

# REFINED SHELL ELEMENTS FOR THE ANALYSIS OF MULTILAYERED STRUCTURES WITH PIEZOELECTRIC LAYERS

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**Key words:** Finite element method, plates and shells, Unified Formulation, Mixed Interpolated Tensorial Components, piezoelectric layers, static and dynamic.

**Summary.** *The present work deals with the static and dynamic analysis of multilayered plates and shells embedding piezoelectric layers. To this aim, the refined shell elements, recently formulated by the authors on the basis of Unified Formulation (UF), have been extended to the electro-mechanical problem. The governing equations are derived using the Principle of Virtual Displacements (PVD) extended to the electro-mechanical case. The Mixed Interpolated Tensorial Components (MITC) method is employed to contrast the membrane-shear locking phenomenon that usually affects shell finite elements. Results are compared with some reference solutions from the literature.*

## 1. INTRODUCTION

Smart structures differ from the conventional ones by the presence of elements able to perform as actuators and/or sensors, allowing the structure itself to adapt and/or sense to the external environment. This capability leads to a wide range of applications, in particular in the

aerospace field. Among materials that can be used, only piezoelectric ones have shown the capability to perform effectively both as actuators and sensors. Another advantage of piezoelectric materials is the simple integration with composite structures. Due to the discontinuity of the mechanical and electrical properties at the layer interfaces, an accurate description of the mechanical and electrical fields in the layers is essential. For these reasons, the use of classical plate theories based on Kirchhoff and Reissner-Mindlin hypotheses can lead to inaccurate results.

Even if analytical models are available, the solution of practical problems often demand the use of computational methods such as the finite element method. The present paper presents shell finite elements for the analysis of multilayered composite structures embedding piezoelectric layers, based on the Principle of Virtual Displacements (PVD) and the Unified Formulation (UF) introduced by Carrera [1]. These shell elements have nine nodes, and the mixed interpolation of tensorial components method is employed to contrast the membrane and shear locking phenomenon. This formulation has already shown all its potentiality as a base for finite elements in the mechanical analysis of multilayered shells [2]. Moreover, plate finite elements based on UF for the analysis of electro-mechanical problem have been already presented in [3]. One of the most interesting features of the unified formulation consists in the possibility to keep the order of the expansion of the state variables along the thickness of the plate as a parameter of the model. In so doing, both equivalent single layer (ESL) and layer-wise (LW) descriptions of the variables are allowed. The electrical potential assumption has been limited to a LW description. This feature is particularly suitable since electric degrees of freedom (dofs) are often avoided for plate/shell elements or a simple through-thickness linear variation is assumed for the electric potential. The linear through-thickness hypothesis for the electric potential, as demonstrated by Benjeddou [4], neglects systematically the contribution of the induced potential leading to a partial electromechanical coupling. Some results from the static and dynamic analysis of plates and shells embedding piezoelectric layers will be provided in order to show the efficiency of models presented.

## References

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