The zig-zag and warping effects on buckling of sandwich plates with functionally graded skins, by sinusoidal shear deformation theories

Introduction
In this work we study the influence of zig-zag and warping effects in the buckling of sandwich plates with functionally graded skins. The sandwich plate is subjected to compressive in-plane forces acting on the mid-plane of the plate.

Methodology

Displacement field of the sinusoidal shear deformation theories

\[
\begin{align*}
\sinus: & \\
\sinus ZZ: & \\
\sinus ZZ0: & \\
\end{align*}
\]

Material properties

The power-law function is used to describe the volume fraction of the metal \(V_m\) and ceramic \(V_c\) phases and the material homogenization technique adopted to the Young’s modulus is the law of mixtures:

\[
E(z) = E_m V_m + E_c V_c \quad \text{with} \quad V_c = \begin{cases} 
\frac{z-h_0}{h_1-h_0}, & z \in [h_0, h_1] \\
1, & z \in [h_1, h_2] \\
\frac{h_3-z}{h_3-h_2}, & z \in [h_2, h_3]
\end{cases}
\]

Carrera’s Unified Formulation

The governing equations and boundary conditions are derived under a generalization of Carrera’s Unified Formulation (CUF) [1] based on the principle of virtual displacements. Although the sandwich presents 3 physical layers, we consider \(N = 91\) virtual (mathematical) layers of constant thickness.

Meshless method

The governing equations are interpolated by global collocation with radial basis functions [2]. We consider the compact-support Wendland function defined as \(\phi(r) = (1 - cr)^{p+1} (32 cr)^{p} + 25 (cr)^{2} + 8 c r + 1\).

The shape parameter \(c\) is optimized [3] and a 17s Chebyshev grid is used.

Results

Simply supported square \((a = b = 2)\) plates with thickness \(h\) and \(a/h = 10\) are analysed. The material properties are \(E_0 = 70 GPa, E_c = 380 GPa\), with \(E_0 = 1 GPa\), and \(v_0 = v_c = 0.3\). The non-dimensional parameter used is

\[
P = \frac{P a}{E_0 h^2}
\]

Uni-axial buckling load: First four buckling modes, \(p = 10\, 2-2-1\)

\[
\begin{align*}
\sinus: & \\
\sinus ZZ: & \\
\sinus ZZ0: & \\
\end{align*}
\]

Bi-axial buckling load: Fundamental bi-axial buckling load

\[
\begin{align*}
\text{Theory} & \quad 1-0-1 & \quad 1-0-1 & \quad 1-0-1 & \quad 1-0-1 \\
0 & \sinus & 6.47656 & 6.47656 & 6.47656 & 6.47656 \\
0 & \sinus0 & 6.50272 & 6.50272 & 6.50272 & 6.50272 \\
0 & \sinus ZZ & 6.47650 & 6.47595 & 6.47650 & 6.47650 \\
0 & \sinus ZZ0 & 6.50266 & 6.50266 & 6.50266 & 6.50266 \\
0.5 & \sinus0 & 3.58115 & 3.83821 & 3.99480 & 4.09640 & 4.27584 & 4.47083 \\
0.5 & \sinus ZZ & 3.59380 & 3.87175 & 4.08555 & 4.11069 & 4.29064 & 4.48642 \\
0.5 & \sinus ZZ0 & 3.58112 & 3.87799 & 4.09592 & 4.27541 & 4.47057 \\
\end{align*}
\]

Conclusions

The zig-zag effects have influence on the buckling loads of simply supported square sandwich plates with functionally graded skins. By comparing \(\sinus\) and \(\sinus ZZ\) theories we see that the first one (without ZZ effect) gives higher buckling loads than the other (with ZZ effects). Same happens to \(\sinus0\) and \(\sinus ZZ0\) theories. The influence of the warping effects is stronger than the ZZ effects.

References

[1] Carrera, E. Theories and finite elements for multilayered plates and shells: a unified compact formula-


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