s_\tau) \qquad \tau = 1,3,5,7$$

$$\begin{aligned} F_\tau = &\frac{1}{2}s_\tau^2(s^2 - s\, s_\tau)(1 - r^2) \\ &+ \frac{1}{2}r_\tau^2(r^2 - r\, r_\tau)(1 - s^2) \qquad \tau = 2,4,6,8 \end{aligned} \tag{5}$$

$$F_\tau = (1 - r^2)(1 - s^2) \qquad \tau = 9$$

Where r and s from -1 to $+1$. More details on refined beam models can be found in the book by Carerra *et al.*[23] The Finite Element Method is used to approximate the displacement over the beam axis. The displacement field can be written introducing the classical FEM one-dimensional shape functions:

$$\mathbf{u} = F_\tau(x,z)N_i(y)\mathbf{q}_{i\tau} \tag{6}$$

Where index i indicates the node of the element. In the present work only cubic shape functions are used therefore each element is considered to have four nodes.

2.3. Governing equations

The governing equation can be derived using the principle of virtual displacements in the dynamic formulation:

$$\delta L_{int} + \delta L_{ine} = \delta L_{ext} \tag{7}$$

Where L_{int} stands for the strain energy, L_{ine} stands fro the inertial energy and L_{ext} is the work of the external loadings. δ stands for the virtual variation. The internal work can be written as:

$$\delta L_{int} = \int_V \delta\boldsymbol{\epsilon}^T \boldsymbol{\sigma} dV \tag{8}$$

Introducing the formulation of the stress and strain presented in Eq.s 2 and 3, and using the displacement field introduced in Eq 6, the variation of the internal work becomes:

$$\begin{aligned} \delta L_{int} = &\delta\mathbf{q}_{sj}^T \int_V F_s(x,z)N_j(y) \;\; \boldsymbol{D}^T\boldsymbol{C}\boldsymbol{D}N_i(y)F_\tau(x,z)dV\mathbf{q}_{\tau i} = \\ = &\delta\mathbf{q}_{sj}^T\boldsymbol{k}^{ij\tau s}\mathbf{q}_{\tau i} \end{aligned} \tag{9}$$

$\boldsymbol{k}^{ij\tau s}$ is the stiffness matrix in form of "fundamental nucleus", a 3×3 matrix with an fixed form. The stiffness matrix of the element can be evaluated varying the indexes

i, j, τ and s. The explicit formulation of the fundamental nucleus can be found in [16].

The variation of the inertial work, δL_{ine}, can be written as:

$$\delta L_{ine} = \int_V \delta \boldsymbol{u}^T \rho \ddot{\boldsymbol{u}} dV \tag{10}$$

Considering the displacement field introduced before this equation becomes:

$$\begin{aligned} \delta L_{ine} =& \delta \mathbf{q}_{sj}^T \int_V F_s(x,z) N_j(y) \rho N_i(y) F_\tau(x,z) dV \mathbf{q}_{\tau i} = \\ =& \delta \mathbf{q}_{sj}^T \boldsymbol{m}^{ij\tau s} \mathbf{q}_{\tau i} \end{aligned} \tag{11}$$

$\boldsymbol{m}^{ij\tau s}$ is the mass matrix in form of “fundamental nucleus”.

The variation of the external work, δL_{ext}, can be derived in the case of a concentrated load as:

$$\delta L_{ext} = \delta \boldsymbol{u}^T \boldsymbol{P} = \delta \mathbf{q}_{sj}^T F_s|_P N_j|_P \boldsymbol{P} = \delta \mathbf{q}_{sj}^T \boldsymbol{p}^{sj} \tag{12}$$

where $F_s|_P$ and $N_j|_P$ are the values of the shape functions evaluated in the point there the load $\boldsymbol{P}$ is applied. The terms $\boldsymbol{p}^{sj}$ is the fundamental nucleus of the loading vector.

3. RESULTS

The model presented in this work has been used to perform of typical aerospace structures. The capabilities of the model allow to investigate small components as well as complex structures. The results aim to highlight the component-wise capability therefore three different structure are investigated: a reinforced panel under a static load, a modal analysis of a thin-walled cylindrical structure and the modal analysis of a launcher structure including stringers and ribs. All the structures are considered built by an isotropic aluminium alloy with a Young’s modulus, E, equal to 73 $[GPa]$, a density, ρ, of $2700[Kg/m^3]$ and a Poisson’s ratio, ν, equal to 0.3.

3.1. Static analysis of a reinforced panel

A reinforced panel under static load has been investigated. Fig. 1 shows the geometry of the panel and the reference system. The structure is composed by a thin panel reinforced with three longitudinal stiffeners with rectangular section. Four different models have been considered, both L4 and L9 elements are used to build a coarse and a refined mesh model. The structure is loaded with a concentrated load, P, in the middle of the panel with a magnitude of $20000[N]$. The structure is considered clamped for $y = 0[m]$ and $y = 2[m]$. Tab. 1 shows the vertical displacement in different points computed using the present models. The results are compared with those from a commercial code computed using a solid model. The results show that a higher-order

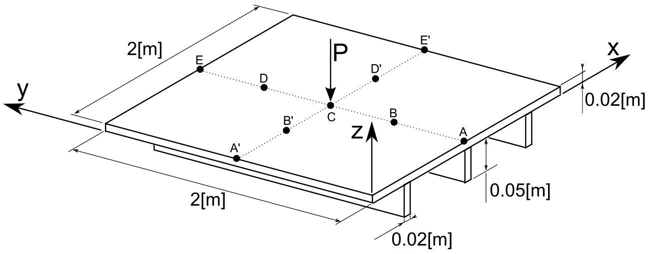


Figure 1: Reference system and geometry of the reinforced panel.

Model	DOF	C	B	A’	B’
FEM_{3D}	63531	-4.218	-2.025	+0.242	-2.034
LE 4	2325	-2.192	-0.991	-1.499	-1.528
LE 4+	13050	-2.913	-1.330	-0.947	-2.101
LE 9	5625	-4.075	-1.955	+0.138	-2.156
LE 9+	14400	-4.154	-1.991	+0.254	-2.068

(+) : *refined Mesh*

Table 1: Vertical displacement, $\times 10^{-3}$ [m]

model is mandatory to compute accurately the vertical displacement, the L9 elements provide the better results but only the refined mesh, LE9+, provides results comparable with those from solid model. Otherwise, the number of degrees of freedom, also for the LE9+ model, is much more lower than in the solid model.

3.2. Dynamic analysis of a thin-walled cylinder

The modal analysis of a thin-walled cylinder has been performed in order to assess the present model in the dynamic analysis. Fig.2 shows the geometry and the reference system of the structure. The structure is considered clamped at both the ends. Two model are considered, in the first case the cross-section has been approximated using 12 L9 elements while in the second 16 L9 elements are used. The results have been compared with

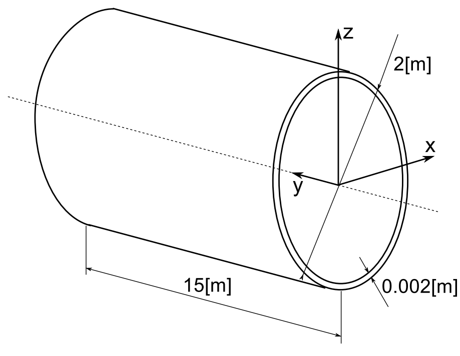


Figure 2: Reference system and geometry of the cylinder.

those from a shell model built using a commercial code. Tab. 2 shows the first flexural and torsional frequencies of the structure. As expected the present model provides accurate results. Tab. 3 show the shell-like frequency

Mode	FEM_{2D}	12L9	16L9
1^{st} $Flex$	46.83	46.48	46.73
1^{st} $Tors$	107.59	108.95	108.95

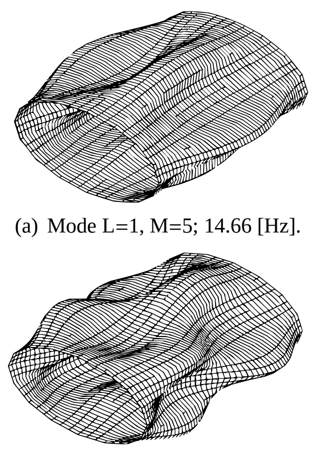
Table 2: First flexural and torsional frequencies , [Hz].

		FEM_{2D}	12 L9	16 L9
$L = 1$	$M = 2$	17.57	17.28	17.41
	$M = 3$	9.43	9.34	9.40
	$M = 4$	9.10	10.13	9.58
$L = 2$	$M = 2$	45.21	44.55	44.89
	$M = 3$	23.26	22.58	22.98
	$M = 4$	15.56	15.30	15.17
$L = 3$	$M = 2$	82.24	81.17	81.71
	$M = 3$	43.49	42.12	42.89
	$M = 4$	27.14	25.35	25.68
$L = 4$	$M = 2$	125.44	124.02	124.71
	$M = 3$	68.71	66.56	67.74
	$M = 4$	42.69	39.25	40.00
$L = 5$	$M = 2$	172.37	170.74	172.78
	$M = 3$	97.79	94.85	96.42
	$M = 4$	61.46	56.25	57.38

$L : half - waves\ on\ cylinde\ axis$
$M : Lobes\ on\ cross - section$

Table 3: Shell-like frequency[Hz]

evaluated using the present model. With shell-like are denoted the modes that involved a large deformation of the cross-section that can not be detected using the classical beam models. With L and M are denoted the number of half-waves on the beam axis and the number of lobes on the cross section of the mode. The present models appear to be able to describe accurately the dynamic behaviour of the structure including the effects due to the deformation of the cross-section. The frequencies are comparable with those from the reference model. Fig.3 shows an example of two shell-like modes, Fig.3*a* shows the mode with one half-wave on the axis and five lobes while Fig.3*b* depicts the same mode but with two half-waves on the axis.



(a) Mode L=1, M=5; 14.66 [Hz].

(b) Mode L=2, M=5; 15.98 [Hz].

Figure 3: Sample of shell-like modes

3.3. Launcher dynamic analysis

The capabilities of the present model allow complex structure to be investigated. In this section the modal analysis of the launcher model, reported in Fig. 4, has been performed. The structure has a central body with two lateral boosters. The dark areas denoted the presence of a ribs while four longitudinal stringers have been introduced in the central body as well as in the lateral boosters. The structure is considered free, therefore none boundary conditions have been imposed. The two boosters have been connected with the main structure at the bottom and in the middle of the central body. The results obtained using the present model have been compared with those from a commercial code. Two different model has been used as reference, the first is a solid model while the second is a mixed model where both shell and beam elements have been used. The shells for the thin panel and the beams for the stringers and ribs. Tab. 4 shows the first 8 natural frequencies. The results obtained using the present model are close to the results obtained using the solid model. The combined 1D-2D model provides good results but does not provide the fifth frequency. The present model lead to a dramatic reduction in the number of the degrees of freedom, in fact, the LE model has about 20 time less DOF than the solid model and more than 10 time less DOF than the combined 1D-2D models. This results make the present model very attractive in the simulation of such structures. Fig. 5 shows an example of some modes of the launcher. It is possible to see that the solution can be compare with a three-dimensional solution. Both global and local modes can be detected.

	FEM3D	FEM2D-1D	LE 9
DOF	565740	408456	29682
1	0.63	0.56 $^{(-11.1\%)}$	0.74 $^{(+17.5\%)}$
2	5.31	4.16 $^{(-21.7\%)}$	4.69 $^{(-11.7\%)}$
3	6.87	6.52 $^{(-5.1\%)}$	9.28 $^{(+35.1\%)}$
4	7.50	6.82 $^{(-9.1\%)}$	6.82 $^{(-9.1\%)}$
5	8.27	-	9.62 $^{(+16.3\%)}$
6	8.55	6.91 $^{(-19.2\%)}$	7.93 $^{(-7.3\%)}$
7	10.42	7.34 $^{(-29.6\%)}$	8.94 $^{(-14.2\%)}$
8	10.66	13.27 $^{(24.5\%)}$	12.49 $^{(+17.2\%)}$

Table 4: First 8 natural frequencies evaluated using different models.

4. CONCLUSIONS

In the present work a refined one-dimensional model is used to investigate complex structures. An advanced approach, component-wise approach, has been adopted to improve the performance of the refined one-dimensional

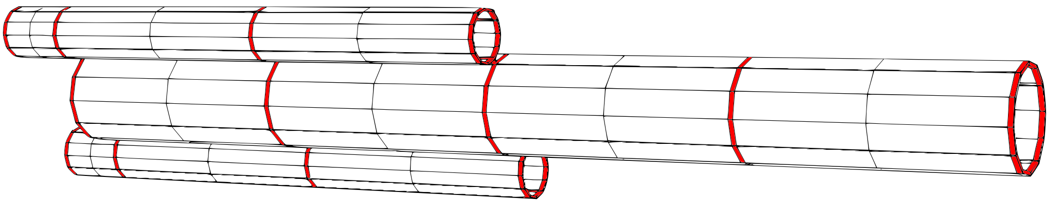
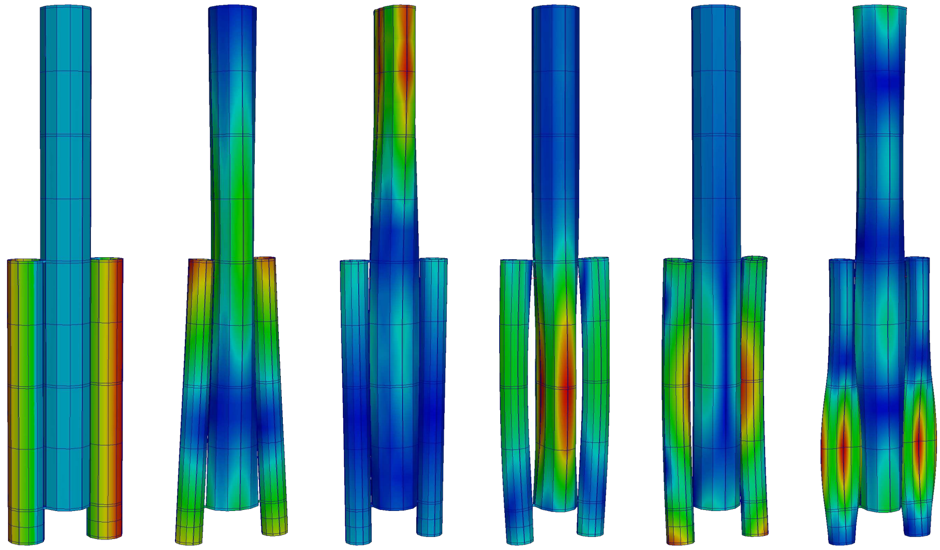


Figure 4: Launcher structure geometry



(a) Mode 1 (b) Mode 2 (c) Mode 3 (d) Mode 5 (e) Mode 8 (f) Mode 12

Figure 5: Example of model of the Launcher structure.

model derived using the CUF formulation. Reinforced and thin-walled structure have been investigated considering both static and dynamic problems. A complete launcher structure has been considered as main example to show the performance of the present model. From the results it is possible to state that:

- The present model is able to deal with the static and dynamic analysis of reinforced structures;
- The present model overcomes the limitation of the classical beam models and provides a *quasi* 3D solution;
- Single components (panels) as well as complete structures (launchers) can be investigated using the component-wise approach;
- The computational cost can be dramatically reduced using the model presented in the present paper.

In conclusion the results present in this work make the present model very attractive for the analysis of complex structure where a large number of degrees of freedom is required. The accuracy of the results and the computational efficiency make this model an valid alternative to the classical analysis approach.

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