Full Aircraft Dynamic Response by Simplified Structural Models

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The finite element method (FEM) is widely used in the structural analysis of aircraft. In order to investigate the static and dynamic response of the whole structure different models are used: one-, two- and three- dimensional models. A new approach in the aircraft structural analysis is presented in this paper. A formulation for one- two- and three-dimensional higher order models is introduced by means of the Carrera Unified Formulation (CUF). The CUF allows to develop refined theories where the order of expansion is a free parameter of the problem. The FEM is used to solve the problem in order to deal with any geometries and boundary conditions. The present model (CUF+FEM) is used to investigate some typical aircraft structure first considering the aircraft as a single beam with a non-uniform cross-section, and then by considering the aircraft built by different beams, one for each part of the aircraft (fuselage, wing, etc...). Two- and three- dimensional elements are used as ‘node’ in order to overcome the problem related to the assembling of higher order beams. In this work a simplified aircraft model is investigated. The different models considered are compared in terms of natural frequencies. The results show that the present model is able describe properly the aircraft dynamic response, by reducing the degrees of freedom (DOFs) needed in the analysis respect to the commercial code.

Nomenclature

*C Material coefficients matrix
\( K^{ij\tau s} \) Stiffness matrix fundamental nucleus
\( M^{ij\tau s} \) Mass matrix fundamental nucleus
\( \delta \) Virtual variation
\( \epsilon \) Strains vector, [-]
\( \Lambda_x, \Lambda_y, \Lambda_z \) Rotation matrix around \( x, y \) and \( z \) axis
\( \rho \) Material density, [Kg/m\(^3\)]
\( \sigma \) Stresses vector, [Pa]
\( \theta, \phi, \xi \) Rotation angle around \( x, y \) and \( z \) axis
\( \zeta_p(i), \zeta_p(j) \) Eigenvectors
\( F_T, F_s \) Approximation polynomials
\( L_{ext} \) External work, [J]
\( L_{inc} \) Inertial work, [J]
\( L_{int} \) Internal work, [J]
\( N_i, N_j \) Shape function (FEM)
\( u \) Displacements vector, [m]
\( q_{ri}, q_{ls} \) Problem unknowns

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Introduction

The analysis of aeronautical structure is a challenging problem. These structures are characterised by a large number of components and so are very complex to investigate. In the past century many models have been proposed in order to analyse parts of this complex structures. In particular two class of theories have been introduced: one-dimensional model (rod/beam) and two-dimensional theories (plate/shell).

The most popular one-dimensional theories are the beam model proposed by Euler (EBBM),\(^1\) that take into account only bending phenomena, and the model introduced by Timoshenko (TBM)\(^2\) also called: first order shear deformation theory (FSDT). The classical one-dimensional theory have been improved in the last years by a large number of new approaches that try to overcome the limits introduced in the EBBM and TBM. Timoshenko\(^3\) and Sokolnikoff\(^4\) proposed to use a shear correction factor in order to take into account the cross-section shape in the shear distribution. El Fatmi \textit{et al.}\(^5,6\) introduced new terms in his formulation in order to deal with the cross-section warping. Berdichevsky \textit{et al.}\(^7\) introduced the asymptotic approach; this method was used by Volovoi\(^8\) and Yu\(^9\) to develop many new beam formulations where a characteristic parameter (e.g. the cross-section thickness for a beam) is exploited to build an asymptotic series. Schardt\(^10,11\) developed the Generalized Beam Theory (GBT) that was extensively used by Silvestre and Camotim\(^12\) in the thin-walled structures analysis.

A solution for the two-dimensional problem (plate) was first proposed by Kirchhoff, that considered only the bending effect, and after by Reissner-Mindlin that introduced the transverse shear deformation. 2D shell models may be classified in two classes according to what above. The Koiter\(^13\) model is based on the Kirchhoff hypothesis. The Naghdi\(^14\) model is based on the Reissner-Mindlin assumptions.

Both, the one- and the two- dimensional models, provide good results for the structural component but are not able to simulate complex structure such as a whole aircraft.

A possible approach in the analysis of complex structures was proposed by Argyris and Kelsey\(^15\) by referring to the so-called force method, see also Bruhn.\(^16\) These methods, even if based on some strong assumptions, were widely employed in the preliminary design of aeronautical structures.

Another modeling approaches are based on the coupling of shell and beam elements in order to properly describe the structural behavior of aircraft structures. Many notable works have been recently presented on this topic; the work by Assan\(^17\) on the static analysis and by Buermann \textit{et al.}\(^18\) on buckling are excellent examples. This approach provides better accuracies in the analysis but it is usually more expensive in terms of computational costs.

The present paper proposes a new approach in the analysis of complex structures such as reinforced thin-walled structures and aircraft. A unified formulation for one-, two- and three-dimensional higher order models is introduced by using the Carrera Unified Formulation (CUF). CUF 2D models have been developed in the last two decades by Carrera\(^19\) and his co-workers.\(^20\) CUF 1D models have been recently developed\(^21,22\) and two classes of models have been proposed, the Taylor-expansion class (TE) and the Lagrange-expansion class (LE). TE models exploit \(N − \text{order}\) Taylor-like polynomials to define the displacement field above the cross-section with \(N\) as a free parameter of the formulation. Static\(^21,24,25\) and free-vibration analyses\(^26,27\) showed the strength of CUF 1D models in dealing with arbitrary geometries, thin-walled structures and local effects. Moreover, asymptotic-like analyses leading to reduced refined models were carried out.\(^20\) The LE class is based on Lagrange-like polynomials to discretize the cross-section displacement field. LE models have only pure displacement variables. Static analyses on isotropic\(^28\) and composite structures\(^31\) revealed the strength of LE models in dealing with open cross-sections, arbitrary boundary conditions and obtaining Layer-Wise descriptions of the 1D model.

In this paper some new improvements have been introduced with respect the past works in order to address the problem of complex structures. A three-dimensional formulation based on the Carrera unified formulation is presented. Two different approaches are used in the analysis of aircraft structures: a singular non-uniform cross-section beam or multiple beam with non-uniform cross-section.

In fig.1a is depicted the first approach that involved a single beam with a non-uniform cross section. In fig.1b is shown the second method, different beams are used to describe the different parts of the aircraft, they must be rotate and can be merged during the matrix assembling phase in different way: the first is to introduce new relation between the structural nodes, the second is to use two- or three-dimensional elements as node where the different beam are connected, a three-dimensional model is presented in terms of unified formulation. Both the method are employed and compared.
In the first part of this paper the models are described from the theoretical point of view. In the second the dynamic analysis of a simplified aircraft structure is performed by using different approaches.

I. Structural Unified Formulation

The whole theoretical model refers to the following notation. The transposed displacement vector is defined as:

\[ \mathbf{u}(x, y, z) = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}^T \]  

(1)

where \( x, y, \) and \( z \) are orthonormal axes as shown in Fig. 2. Stress, \( \sigma \), and strain, \( \epsilon \), components are grouped as follows:

\[ \sigma = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix}^T, \epsilon = \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{xz} \\ \epsilon_{yz} \end{bmatrix}^T \]  

(2)

Linear strain-displacement relations are used,

\[ \epsilon = D \mathbf{u} \]  

(3)

Where

\[ D = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix} \]  

(4)
Constitutive laws are exploited to obtain stress components,

\[ \sigma = C \epsilon \] (5)

The components of \( C \) are the material coefficients whose explicit expressions are not reported here for the sake of brevity, they can be found in.\(^3\)

### A. Displacement models

The generic three-dimensional displacements model can be written as follow:

\[ \mathbf{u}(x, y, z) = \mathbf{f}(x, y, z; t) \] (6)

The three-dimensional formulation can be reduced to the two- and one-dimensional formulation by introducing the function \( F_\tau \). This function introduce an expansion in the thickens of the structure (2D formulation) or in the cross-section (1D formulation).

\[
\begin{align*}
3D \rightarrow u(x, y, z) &= f(x, y, z; t)F_\tau^{3D}, \quad \tau = 1, 2, \ldots, N \\
2D \rightarrow u(x, y, z) &= f(x, y; t)F_\tau^{2D}(z), \quad \tau = 1, 2, \ldots, N \\
1D \rightarrow u(x, y, z) &= f(y; t)F_\tau^{1D}(x, z), \quad \tau = 1, 2, \ldots, N
\end{align*}
\] (7)

In eq. 7 the function \( f_\tau \) has been introduced also in the 3D formulation in order to have the same notation for all the models, in this case the function \( f_\tau^{3D} \) can be considered equal to 1.

In the formulation of \( F_\tau^{2D} \) and \( F_\tau^{1D} \) can be used different function. A complete overview of the function used in the 1D formulation can be found in the book by Carrea et al.\(^2\) In this work two different expansion are used. The first one based on the Taylor formulation (TE), the second one based on the Lagrange formulation\(^3\) were the unknowns are the displacements only. The formulation of the 2D expansion\(^4\) can be derived with a Equivalent Single Layer (ESL) approach or in a Layer Wise (LW) formulation. In this work the second approach, layer wise formulation, is used.

### B. FE Formulation and the Fundamental Nucleus

The model derived by the introduction of the \( F_n^{n-D} \) can be solved by using different approaches. In this work the FEM approach is used. By introducing the shape functions, \( N_i \), the eq. 7 can be written in the following formulation:

\[
\begin{align*}
3D \rightarrow u(x, y, z) &= q_\tau(t)N_i(x, y, z)F_\tau^{3D}, \quad i = 1, 2, \ldots, K \\
2D \rightarrow u(x, y, z) &= q_\tau(t)N_i(x, y)F_\tau^{2D}(z), \quad i = 1, 2, \ldots, K \\
1D \rightarrow u(x, y, z) &= q_\tau(t)N_i(y)F_\tau^{1D}(x, z), \quad i = 1, 2, \ldots, K
\end{align*}
\] (8)

Where the function \( N_i \) are the Lagrange function and \( K \) is the number of node of the element used. The stiffness and mass matrices can be derived in the frameworks of the CUF formulation introduced by Carrera\(^2\) that allow to derive a unified formulation where \( K \) and \( N \) can be considered as an input of the problem. The matrices are obtained via the Principle of Virtual Displacements,

\[ \delta L_{\text{int}} = \delta L_{\text{ext}} \] (9)

Where \( L_{\text{int}} \) stands for the strain energy, and \( L_{\text{ext}} \) is the work of the external loadings. \( \delta \) stands for the virtual variation. The virtual variation of the strain energy is given by:

\[
\delta L_{\text{int}} = \int_V (\delta \mathbf{e}^T \sigma) dV = \int_V \delta q_j^T \left[ D^T \left( N_i F_\tau I \right) \right] C \left[ D \left( N_i F_\tau I \right) \right] q_i dV
\] (10)
The variation of the internal work is then written by means of the CUF fundamental nucleus,

\[
\delta L_{\text{int}} = \delta q_i^T \int_V \left[ N_j F_s I \right] \begin{bmatrix} [D]^T & [C] & [D] \end{bmatrix} \begin{bmatrix} 3 \times 6 & 6 \times 6 & 6 \times 3 \end{bmatrix} I F_r N_i dV q_i \tau \tag{11}
\]

\[
\delta L_{\text{int}} = \delta q_i^T K^{ij\tau \sigma} q_i \tau \tag{12}
\]

Where \( K^{ij\tau \sigma} \) is the stiffness matrix in the form of the fundamental nucleus. The explicit forms of the 9 components of \( K^{ij\tau \sigma} \) are not reported here, they can be found in literature.\(^{13}\)

The mass matrix can be derived in the same way by considering the inertial energy expressed by:

\[
\delta L_{\text{ine}} = \int_V \left( \rho \dddot{u} \delta u^T \right) dV \tag{13}
\]

where \( \rho \) stands for the density of the material, and \( \dddot{u} \) is the acceleration vector. Eq. 13 is rewritten using Eq.s 3:

\[
\delta L_{\text{ine}} = \int_V \delta q_j^T N_i \rho (F_s I)(F_s I) N_j \dddot{q}_i \tau dV \tag{14}
\]

where \( \dddot{q} \) is the nodal acceleration vector. The last equation can be rewritten in the following compact manner:

\[
\delta L_{\text{ine}} = \delta q_j^T M^{ij\tau \sigma} \dddot{q}_i \tag{15}
\]

where \( M^{ij\tau \sigma} \) is the mass matrix in the form of the fundamental nucleus. No assumptions on the approximation order have been done to obtain the fundamental nucleus. It is therefore possible to obtain refined 1D models without changing the formal expression of the nucleus components. This is the key-point of CUF which permits, with only nine FORTRAN statements, to implement any-order one-dimensional theories.

C. Fundamental Nucleus rotation

The formulation introduced can be generalized for elements arbitrary oriented in the space. The rotation matrix around the 3 axis can be written as:

\[
\Lambda_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}, \quad \Lambda_y = \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{bmatrix}, \quad \Lambda_z = \begin{bmatrix} \cos(\xi) & -\sin(\xi) & 0 \\ \sin(\xi) & \cos(\xi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{16}
\]

where \( \theta, \phi \) and \( \xi \) are the rotation around the axis: \( x,y, \) and \( z \). As showd in 3. The complete rotation matrix can be written as:

\[
[A] = [\Lambda_x][\Lambda_y][\Lambda_z] \tag{17}
\]

the generic displacement vector \( u \) expressed in the local system of reference, \( u_{\text{loc}} \), can be expressed in a global system of reference, \( u_{\text{glob}} \), by using the formulation:

\[
u_{\text{glob}} = [A]u_{\text{loc}} \tag{18}\]
The fundamental nucleus expressed in the local system, \([K_{ijτs}]_{loc}\), of reference so can be turn in the global system of reference, \([K_{ijτs}]_{glob}\), as show in the following formulation:

\[
[K_{ijτs}]_{glob} = [Λ][K_{ijτs}]_{loc}[Λ]^T
\]  

(19)

D. Assembling approach

The different models one-, two-, and three-dimensional can be assembled very easily because all the formulations (except the one based on Taylor formulation) are expressed in terms of displacements. In fig. 4 is shown as 2 different element can be linked. The assembling can be done by adding the stiffness of the nodes that must be linked. To use this approach the different matrices must be expressed in the same system of reference.

II. Results

In this section the results for different structure are presented. The first example are devoted to the assessment of the model for simple structure. A more complex structure is presented at the end in order to show the potentialities of the present model.
A. Analysis of transversal ribs by 1D CUF elements

Ribs could be considered as a 'very low' aspect ratio one-dimensional structure. The static response of a single rib is investigated in this section to assess the present model for the subsequent analysis. The geometry of the structure is shown in Fig. 5. The length, \( L \), is equal to 0.2 [m]; the radius, \( R \), of the circular cross-section is 1[m]. The structure is clamped in the centre (C) along the whole y-axis. The structure is loaded with a uniform pressure with a magnitude of 100 [Pa] acting along the radial direction above the whole y-axis and in the intervals: \( \frac{3}{8} \pi < \theta < \frac{5}{8} \pi \) and \( \frac{11}{8} \pi < \theta < \frac{13}{8} \pi \). The results are compared with a solid model from MSC NASTRAN®. The material considered is aluminium.

<table>
<thead>
<tr>
<th>Order</th>
<th>DOFs</th>
<th>Displacements</th>
<th>Stress</th>
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<td></td>
<td></td>
<td>( u_{xz} \times E - 8 )</td>
<td>( u_{y_{cr}} \times E - 10 )</td>
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<tr>
<td>N=1</td>
<td>27</td>
<td>-0.5789</td>
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<tr>
<td>Solid</td>
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<td>-1.1498</td>
<td>0.8184</td>
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</table>

Table 1. Displacements and stress of the 'very low' aspect ratio beam evaluated in different points by means of higher order beam model and a solid model.

In Tab.1 the results are presented in terms of displacements at different points: \( a[R,L/2,R] \), \( a'[R,L,R] \) and \( b[R,L/2,0] \); stress values are evaluated at the points \( a[R,L/2,R] \). The displacements related to the sixth order model at \( y = L/2 \) are depicted in Fig.6. Some stress distributions are reported in Fig.7. The results show that the present model is able to properly predict the displacements and the stress of a 'very low' aspect ratio beam. The results clearly show the capability of the present higher-order model to detect 3D responses of the thin ribs considered. At least a fifth-order model is needed to obtain good accuracy in the results. It is concluded that the present model is able to describe properly the transverse stiffeners.

B. Variable kinematics model assessment

The dynamic response of a simplified model of aircraft is considered in order to investigate the capabilities of the model to combine different structural models. The geometry of the structure is depicted in fig.8. The geometry is function of the parameter \( a \) that is considered equal to 0.5. The structure has a constant thickness of \( 0.2 \times a \). No boundary condition are applied, so the structure is completely free. The material used is aluminium. Different approach have been considered:

Figure 5. 'Very low' aspect ratio beam model.
Singular beam non-uniform section approach:
- Fourth order 1D model (TE) (fig 13a)
- Fifth order 1D model (TE) (fig 13a)
- 1D LE model whit not oriented beam (fig 13b)

multi beam approach:
- 1D LE model whit oriented beam (fig 13c)
- 1D LE model whit oriented beam + 2D Shell element (fig 13d)
- 1D LE model whit oriented beam + 3D element (fig 13e)

The results in terms of natural frequencies are reported in tab. 2. A 2D Nastran Model is used as comparisons. The Degrees of Freedom of each model are reported in the table.

The results show that the analysis of a simplified aircraft structure may be carried out by means of higher order one-dimensional models. All the 1D model based on Lagrange Expansion (LE) are able to detect the first ten frequencies with a good accuracy. The introduction of 2D elements and 3D elements based on CUF formulation allow to connect higher order 1D model generally oriented in the space so make this approach suitable for complex structure.
C. Sweep and Dihedral angle effects

In this section is used the same model presented in the previews one. The approach presented in Figure 13c is adopted. Two new parameters are introduced, the sweep and the dihedral angles are introduced in order to increase the complexity of the problem as shown in Figure 10.

Three case are considered: the base model, a model with a sweep angle (ξ) equal to 20 degrees and the model with the dihedral angle (ϕ) equal to 10. The results are compared with those from the two-dimensional model used in the section above.

Table 3 shows the first ten natural frequencies of the different models. The results are compared with those from the commercial code Nastran, in superscript is reported the percentage difference between the two results. The frequency evaluated using the present approach appear to be very close to the reference. With error that are between 1% and 5%, only the sixth mode shows a error higher than 10%.

In Figure 12 are reported the first five modes for the three models considered. In order to compare the results with those form Nastran in terms of modal shape the Modal assurance criterion (M.A.C.) is used. The M.A.C. can be defined as:

\[
M.A.C. = \frac{[\zeta_p(i)^T \zeta_r(j)]^2}{[\zeta_p(i)^T \zeta_p(i)][\zeta_r(j)^T \zeta_r(j)]}
\]  

where \(\zeta_p(i)\) is the \(i-th\) eigenvector from the present model and \(\zeta_r(j)\) is the \(j-th\) eigenvector from the reference model. The M.A.C. value is in between 0 and 1. Closer to one is the M.A.C., more similar
<table>
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<th>TE4</th>
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<td>V</td>
<td>73.24</td>
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<td>X</td>
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<td>88.59</td>
<td>88.59</td>
<td>87.44</td>
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Table 2. First ten natural frequencies of the simplified aircraft model.

![Figure 10. Sweep angle and dihedral angles definition.](image)

(a) Base model  
(b) Sweep angle  
(c) Dihedral angle

![Figure 11. Results of the M.A.C. analysis.](image)

(a) $\varphi = 0; \xi = 0$  
(b) $\varphi = 0; \xi = 20$  
(c) $\varphi = 10; \xi = 0$  

Figure 11. Results of the M.A.C. analysis. On $x$ axis there are the frequencies of the first ten modes of the present model, on $y$ axis the reference frequency. White means zero, black means one.
\[ \varphi = 0; \xi = 0 \]
\[ \varphi = 0; \xi = 20 \]
\[ \varphi = 10; \xi = 0 \]

Mode  1D-LE  2D-Nastran  1D-LE  2D-Nastran  1D-LE  2D-Nastran
I    10.931.1%  10.80     10.794.0%  10.47   11.141.6%  10.96
II   18.175.2%  17.26     18.654.9%  17.77   18.384.1%  17.66
III  20.200.7%  20.05     20.094.7%  19.19   20.051.6%  19.73
IV   50.550.4%  50.37     49.120.1%  49.22   44.729.9%  43.45
V    52.983.3%  52.31     56.3921.8%  56.28   59.518.9%  50.05
VI   67.482.9%  65.55     70.166.6%  65.83   66.971.3%  66.11
VII  71.782.6%  69.94     72.214.7%  68.95   74.291.4%  73.25
VIII 76.601.1%  75.77     78.321.3%  73.69   78.912.2%  77.95
IX   88.551.3%  87.44     89.140.5%  84.46   89.045.0%  84.80

Table 3. First ten natural frequencies of the simplified aircraft model for different configuration. In superscript is presented the percentage error with respect the Nastran solution.

are the modal shapes. The results are reported in Figure 11. The results fo the base model show a very good correlation for the first ten modes. The model with the sweep angle equal to 20 degrees show a good agreement except for the sixth modes, the same that show a discrepancy in the frequency. The model with the dihedral angle equal to 10 degrees show a very good correlation except for a switch between the fifth and the sixth modes. In conclusion the results show that the present model can be used with structure with an arbitrary rotation.

D. Typical aircraft structure

In this section a typical aircraft structure is considered. The structure is shown in Figure 13. It represents a part of fuselage with the wing connection. Ribs, longeron and thin skin are present in the same structure. The main dimensions of the structure are reported in Table 4.

<table>
<thead>
<tr>
<th>Dimensions [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a= 3.000</td>
</tr>
<tr>
<td>b= 3.160</td>
</tr>
<tr>
<td>c= 6.000</td>
</tr>
<tr>
<td>d= 0.040</td>
</tr>
<tr>
<td>e= 1.080</td>
</tr>
<tr>
<td>f= 0.002</td>
</tr>
<tr>
<td>g = 0.224</td>
</tr>
<tr>
<td>h = 0.080</td>
</tr>
<tr>
<td>i = 0.035</td>
</tr>
</tbody>
</table>

Table 4. First ten natural frequencies of the simplified aircraft model.

All the structure is considered built in aluminium alloy with a Young modulus equal to 71.7 [MPa], the poisson ration equal to 0.3 and a density of 2700[Kg/m³].

The structure is considered symmetric, so for \( y = 0 \) and \( y = b \) are not allowed the displacement in \( y \) direction, for \( x = 0 \) are not allowed the displacement in \( x \), the displacement in \( z \) are free, so the solution will provide a rigid-body mode.

The whole structure is solved using a one-dimensional model based on Lagrange expansion. The fuselage is considered as a beam axis in the \( y \) direction and a variable cross-section in order to consider the ribs. Also the wing is considered as a beam with the axis in the \( x \) direction. The structure are joined in the shared nodes. The complete model has 19056 degrees of freedom.

The results in terms of frequency are reported in 5, the rigid-body mode is not considered. Some modal shape are reported in figure 14. The results are compared with those from the commercial code Nastran evaluated by means of a solid element model with about 30000 DOF.

The results show as the present model is able to predict pretty well the complex dynamic behaviour this structure. Many frequencies show a good agreement with those from the commercial code. The modal shapes in Figure 14 show that the model can predict very complex deformation that usually are not predicted by classical beam element.
Figure 12. First five modal shapes for the different model considered.
III. Conclusion

In the present work the dynamic response of aircraft structure is investigated. A unified approach based on Carrera Unified Formulation is introduced in order to easily handle different structural models: one-, two- and three-dimensional structural theories are used. In the first part of the article the different models have been assessed by solving different problems. In the second part of the paper a typical aircraft structure is investigated in order to show the potentiality of the present approach.

From the results it is possible to see that, the introduction of higher order models with only displacements as unknowns allows to connect different models very easily and with very good results. In this work the one-dimensional models, that are very attractive in the aerospace engineering because of the slender shape of many structures, have been coupled with two- and three- dimensional models that allow to describe properly more complex structures. The results obtained for the simplified aircraft show a very good correlation with those from commercial codes.

The same approach was used in the analysis of a more complex structure. The results show that the present model provides a solution that can be compared with those from a 3D model.

In conclusion the present approach allows to use simplified structural model to predict the dynamic response of complex structure.
Figure 14. Five modal shape of the typical aircraft structure.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Present (1D)</th>
<th>Nastran (3D)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>19065 DOF</td>
<td>~30000 DOF</td>
</tr>
<tr>
<td>I</td>
<td>21.96</td>
<td>22.02</td>
</tr>
<tr>
<td>II</td>
<td>23.65</td>
<td>24.22</td>
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<td>III</td>
<td>32.78</td>
<td>28.05</td>
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<td>IV</td>
<td>35.63</td>
<td>-</td>
</tr>
<tr>
<td>V</td>
<td>59.99</td>
<td>48.95</td>
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<td>VI</td>
<td>71.50</td>
<td>66.26</td>
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<tr>
<td>VII</td>
<td>86.60</td>
<td>-</td>
</tr>
<tr>
<td>VIII</td>
<td>95.82</td>
<td>-</td>
</tr>
<tr>
<td>IX</td>
<td>101.27</td>
<td>90.81</td>
</tr>
</tbody>
</table>

Table 5. First ten natural frequencies of the typical aircraft structure.
References