

# Free vibration analysis of thin-walled structures through 1D and 2D refined models

Erasmus Carrera<sup>1</sup>, Maria Cinefra<sup>1</sup>, Marco Petrolo<sup>1</sup>, Enrico Zappino<sup>1</sup>

<sup>1</sup>*Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Italy*  
E-mail: *erasmo.carrera@polito.it, maria.cinefra@polito.it, marco.petrolo@polito.it, enrico.zappino@polito.it*

*Keywords:* Dynamics, Thin-Walled, Refined Theories, Beam, Shell.

Structural models can be built according to various approaches. The development of a structural model is aimed to the detection of the minimum number of unknown variables to be used to solve a given problem according to a fixed accuracy. In a so-called axiomatic approach and exploiting displacement variables, the unknowns are introduced by postulating an expansion for the three unknown displacements. The expansion can be made in many different ways. The generic displacement component  $u$  in an orthogonal reference system  $(x, y, z)$  can be expanded in terms of one-dimensional (1D) or two-dimensional (2D) variables, respectively

$$\begin{aligned} 1D: u &= u_i^{1D}(z) F_i^{1D}(x, y), & i &= 1, N_{1D}; \\ 2D: u &= u_i^{2D}(x, y) F_i^{2D}(z), & i &= 1, N_{2D} \end{aligned}$$

$u_i^{1D}$  and  $u_i^{2D}$  are the unknown variables,  $F_i^{1D}$  and  $F_i^{2D}$  are the base functions exploited for the 1D and 2D expansions. It is assumed that in the 1D case the expansion is made in terms of the two coordinates  $x, y$  (over a given section at a fixed  $z$ -value) while in the 2D case the expansion is made in terms of  $z$  (over the thickness at a fixed  $x, y$ ). The base functions can be represented by any polynomials, power of  $z$  (or  $x, y$  in the 1D case), harmonics, Lagrange, Legendre, etc. The Principle of Virtual Displacements can be used to derive the governing equations (in both weak and strong form).

The present paper compares 1D and 2D assumptions in the free vibration analysis of thin-walled structures. The values of  $N_{1D}$  and  $N_{2D}$  are free parameters, this means that classical beam/plate/shell theories can be refined at any extent. Finite Element FE approximations are used to deal with different boundary conditions (geometrical and mechanical). The Carrera Unified Formulation (CUF) is exploited to obtain the finite element matrices of 1D and 2D models. In CUF, these matrices are given in terms of the so-called fundamental nuclei [1-2]. Well established benchmark problems which are often used to assess shell problems are solved by implementing both 1D and 2D assumptions. Accuracy of both models is evaluated against their computational costs. Also, the adoption of different expansion polynomials is investigated.

## *References*

- [1] Carrera, E., Giunta, G. and Petrolo, M., *Beam Structures Classical and Advanced Theories*, Wiley, (2011).
- [2] Carrera, E., Cinefra, M., Petrolo, M. and Zappino E., *Finite Element Analysis of Structures by Unified Formulation*, Wiley, (In Press).