A finite element model using a unified formulation for the analysis of viscoelastic sandwich laminates

A. J. M. Ferreira [a], A.L. Araújo [b] A. M. A. Neves [c], J. D. Rodrigues [d], E. Carrera [e], M. Cinefra [e], C. M. Mota Soares [b]

[a] *(Corresponding author: ferreira@fe.up.pt)*
Departamento de Engenharia Mecânica, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal

[b] *IDMEC/IST, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal*

[c] *Departamento de Engenharia Mecânica, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal*

[d] *INEGI, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal*

[e] *Departament of Aeronautics and Aerospace Engineering, Politecnico di Torino, Corso Duca degli Abruzzi, 24, 10129 Torino, Italy*

1 Introduction

Sandwich plates with viscoelastic core are very effective in reducing and controlling vibration response of lightweight and flexible structures, where the soft core is strongly deformed in shear, due to the adjacent stiff layers. The theoretical work on constrained layer damping can be traced to DiTaranto [1] and Mead and Markus [2] for the axial and bending vibration of sandwich beams. Since then, different formulations and techniques have been reported for modelling and predicting the energy dissipation of the viscoelastic core layer in a vibrating passive constrained layer damping structure [3–5]. Other proposed formulations include thickness deformation of the core layer [6] and deal with the cases where only a portion of the base structure receives treatment [7].

Due to the high shear developed inside the core of the sandwich, equivalent single layer plate theories, even those based on higher order deformations, are not adequate to describe the behaviour of these sandwiches, also due to the high deformation discontinuities that arise at the interfaces between the viscoelastic core material and the surrounding elastic constraining layers. The usual approach to analyse the dynamic response of sandwich plates uses a layered scheme of plate and brick elements with nodal linkage. This approach leads to a time consuming spatial modelling task. To overcome these difficulties, the layerwise theory has been considered for constrained viscoelastic treatments, and most recently, Moreira et al. [8,9], among others, presented generalized layerwise formulations in this scope.

A review and assessment of various theories for modeling sandwich composites with application to sandwich beams can be found in the work of Hu et al. [10].

More recently, Araújo et al. [11–14] have presented and used for optimisation and viscoelastic material identification purposes a sandwich finite element model based on an eight nodded serendipity plate element. The viscoelastic core layer is modelled according to a higher order shear deformation theory and adjacent elastic and piezoelectric layers are modelled using the first order shear deformation theory. All materials are considered to be orthotropic, with elastic layers being formulated as laminated composite plies. Passive damping is accounted for by using the complex modulus approach, allowing for frequency dependent viscoelastic materials and active damping is incorporated through feedback control laws for co-located control. Also in this framework, Moita et al. [15] developed a simple and efficient non conforming triangular finite element where the viscoelastic core is modelled according to Reissner-Mindlin laminated plate theory and the face layers are modelled according the Kirchhoff-Love plate theory. Another sandwich plate model presented by Moita et al. [16] is based on Reddy's third order shear deformation theory for the core and the face layers are also modelled according to the classical laminated plate theory. These models also contemplate hybrid active-passive damping. A similar model was also presented by Bilasse et al. [17] for non linear vibrations of sandwich plates.

In the present work the stiffness and mass matrices are obtained by Carrera's Unified Formulation (CUF), firstly proposed in [18–20] for laminated plates and shells and extended to functionally graded (FG) plates in [21–23]. The present formulation considers a displacement-based layerwise formulation, with linear expansion of displacements in each layer, with degrees of freedom u_x, u_y, u_z at each lamina interface. To the authors knowledge, it is the first time that CUF is applied to this class of problems with viscoelastic behaviour. The main advantage of the present formulation is its versatility allowing for great flexibility in the through the thickness approximations, which is an important feature in sandwich structures.

The dynamic response of the finite element model is validated using a few reference solutions from the literature.

2 Acknowledgements

The authors thank the financial support of FCT, through POCTI and POCI(2010) / FEDER. In particular the support of LAETA to project Composites in Mechanical Design and the support to PTDC/EME-PME/120830/2010 is gratefully acknowledged. The authors also acknowledge the kind support of FCT to project PTDC/EME-PME/109116/2008.

References

[1] DiTaranto, R.A. (1965). Theory of vibratory bending for elastic and viscoelastic layered finite-length beams, *ASME Journal of Applied Mechanics*, **32**:881–886.

[2] Mead, D.J. and Markus, S. (1969). The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions, *AIAA Journal*, **10**:163–175.

[3] Rao, D.K. (1978). Frequency and loss factors of sandwich beams under various boundary conditions, *International Journal of Mechanical Engineering Science*, **20**:271–278.

[4] Yan, M.J. and Dowell, E.H. (1972). Governing equations of vibrating constrained-layer damping sandwich plates and beams, *ASME Journal of Applied Mechanics*, **39**:1041–1046.

[5] Rao, M.D. and He, S. (1993). Dynamic analysis and design of laminated composite beams with multiple damping layers, *AIAA Journal*, **31**:736–745.

[6] Douglas, B.E. and Yang, J.C.S. (1978). Transverse compressional damping in the vibratory response of elastic-viscoelastic beams, *AIAA Journal*, **16**:925–930.

[7] Lall, A.K., Asnani, N.T. and Nakra, B.C. (1988). Damping analysis of partially covered sandwich beams, *Journal of Sound and Vibration*, **123**:247–259.

[8] Moreira, R.A.S., Rodrigues, J.D., Ferreira, A.J.M. A generalized layerwise finite element for multi-layer damping treatments. Computational Mechanics. 37, 426-444 (2006)

[9] Moreira, R.A.S., Rodrigues, J.D. A layerwise model for thin soft core sandwich plates. Computers and Structures. 84, 1256-1263 (2006)

[10] Hu, H., Belouettar, S., Potier-Ferry, M., Daya, E.M. Review and assessment of various theories for modeling sandwich composites. Composite Structures. 84(3), 282-292 (2008)

[11] Araújo, A.L., Mota Soares, C.M., Mota Soares, C.A. Finite element model for hybrid active-passive damping analysis of anisotropic laminated sandwich structures. Journal of Sandwich Structures and Materials. Vol. 12(4):397-419 (2010)

[12] Araújo, A.L., Mota Soares, C.M., Mota Soares, C.A., Herskovits, J. Optimal design and parameter estimation of frequency dependent viscoelastic laminated sandwich composite plates. Composite Structures. 92, 2321-2327 (2010)

[13] Araújo, A.L., Mota Soares, C.M., Mota Soares, C.A. A Viscoelastic Sandwich Finite Element Model for the Analysis of Passive, Active and Hybrid Structures. Applied Composite Materials. 17, 529-542 (2010)

[14] Araújo, A.L., Martins, P., Mota Soares, C.M., Mota Soares, C.A., Herskovits, J. Damping Optimisation of Hybrid Active-Passive Sandwich Composite Structures. Advances in Engineering Software. 46, 69-74 (2012)

[15] Moita, J.S., Araújo, A.L., Martins, P., Mota Soares, C.M., Mota Soares, C.A. A Finite Element Model for the Analysis of Viscoelastic Sandwich Structures. Computers & Structures. 89, 1874-1881 (2011)

[16] Moita, J.S., Araújo, A.L., Martins, P., Mota Soares, C.M., Mota Soares, C.A. Analysis of active-passive plate structures using a simple and efficient finite element model. Mechanics of Advanced Materials and Structures. 18, 159-169 (2011)

[17] Bilasse, M., Azrar, L., Daya, E.M. Complex modes based numerical analysis of viscoelastic sandwich plates vibrations. Computers & Structures. 89, 539-555 (2011)

[18] E. Carrera. Developments, ideas, and evaluations based upon reissner's mixed variational theorem in the modelling of multilayered plates and shells. *Applied Mechanics Reviews*, 54:301–329, 2001.

[19] E. Carrera. Evaluation of layer-wise mixed theories for laminated plate analysis. *AIAA Journal*, (36):830–839, 1998.

[20] Erasmo Carrera. Theories and finite elements for multilayered plates and shells: A unified compact formulation with numerical assessment and benchmarking. *Archives of Computational Methods in Engineering*, 10:215–296.

[21] S. Brischetto and E. Carrera. Advanced mixed theories for bending analysis of functionally graded plates. *Computers and Structures*, 88(23-24):1474 – 1483, 2010.

[22] S. Brischetto. Classical and mixed advanced models for sandwich plates embedding functionally graded cores. *J Mech Mater Struct*, 4:13–33, 2009.

[23] E. Carrera, S. Brischetto, and A. Robaldo. Variable kinematic model for the analysis of functionally graded material plates. *AIAA Journal*, 46:194–203, 2008.

[24] Murakami, H., 1985, Laminated composite plate theory with improved in-plane responses, ASME Proceedings of PVP Conference, New Orleans, June 24-26, PVP-Vol. 98-2, 257-263.

[25] Toledano A, and Murakami H, 1987a, A high-order laminated plate theory with improved in-plane responses, *International Journal of Solids and Structures* , vol 23, pp 111-131.

[26] Toledano A and Murakami H. 1987b, A composite plate theory for arbitrary laminate configurations, *Journal of Applied Mechanics* vol 54 , 181-189.

[27] Carrera E, 1995, A class of two-dimensional theories for anisotropic multilayered plates analysis , *Accademia delle Scienze di Torino*, Memorie Scienze Fisiche, 19-20 (1995-1996), pp 1-39.

[28] Rikards, R., Chate, A., Barkanov, E. Finite element analysis of damping the vibrations of laminated composites. Computers & Structures. 47, 1005-1015 (1993)

[29] Barkanov, E., Skukis, E., Petitjean, B. Characterisation of viscoelastic layers in sandwich panels via an inverse technique, Journal of Sound and Vibration. 327, 402-412 (2009)

[30] Alam, N., Asnani, N.T. Vibration and damping analysis of multilayered rectangular plates with constrained viscoelastic layers. Journal of Sound and Vibration. 97, 597-614 (1984)

[31] Sadasiva Rao, Y.V., Nakra, B.C. Vibrations of unsymmetrical sandwich beams and plates with viscoelastic cores. Journal of Sound and Vibration. 34, 309-326 (1974)

[32] Bischoff, M., Ramm, E. On the physical significance of higher order kinematic and static variables in a three-dimensional shell formulation. International Journal of Solids and Structures. 37, 6933-6960 (2000)

[33] Trindade M, Benjeddou A, Ohayon R. Modeling of frequency dependent viscoelastic materials for active-passive vibration damping. J Vibration Acoust 2000;122(2):169-74.

[34] Johnson C, Kienholz D. Finite element prediction of damping in structures with constrained viscoelastic layers. Am Inst Aeronaut Astronaut J 1982;20:1284Ð90.

[35] Duigou L, Daya EM, Potier-Ferry M. Iterative algorithms for nonlinear eigenvalue problems. Application to vibrations of viscoelastic shells. Comput Methods Appl Mech Eng 2003;192:1323Ð35.