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STRUCTURAL ANALYSIS OF DAMAGED STRUCTURES THROUGH 1D REFINED MODELS

M. Petrolo*, E. Carrera

Department of Mechanical and Aerospace Engineering, Politecnico di Torino,
Corso Duca degli Abruzzi 24, 10129, Torino (Italy)

*marco.petrolo@polito.it

ABSTRACT

This paper deals with the structural analysis of damaged structures through refined beam models. Damages are locally introduced and their effects are evaluated in terms of variations in natural frequencies and modal shapes. Computationally cheap 1D structural models are adopted. These models are based on the Carrera Unified Formulation (CUF) and provide shell- and solid-like accuracies with far less computational costs than shell and solid finite elements. 1D CUF models are built according to arbitrary 2D expansions of the unknown variables. The order and the type of the expansion are free parameters of the analysis. In this work, Taylor polynomials are used. Results show that 1. Damage effects can deeply vary depending on the damage intensity and location, 2. 1D CUF models can detect typical behaviours of damaged structures such as the bending-torsion coupling, 3. 1D CUF low computational costs are of particular interest for the damage detection via modal analysis.
Keywords: beam, refined models, unified formulation, damage

1 INTRODUCTION

Beam models are widely adopted for many structural engineering applications. Beam models are, in fact, computationally cheap and less cumbersome than plate, shell and solid finite elements. The classical beam theories are those by Euler-Bernoulli [1] and Timoshenko [2]. None of these theories can detect non-classical effects such as warping, out- and in-plane deformations, torsion-bending coupling or localized boundary conditions (geometrical or mechanical). These effects are important when, for instance, small slenderness ratios, thin walls and the anisotropy of the materials are considered.
Many methods have been proposed over the last decades to overcome the limitations of classical theories and to extend the application of 1D models to any geometry or boundary condition. Most recent developments in 1D models have been obtained by means of the following approaches:
1. The introduction of shear correction factors [2].
2. The use of warping functions based on the de Saint-Venant's solution [3-4].
3. Asymptotic approaches [5].
4. Generalized beam theories (GBT) [6].
5. Higher-order beam models [7-8].

This work is embedded in the framework of the Carrera Unified Formulation (CUF) for higher-order 1D models [9]. CUF has been initially developed for plates and shells [10-11], more recently for beams [9,12]. In CUF models the displacement field above the cross-section is modelled through expansion functions whose order is a free parameter of the analysis. This means that any-order structural models can be implemented with no need of formal changes in the problem equations and matrices. CUF can therefore deal with arbitrary geometries, boundary conditions and material characteristics with no need of ad hoc formulations.
CUF 1D models have recently been applied to static [12-14] and free-vibration [15] analyses. The most recent extension of CUF model is the so-called Component-Wise approach (CW). Lagrange polynomials are exploited in a 1D CW model [16,17] and multicomponent structures (e.g. aircraft wings or fibre reinforced composites) are modelled through a unique 1D formulation [18-21]. Solid-like accuracies were obtained with far less computational costs.
In this work, CUF 1D models are exploited to analyse damaged structures through the free vibration analysis. This paper is aimed to 1. The outlining of guidelines for the damage analysis of structures, 2. Show the enhanced capabilities of the present formulation which can be easily adopted to detect the structural behaviour of damaged structures.

2 1D CARRERA UNIFIED FORMULATION

The beam cross-section displacement field is described by an expansion of generic functions (F_τ)

$$\boldsymbol{u} = F_\tau \boldsymbol{u}_\tau, \qquad \tau = 1,2,\ldots,M \tag{1}$$

where F_τ are functions of the cross-section coordinates (x, z), $\boldsymbol{u}_\tau$ is the displacement vector and M stands for the number of terms of the expansion. In this work, Taylor-like polynomial expansions of the displacement field above the cross-section of the structure were used. The order of the expansion is arbitrary and is set as an input of the analysis. For example, the second-order model (*N* = *2*) is based on the following displacement field:

$$\begin{aligned} u_x &= u_{x_1} + x\ u_{x_2} + z\ u_{x_3} + x^2\ u_{x_4} + xz\ u_{x_5} + z^2\ u_{x_6} \\ u_y &= u_{y_1} + x\ u_{y_2} + z\ u_{y_3} + x^2\ u_{y_4} + xz\ u_{y_5} + z^2\ u_{y_6} \\ u_z &= u_{z_1} + x\ u_{z_2} + z\ u_{z_3} + x^2\ u_{z_4} + xz\ u_{z_5} + z^2\ u_{z_6} \end{aligned} \tag{2}$$

The 1D model described by Eq. (2) has 18 generalized displacement variables: three constant, six linear, and nine parabolic terms.
The governing equations were derived by means of the Principle of Virtual Displacements (PVD). A compact form of the virtual variation of the strain energy can be obtained as shown in [9],

$$\delta L_{int} = \delta \boldsymbol{q}_{\tau i}^T \boldsymbol{K}^{ij\tau s} \boldsymbol{q}_{sj} \tag{3}$$

where $\boldsymbol{K}^{ij\tau s}$ is the stiffness matrix written in the form of the fundamental nuclei. Superscripts indicate the four indexes exploited to assemble the matrix: i and j are related to the shape functions, τ and s are related to the expansion functions. The fundamental nucleus is a 3×3 array which is formally independent of the order of the beam model. In a compact notation, the stiffness matrix for a given material property set can be written as:

$$
\begin{aligned}
\mathbf{K}^{ij\tau s} = \quad & I_l^{ij} \lhd (\mathbf{D}_{np}^T F_\tau \mathbf{I})[\tilde{\mathbf{C}}_{np} (\mathbf{D}_p F_s \mathbf{I}) + \tilde{\mathbf{C}}_{nn} (\mathbf{D}_{np} F_s \mathbf{I})] + \\
& (\mathbf{D}_p^T F_\tau \mathbf{I})[\tilde{\mathbf{C}}_{pp} (\mathbf{D}_p F_s \mathbf{I}) + \tilde{\mathbf{C}}_{pn} (\mathbf{D}_{np} F_s \mathbf{I})] \rhd_\Omega + \\
& I_l^{ij,y} \lhd [(\mathbf{D}_{np}^T F_\tau \mathbf{I}) \tilde{\mathbf{C}}_{nn} + (\mathbf{D}_p^T F_\tau \mathbf{I}) \tilde{\mathbf{C}}_{pn}] F_s \rhd_\Omega \mathbf{I}_{\Omega y} + \\
& I_l^{i,yj} \mathbf{I}_{\Omega y} \lhd F_\tau [\tilde{\mathbf{C}}_{np} (\mathbf{D}_p F_s \mathbf{I}) + \tilde{\mathbf{C}}_{nn} (\mathbf{D}_{np} F_s \mathbf{I})] \rhd_\Omega + \\
& I_l^{i,yj,y} \mathbf{I}_{\Omega y} \lhd F_\tau \tilde{\mathbf{C}}_{nn} F_s \rhd_\Omega \mathbf{I}_{\Omega y}
\end{aligned} \tag{4}
$$

where

$$
\mathbf{I}_{\Omega y} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \lhd \dots \rhd_\Omega = \int_\Omega \dots \, d\Omega \tag{5}
$$

$$
(I_l^{ij}, I_l^{ij,y}, I_l^{i,yj}, I_l^{i,yj,y}) = \int_l (N_i N_j, N_i N_{j,y}, N_{i,y} N_j, N_{i,y} N_{j,y}) \, dy \tag{6}
$$

$\tilde{\mathbf{C}}$ is the material coefficient matrix and $\mathbf{D}$ is the differential operator matrix. It should be underlined that the formal expression of $\boldsymbol{K}^{ij\tau s}$

1. does not depend on the expansion order,
2. does not depend on the choice of the F_τ expansion polynomials.

These are the key-points of CUF which permits, with only nine FORTRAN statements, to implement any-order of multiple class theories.

The virtual variation of the work of the inertial loadings is

$$
\delta L_{ine} = \int_V \rho \ddot{\boldsymbol{u}} \delta \boldsymbol{u}^T dV \tag{7}
$$

where ρ stands for the density of the material, and $\ddot{\boldsymbol{u}}$ is the acceleration vector. Equation (7) can be rewritten in a compact manner as follows:

$$
\delta L_{ine} = \delta \boldsymbol{q}_{\tau i}^T \boldsymbol{M}^{ij\tau s} \ddot{\boldsymbol{q}}_{sj} \tag{8}
$$

where $\boldsymbol{M}^{ij\tau s}$ is the mass matrix in the form of the fundamental nucleus whose components can be found in [9].

2.1 Damage modelling

A basic damage modelling approach was adopted in this work. Fig. 1 shows an example of locally damaged structure. In the damaged zone, the material characteristics were modified according to the following formulas:

$$
E_i = D_i \times E \; with \; 0 \le D_i \le 1, \qquad D_0 = 1; \; D_1 = 0.9; \dots; \; D_9 = 0.1
$$

Damages were introduced in different portions of the structure as will be shown in the result section of this paper.

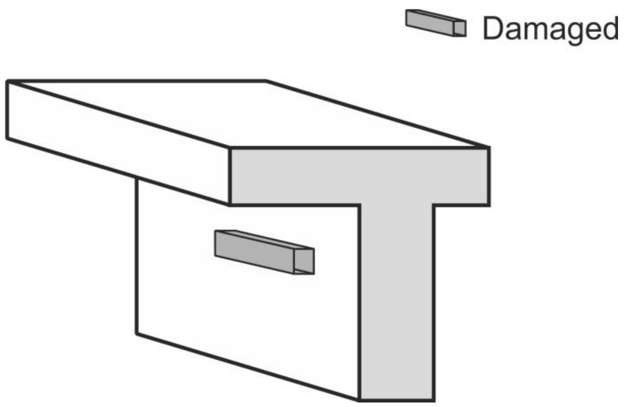

Figure 1: Locally damaged structure

3 RESULTS AND DISCUSSION

3.1 Damage along the entire axis

A rectangular cross-section beam was considered. The length of the beam (L) equal to 2 m, the width (b) equal to 0.1 m and the thickness (h) equal to 1 mm. An orthotropic material was considered, with E_L = 40 GPa, $E_T = E_Z$ = 4 GPa, G = 4 GPa, ν = 0.25 and ρ = 1600 Kg/m^3. Fig. 2 shows the damage distribution along the thickness.

Table 1 presents the natural frequencies for different damage levels, results were obtained through a fourth-order model (N = 4). Fig. 3 shows the type of modal shapes for different beam models. It can be stated that

- Damages influence the natural frequencies of the beam.
- Refined models are needed to detect torsional modes.

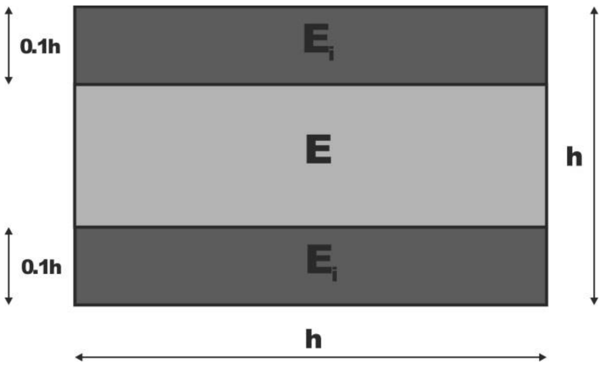

Figure 2: Damage distribution above the cross-section

E_0	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
				$f_{1b}[Hz]$					
0.5962	0.5903	0.5823	0.5715	0.5572	0.5380	0.5121	0.4766	0.4262	0.3501
				$f_{2b}[Hz]$					
3.730	3.695	3.640	3.584	3.494	3.377	3.210	2.980	2.674	2.193
				$f_{3t}[Hz]$					
6.214	6.154	6.087	6.001	5.882	5.748	5.568	5.325	5.021	4.615
				$f_{4b}[Hz]$					
10.45	10.35	10.21	10.02	9.773	9.435	8.987	8.366	7.472	6.142
				$f_{5b}[Hz]$					
16.65	16.53	16.39	16.25	16.10	15.93	15.75	15.54	14.65	12.03

Table 1: Natural frequencies of a damaged orthotropic beam via an N = 4 model

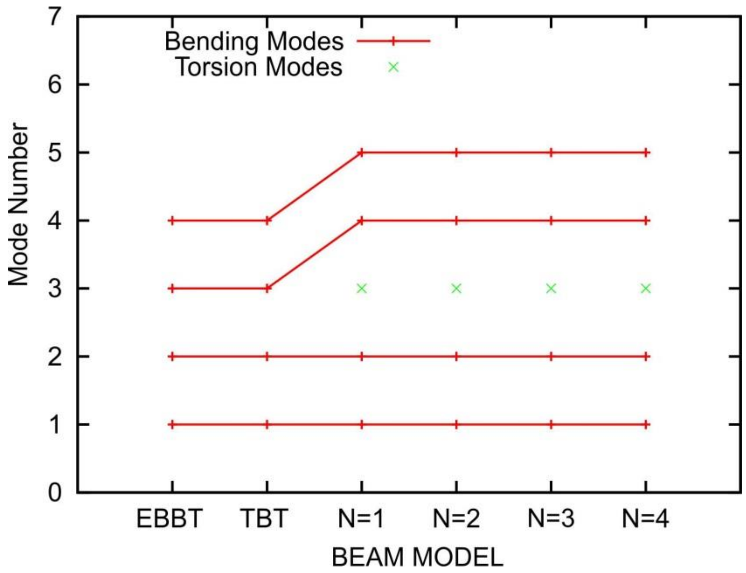

Figure 3: Modal shapes for different beam models

3.2 Damage along a portion of the axis

In a second assessment, damage was localized in a portion of the beam. Fig. 4 shows the damaged clamped root portion of the beam. Table 2 presents the natural frequencies of the damaged beam via a fourth-order model. Fig. 5 shows a modal shape of a beam whose mid-span portion was damaged.
These results suggest the following:

- As the damage increases, changes in the modal shapes of the structure were observed.
- Since torsional modes can appear or bending/torsion coupling can take place, refined 1D models are needed to detect these behaviours.
- Damages can lead to local effects which can be detected by refined models only.

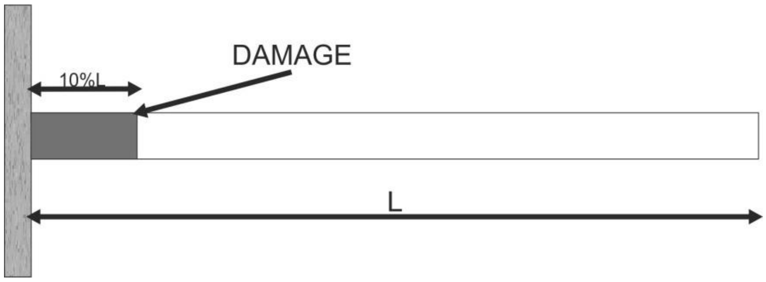

Figure 4: Damage distribution along a portion of the beam

E_0	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
				$f_{1b}[Hz]$					
0.5962	0.5905	0.5838	0.5756	0.5654	0.5523	0.5349	0.5105	0.4738	0.4111
				$f_{2b}[Hz]$					
3.734	3.712	3.687	3.654	3.611	3.564	3.501	3.432	3.332	3.187
				$f_{3b}[Hz]$					
6.201	6.184	6.161	6.132	6.098	6.057	6.001	5.921	5.824	5.664
				$f_{4b}[Hz]$					
10.45	10.41	10.35	10.29	10.22	10.14	10.04	9.914	9.742	9.512
$f_{5_b}[Hz]$					$f_{5_t}[Hz]$				
16.65	16.51	16.35	16.18	15.98	15.76	15.52	15.24	14.91	14.53

Table 2: Natural frequencies of a damaged orthotropic beam via an N = 4 model, damage located at the clamped root of the beam

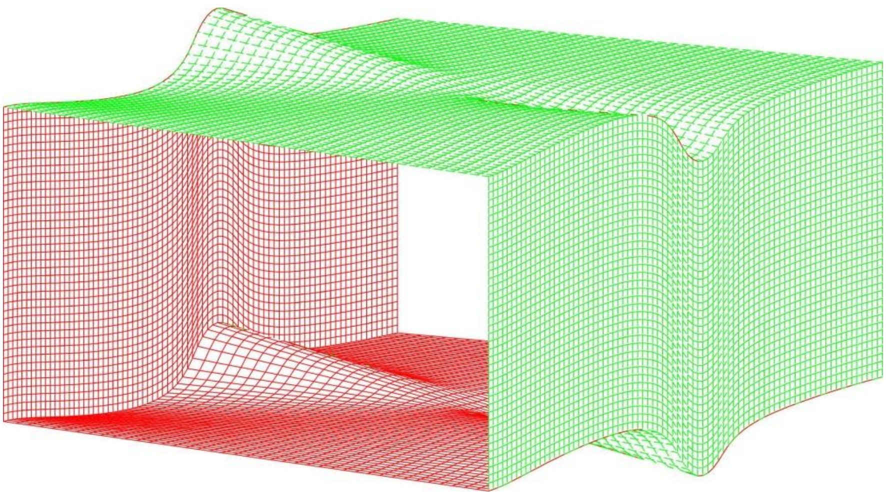

Figure 5: Damage distribution along a portion of the beam

3.3 I-section beam

An isotropic I-section beam was considered, see Fig. 6. The length of the beam equal to 1 m, the Young modulus equal to 75 GPa, the Poisson ratio equal to 0.33 and the density equal to 2700 Kg/m^3.

Table 3 shows the natural frequencies obtained through an N = 4 model. As previously observed, as the damage increases, different modal shapes are detected with predominant bending and torsion modes. This means that refined 1D models must be adopted to detect the modal shapes of damaged structures.

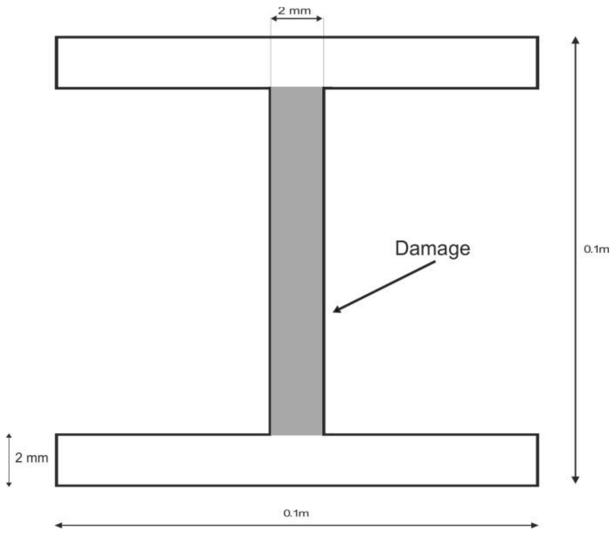

Figure 6: Damaged I-section

E_0	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
				$f_{1b}[Hz]$					
70.013	70.005	69.996	69.986	69.974	69.960	69.944	69.923	69.897	69.861
			$f_{2_b}[Hz]$					$f_{2_t}[Hz]$	
124.06	122.91	121.69	120.35	118.87	117.14	115.03	112.16	106.46	95.488
			$f_{3_b}[Hz]$					$f_{3_t}[Hz]$	
152.24	148.47	144.36	139.85	134.83	129.20	122.79	115.34	107.61	97.627
				$f_{4b}[Hz]$					
425.57	425.35	425.09	424.80	424.46	424.05	423.52	422.81	421.73	362.05
				$f_{5t}[Hz]$					
551.62	543.35	534.53	525.09	514.94	503.96	492.03	481.66	470.46	448.50

Table 3: Natural frequencies of the damaged I-section

4 CONCLUSIONS

This paper has presented a preliminary study to analyse damaged structures via refined 1D models. These models were obtained through the Carrera Unified Formulation (CUF) whose hierarchical capabilities allow us to deal with any-order models with no need of ad hoc formulations. Different structures have been analysed and the results suggest the following:

- 1D CUF structural models are powerful an computationally cheap tools to analyze structures for different applications, including aerospace and civil structures and composites.
- CUF models provide 3D-like accuracies with low computational costs.
- 1D CUF models can deal with damaged structures since they can detect non-classical effects such as torsion/bending couplings.

- Refined 1D models are mandatory for the analysis of damaged structures since non-classical effects can take place.

5 ACKNOWLEDGEMENTS AND REFERENCES

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