Analysis of functionally graded structures by radial basis functions using several theories under Carrera's Unified Formulation

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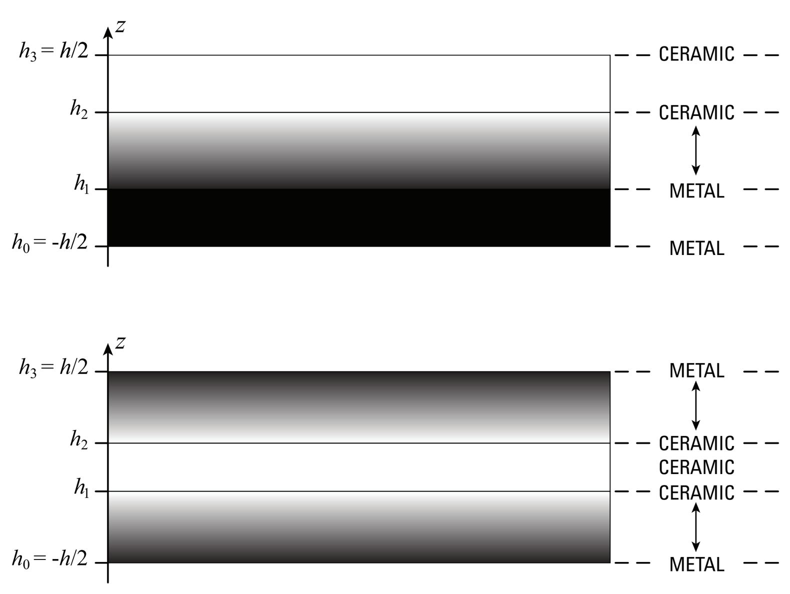
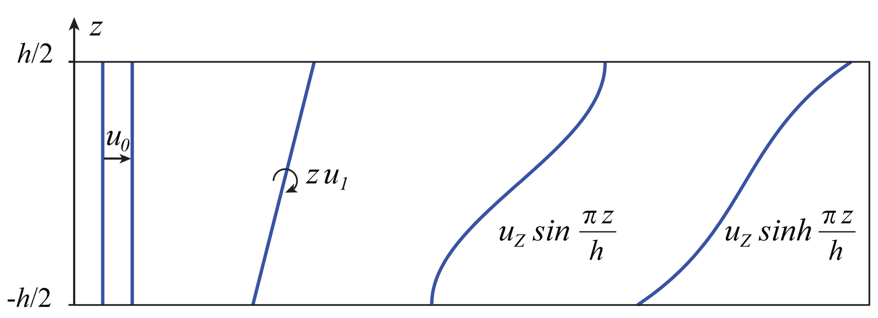
Topics

static, free vibration, and buckling analysis of functionally graded plates and shells, including sandwiches with functionally graded core or skins

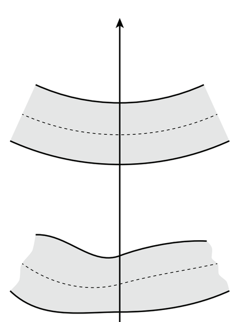


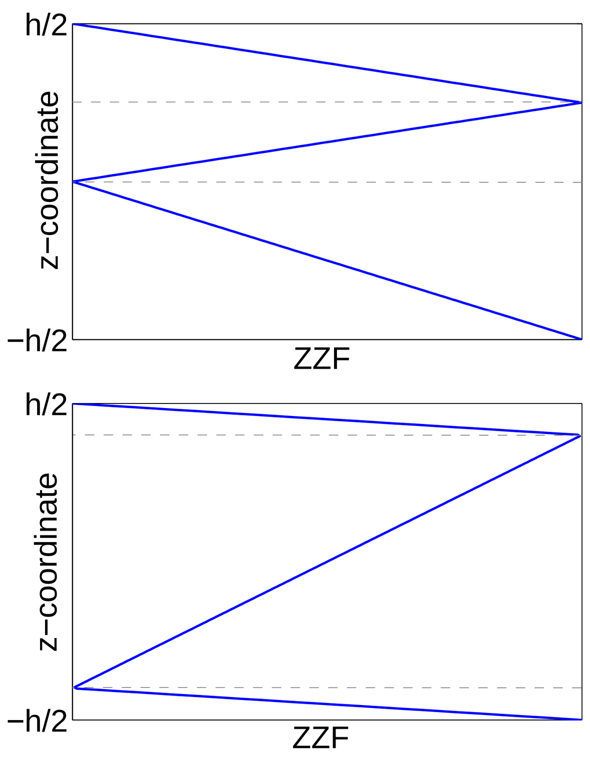
Some theories used



- Equivalent single layer displacement based theories, Hybrid CUF
- $u_x = u_y = u_0 + z u_1 + f(z) u_H + f(ZZ) u_{ZZ}$
- $w = w_0 + z w_1 + z^2 w_2$

$$\begin{cases} f(z) = z^3 \\ f(z) = \sin(\pi z/h) \\ f(z) = \sinh(\pi z/h) \\ f(ZZ) = (-1)^k \frac{2}{h_k}\left(z - \frac{1}{2}(z_k + z_{k+1})\right) u_Z \end{cases}$$





Methodology

Material properties The power-law function is adapted to describe the volume fraction of the metal (V_m) and ceramic (V_c) phases and the material homogeneization technique adopted to the Young's modulus is the law of mixtures:

$$E(z) = E_m V_m + E_c V_c \quad \text{with} \quad V_c = \begin{cases} \left(\frac{z-h_0}{h_1-h_0}\right)^p, & z \in [h_0, h_1] \\ 1, & z \in [h_1, h_2] \\ \left(\frac{z-h_3}{h_2-h_3}\right)^p, & z \in [h_2, h_3] \end{cases}$$

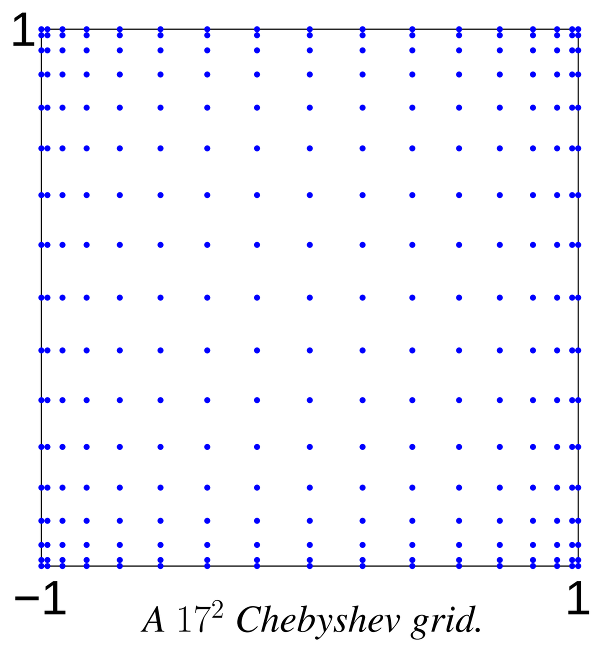
Carrera's Unified Formulation

The governing equations and boundary conditions are derived under a generalization of Carrera's Unified Formulation (CUF) [1] based on the principle of virtual displacements. Although the sandwiches present 3 physical layers, we consider $N_l = 91$ virtual (mathematical) layers of constant thickness.

Meshless method

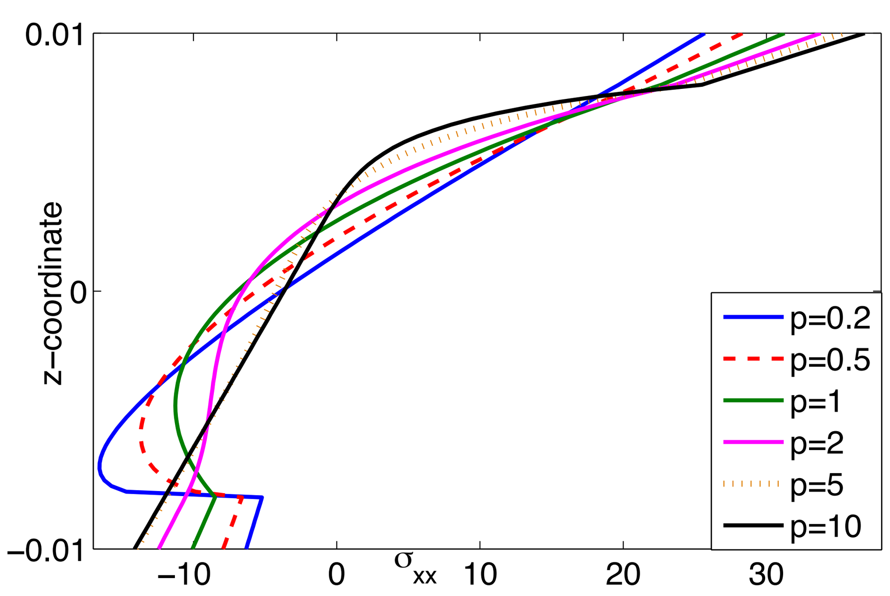
The governing equations are interpolated by global collocation with radial basis functions [2]. We consider the compact-support Wendland function defined as $\phi(r) = (1 - c\,r)^8_+ \left(32(c\,r)^3 + 25(c\,r)^2 + 8c\,r + 1\right)$.

The shape parameter (c) is optimized [3] and a Chebyshev grid with a variable number of points is used.



A 17^2 Chebyshev grid.

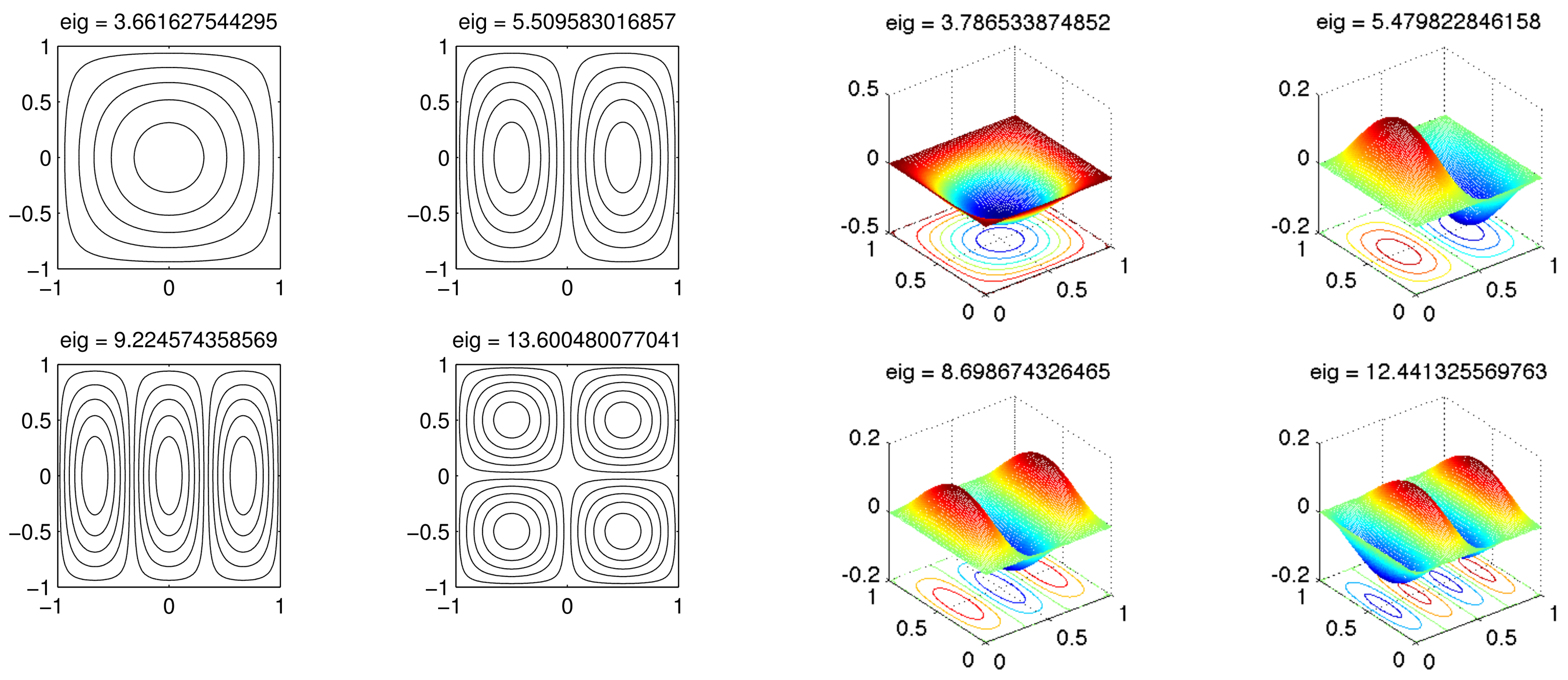
Results



SSSS square sandwich plate with FG core, $f(z) = \sinh(\pi z/h)h$, with $f(ZZ)$, $a/h = 100$, 15^2 points

Some results on free vibration and buckling:

First four modes



Conclusions

- Carrera's Unified (GENERALIZED) Formulation was used as an AUTOMATIC tool to implement (and obtain) the governing equations AND boundary conditions.
- Several new theories and cases were tested successfully. The explicit governing equations and boundary conditions of these theories can be found in some published papers by the authors listed below.
- The present approach is a simple yet powerful alternative to other techniques to perform the analysis of thin and thick FGM plates and shells behaviour.
- Obtained results with the present approach are in good agreement with references considered and can be found in some published papers by the authors:

- A. M. A. Neves, A. J. M. Ferreira, E. Carrera, M. Cinefra, C. M. C. Roque, R. M. N. Jorge, C. M. M. Soares, *Free vibration analysis of functionally graded shells by a higher-order shear deformation theory and radial basis functions collocation, accounting for through-the-thickness deformations, European Journal of Mechanics -A/Solids, Volume 37, 2013, pág. 24-34.*
- *A. M. A. Neves, A. J. M. Ferreira, E. Carrera, C. M. C. Roque, M. Cinefra, R. M. N. Jorge, C. M. M. Soares,* A quasi-3D sinusoidal shear deformation theory for the static and free vibration analysis of functionally graded plates, Composites Part B: Engineering, Volume 43, 2012, pág. 711-725.
- A. M. A. Neves, A. J. M. Ferreira, E. Carrera, M. Cinefra, C. M. C. Roque, R. M. N. Jorge, C. M. M. Soares, *A quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates, Composites Structures, Volume 94, 2012, pág. 1814-1825.*
- *A. M. A. Neves, A. J. M. Ferreira, E. Carrera, M. Cinefra, R. M. N. Jorge, C. M. M. Soares,* Static analysis of functionally graded sandwich plates according to a hyperbolic theory considering Zig-Zag and warping effects, Advances in Engineering Software, Volume 52, 2012, pág. 30-43.
- A. M. A. Neves, A. J. M. Ferreira, E. Carrera, M. Cinefra, C. M. C. Roque, R. M. N. Jorge, C. M. M. Soares, *Buckling behaviour of cross-ply laminated plates by a higher-order shear deformation theory, Science and Engineering of Composite Materials, Volume 19, 2012, pág. 119-125.*
- *A. M. A. Neves, A. J. M. Ferreira, E. Carrera, M. Cinefra, C. M. C. Roque, R. M. N. Jorge, C. M. M. Soares,* Static, free vibration and buckling analysis of functionally graded plates using a quasi-3D higher-order shear deformation theory and a meshless technique, Composites Part B: Engineering, Volume 44, 2012, Pág. 657-674.
- A. M. A. Neves, A. J. M. Ferreira, E. Carrera, M. Cinefra, R. M. N. Jorge, C. M. M. Soares, *Buckling analysis of sandwich plates with functionally graded skins using a new quasi-3D hyperbolic sine shear deformation theory and collocation with radial basis functions, Zeitschrift für Angewandte Mathematik und Mechanik, Volume 92, 2012, Pág. 749-766.*
- *Ana M. A. Neves, A. J. M. Ferreira, E. Carrera, C. M. C. Roque, M. Cinefra, R. M. N. Jorge, C. M. M. Soares,* Bending of FGM plates by a sinusoidal plate formulation and collocation with radial basis functions, Mechanics Research Communications, Volume 38, 2011, Pág. 368-371.

References

[1] Carrera, E. Theories and finite elements for multilayered plates and shells: a unified compact formulation with numerical assessment and benchmarking. *Archives of Computational Methods in Engineering* 10(3):215-296, 2003.

[2] Kansa, E. J. Multiquadrics- A scattered data approximation scheme with applications to computational fluid dynamics. I: Surface approximations and partial derivative estimates. *Computers and Mathematics with Applications* 19(8/9):127-145, 1990.

[3] Ferreira, A. J. M., and Fasshauer, G. E. Computation of natural frequencies of shear deformable beams and plates by a RBF-Pseudospectral method. *Computer Methods in Applied Mechanics and Engineering,* 196:134-146, 2006.