Effects of Thermo-Mechanical Loads on the Aeroelastic Instabilities of Metallic and Composite Panels

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Abstract

Panel flutter phenomena can be strongly affected by thermal loads. In this paper a refined aeroelastic model is presented. An higher-order shell theory is used in the structural model. The aerodynamic forces are described using the Piston Theory. Composite and sandwich structure are considered and different boundary condition are take into account. The effects of the thermal load on the aeorelastic behaviour are investigated.

1 Introduction

Panel flutter is an aeroelastic phenomena that can cause failure of panels of wings, fuselages, missiles. The panel flutter phenomena involves mainly the aeronautic structures but it appears also on space structure during the coasting phase. The new launchers generations try to improve the performances by introducing new panels, they must protect the cryogenics stage during the coasting phase. These panels, called Versatile Thermal Insulation (VTI panel), are bigger than the common aeronautical panels and usually are connected with the main structure by means of pinched points. The dimension, the boundary conditions (BC) and the weight requirements make the VTI panels very flexible and so they may easily occur in aeroelastic phenomena. The aerodynamic heating on the external surface and the cryogenic fluid on the inner surface, create a high thermal gradient along the thickness of the panel. The stress field due to the differential thermal loads could affect strongly the dynamic behaviour of the panel and can plays an important role in the aeroelastic instability, as shown by Dixon et al.[1].

In this paper an aero-thermo-mechanical analysis is performed by using a refined shell theories [2] for the structural model including the thermal effects, and the Piston theory [3] in its linear formulation for the aerodynamic loads. A cylindrical shell finite element based on CUF [4] is adopted. The higher-order models derived by means of the CUF approach allow to address the thermo-mechanical problem with a very good accuracy Different material laminations are considered: isotropic, composite, and sandwich material. Only supersonic regimes are investigated. The results show that the thermal loads can afflict the aeroelastic behavior of the panel. The results show also the effect of the use of the refined shell elements respect to the classical one. The advantages of these models are pointed out mainly in the composite and sandwich panels.

2 Aeroelastic model

The aeroelastic model used in the present work can be expressed in terms of equilibrium of the works virtual variations. From the Principle of Virtual Displacement (PVD) it is possible to write:

$$\delta L_{int} + \delta L_{ine} = \delta L_a + \delta L_{heat} \tag{1}$$

where L_{int} is the work due to the elastic forces, L_{ine} is the inertial work, L_a is the work made by the aerodynamic forces and L_{heat} is the thermal work. δ denotes the virtual variation. The solution via FEM introduce the matrix formulation, if the solution of the dynamic problem is supposed to be harmonic eq.1 becomes:

$$[-\omega^2[\boldsymbol{M}] + i\omega[\boldsymbol{D}_a] + ([\boldsymbol{K}] + [\boldsymbol{K}_a] + [\boldsymbol{K}_{heat}])]\{\bar{q}\}e^{i\omega t} = 0 \tag{2}$$

From left to right, you can see the Mass Matrix of the structure, the Aerodynamic Dumping matrix, the structural Stiffness matrix, the Aerodynamic Dumping matrix and finally, the Thermal Stress Matrix. The matrices are derived in terms of $fundamental\ nuclei$, a 3×3 matrix that is independent of the used model.

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The Carrera Unified Formulation approach is used both for structural and aerodynamic matrices, more details on CUF can be found in [4]).

2.1 Unified formulation

The generic three dimensional displacements model can be written as follow:

$$\mathbf{u}(x,y,z) = f(x,y,z;t) \tag{3}$$

The three dimensional formulation can be reduced to the two-dimensional formulation by introducing the function F_τ. This function introduce an expansion in the thickness of the structure (2D-SHELL formulation).

$$u(x,y,z) = f(x,y;t)F_\tau(z), \qquad \tau = 1,2,....,N \tag{4}$$

In the formulation of F_τ can be used different polynomials. The formulation of the 2D expansion can be derived with a Equivalent Single Layer (ESL) approach or in a Layer Wise (LW) formulation (see [4]). The FEM approach is used to solve the present problem. By introducing the shape functions, N_i, the eq.4 can be written in the following formulation:

$$u(x,y,z) = q_{i\tau}(t)N_i(x,y)F_\tau(z), \qquad i = 1,2,....,K \tag{5}$$

Where the function N_i are the Lagrange function and K is the number of node of the element used.

2.1.1 Elastic Work

The elastic work can be derived by the classical formulation of stress and strain:

$$\boldsymbol{\epsilon} = [\boldsymbol{D}]\mathbf{u}; \quad \boldsymbol{\sigma} = [\boldsymbol{C}]\boldsymbol{\epsilon} \tag{6}$$

The expression of $[\boldsymbol{D}]$ can be found in [4]. The components of $[\boldsymbol{C}]$ are the material coefficients whose explicit expressions are not reported here for the sake of brevity, they can be found in [5]. The internal work can be written as follow:

$$\delta L_{int} = \int_V (\delta\boldsymbol{\epsilon}^T\boldsymbol{\sigma})dV = \delta\mathbf{q}_{\tau i}^T\Big[\int_V \Big[\boldsymbol{D}^T\Big(N_iF_\tau\boldsymbol{I}\Big)\Big]\boldsymbol{C}\Big[\boldsymbol{D}\Big(N_jF_s\boldsymbol{I}\Big)\Big]\mathbf{d}V\Big]q_{sj} = \delta\mathbf{q}_{\tau i}^T\big[\boldsymbol{K^{ij\tau s}}\big]q_{sj} \tag{7}$$

2.1.2 Inertial Work

The mass matrix formulation derives from the variation of the work made by the inertial forces:

$$\delta L_{ine} = \int_V \delta\mathbf{u}\ddot{\mathbf{u}}\rho dV = \delta\mathbf{q}_{i\tau}^T\Big[\int_V \Big(F_\tau\boldsymbol{I}N_iN_jF_s\boldsymbol{I}\rho\Big)dV\Big]\ddot{\mathbf{q}}_{sj} = \delta\mathbf{q}_{i\tau}^T\big[\mathbf{M}^{ij\tau s}\big]\ddot{\mathbf{q}}_{sj}$$

A dot denotes derivatives with respect to time double dots denotes acceleration.

2.1.3 Thermal Work

The strain and the stress field related to the thermal load is assumed as:

$$\epsilon^h = \alpha_p\Delta T; \qquad \sigma^h = \boldsymbol{C}_p\alpha_p\Delta T = \boldsymbol{\lambda}_p\Delta T \tag{8}$$

where $\boldsymbol{\alpha_p}$ contains the in-plane thermal expansion coefficients along x, y and xy direction, ΔT is a vector (3×1) containing the temperature gradient and $\boldsymbol{C_p}$ is the in-plane material matrix. In order to obtain the fundamental nuclei, has been used the Von Karman non linear differential matrix:

$$\begin{aligned}\delta L_{heat} = \int_V (\delta\boldsymbol{\epsilon_{xx}^{nl}}^T\boldsymbol{\sigma_{xx}^h} + \delta\boldsymbol{\epsilon_{yy}^{nl}}^T\boldsymbol{\sigma_{yy}^h} + \delta\boldsymbol{\epsilon_{xy}^{nl}}^T\boldsymbol{\sigma_{xy}^h})dV = \delta\mathbf{q}_{i\tau}^T\Big[\int_V\Big[\big((F_\tau N_{i,x}N_{j,x}F_s\big)\boldsymbol{\sigma_{xx}^h} + \\ + \big((F_\tau N_{i,y}N_{j,y}F_s\big)\boldsymbol{\sigma_{yy}^h} + \big((F_\tau N_{i,x}N_{j,y}F_s\big)\boldsymbol{\sigma_{xy}^h}\Big]dV\Big]\mathbf{q}_{sj} = \delta\mathbf{q}_{i\tau}^T\big[\mathbf{K}_{heat}^{ij\tau s}\big]\mathbf{q}_{sj}\end{aligned} \tag{9}$$

For thin structures, the fundamental nuclei of Thermal Stress matrix (3×3) has only the third diagonal element different from zero. The complete formulation can be found in [6].

2.1.4 Aerodynamic Work

The aerodynamic forces are described using the approximate model called Piston Theory. Piston Theory was introduced for the first time by *Ashley and Zartarian* [3], it results easier to implement and solve but it is valid only starting from Mach 1.5 and correctly predicts occurrences of coupled mode flutter.

Piston Theory assumes the pressure distribution as:

$$\Delta p(y,t) = \frac{2q}{\sqrt{M^2-1}}\left(\frac{\partial u_z}{\partial x} + \frac{M^2-2}{M^2-1}\frac{1}{V}\frac{\partial u_z}{\partial t}\right) = A\frac{\partial u_z}{\partial x} + B\frac{\partial u_z}{\partial t} \tag{10}$$

$$\delta L_a = \int_{\Lambda} (\delta \mathbf{u} \Delta p)\, d\Lambda = \delta q_{i\tau}^T \left[\boldsymbol{K}_a^{ij\tau s}\right] q_{sj} + \delta q_{i\tau}^T \left[\boldsymbol{D}_a^{ij\tau s}\right] \dot{q}_{sj} \tag{11}$$

The complete formulation has not been reported for sake of brevity.

3 Results

3.1 Thermo-mechanical model validation

In order to validate the thermo-mechanical solver, were chosen results concerning a 5 layer composite panel with stacking sequence $[\theta/-\theta/\theta/-\theta/\theta]$. The reference test case adopted a Ritz/Galerkin approach for evaluating the thermal buckling load, varying the fibers orientation [6]. The panel has the following characteristics:a=1 m, b=1 m, h=0.01 m, E_1=40 $\times 10^9$ Pa, E_2=E_3=1 $\times 10^9$ Pa, G_{12}=G_{13}=0.6 $\times 10^9$ Pa, G_{23}=0.5 $\times 10^9$ Pa, ν_{12}=ν_{13}=ν_{23}=0.25, α_1=2 $\times$ 10^{-8} K^{-1}, α_2=2.25 $\times$ 10^{-5} K^{-1}. Structural theory used was Layer Wise of second order, 3042 Dofs. As expected, in correspondence of θ= 0 or 90 deg, both results obtained using Galerkin and FEM approach, are in very good agreement. Varying the fiber orientation, FEM formulation provides more conservative results. Its known that Ritz/Galerkin-based solvers, are less capable to correctly implement the BCs for angle-ply thin structures. Figure 1 shows critical temperature trends with both the approaches:

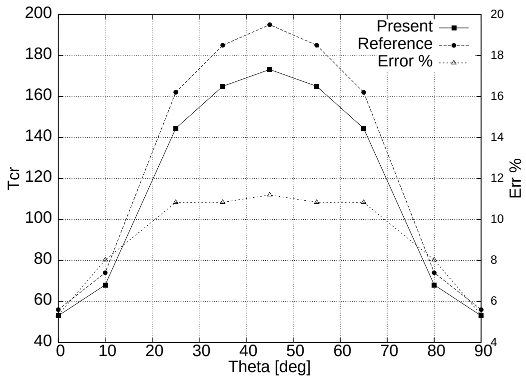


Figure 1: Critical temperature vs fibers orientation angle

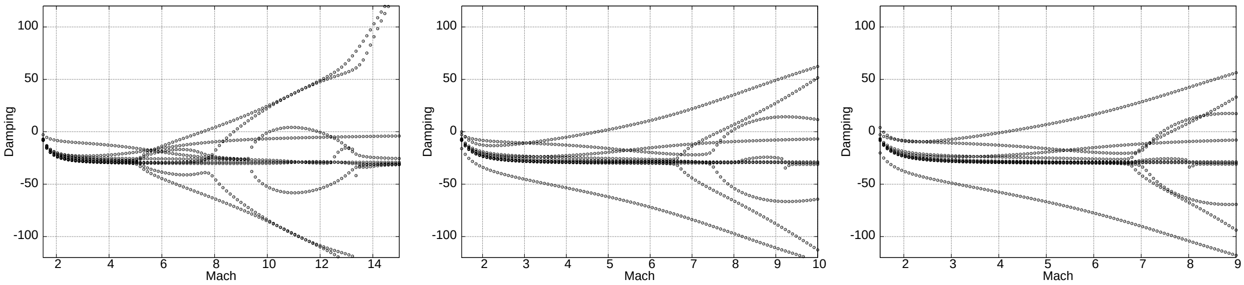
3.2 Thermal stress on VTI panel configuration

Was considered 1/3 of cylinder arc length VTI sandwich panel. Characteristics of this configuration are: a=1.5 m, b=3.12 $[m]$, h_{tot}=0.012 $[m]$, h_{l-skin}=0.0005 $[m]$, h_{core}=0.01 $[m]$, R=1.49 $[m]$, Core - E = 5.4 $\times$ 10^7 Pa, G = 2.3 $\times$ 10^7 Pa, ν = 0.1739, ρ = 80 Kg/m^3, α = 10^{-6} K^{-1}, Skins - E_{11}=85.0 $\times$ 10^9 Pa, E_{22}=E_{33}=1.5 $\times$ 10^9 Pa, G_{12}=G_{13}=1.6 $\times$ 10^9 Pa, G_{23}=1.8 $\times$ 10^9 Pa, ν_{12}=ν_{13}=0.3, ν_{23}=0.45, α_1=-0.9 $\times$ 10^{-6} K^{-1}, α_2=27.0 $\times$ 10^{-6} K^{-1}. For this analysis was used a Layer Wise theory of the second order, at which corresponds 2754 Dofs. The panel is simply supported along the sides in the spanwise direction. The combined effects of curvature, thickness, and sandwich configuration, provides an elevated momentum of inertia of the panel. So, this involves a very high critical temperature, for thermal buckling. Results obtained indicate T_{cr} = 17080 K. This temperature is obviously unrealistic for any current application. Hence, were chosen lower temperature. The following table shows for a reference density of $\rho = 0.8$ Kg/m^3, the trend of critical Mach number in correspondence of increasing thermal stress.

As you can see, operative temperatures haven't significant effects on the aeroelastic behavior in terms of stability boundary. Applying higher thermal stress, expressed as a fraction of the critical temperature, results notified a relevant reduction of the stability margin. Figures 2 show the modal damping trend for increasing temperature:

ΔT $[K]$	0.0	300	400	500
M_{cr}	7.55	7.54	7.54	7.54
f_{cr} $[Hz]$	121.15	120.89	120.71	120.63
$\Delta M_{cr}\%$	//	0.13	0.13	0.13

Table 1: Critical Mach number vs panel temperature $\rho = 0.8$ Kg/m^3, simply supported VTI panel.



(a) $\frac{\Delta T}{\Delta T_{cr}} = 0.0$ (b) $\frac{\Delta T}{\Delta T_{cr}} = 0.3$ (c) $\frac{\Delta T}{\Delta T_{cr}} = 0.4$

Figure 2: Modal damping trend for simply supported panel, with $\rho = 0.8$ Kg/m^3 and increasing thermal load

	$\frac{\Delta T}{\Delta T_{cr}}$	0.0	0.3	0.4	0.5
PP	M_{cr}	7.55	4.76	4.15	3.69
	f_{cr} $[Hz]$	121.33	96.40	94.05	91.25
	$\Delta M\%$	//	36.95	45.03	51.12

Table 2: Flutter critical Mach vs Thermal load, $\rho = 0.8$ Kg/m^3, VTI ss panel

4 Conclusion

The present aeroelastic model shows good agreement with the reference results. The higher-order formulation allows to investigate sandwich and composite material. The tests carried out for the VTI panel, shown the influence of the thermal loads on the flutter boundary. Typical operative conditions, with temperature in the operational range of the materials, entail a minimal reduction of the critical Mach, expressed in percentage, compared to the un-stressed condition. Future developments will tend to implement a thermal theory which could represents thermal gradients along the thickness and correctly predicts the in-plane stress related to pinched points constraints.

References

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