The zig-zag and warping effects on buckling of sandwich plates with functionally graded skins, by sinusoidal shear deformation theories

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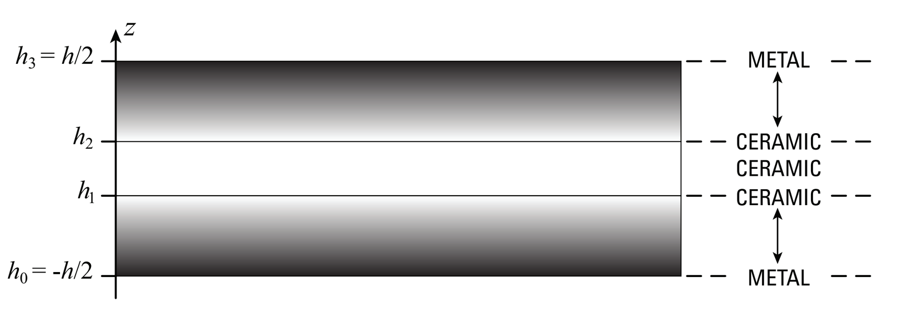
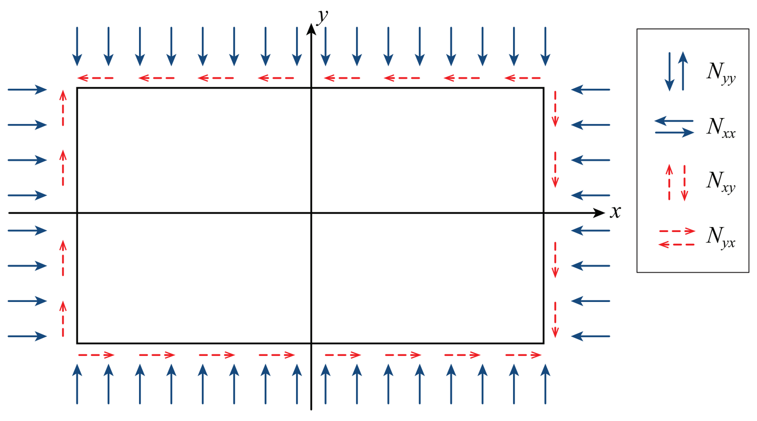
Introduction

In this work we study the influence of zig-zag and warping effects in the buckling of sandwich plates with functionally graded skins. The sandwich plate is subjected to compressive in-plane forces acting on the mid-plane of the plate.





Methodology

Displacement field of the sinusoidal shear deformation theories

sinus:

$$\begin{cases} u = u_0 + z u_1 + \sin\left(\frac{\pi z}{h}\right) u_s \\ v = v_0 + z v_1 + \sin\left(\frac{\pi z}{h}\right) v_s \\ w = w_0 + z w_1 + z^2 w_2 \end{cases}$$

sinus0:

$$\begin{cases} u = u_0 + z u_1 + \sin\left(\frac{\pi z}{h}\right) u_s \\ v = v_0 + z v_1 + \sin\left(\frac{\pi z}{h}\right) v_s \\ w = w_0 \end{cases}$$

sinusZZ:

$$\begin{cases} u = u_0 + z u_1 + \sin\left(\frac{\pi z}{h}\right) u_s + (-1)^k \frac{2}{h_k}\left(z - \frac{1}{2}(z_k + z_{k+1})\right) u_Z \\ v = v_0 + z v_1 + \sin\left(\frac{\pi z}{h}\right) v_s + (-1)^k \frac{2}{h_k}\left(z - \frac{1}{2}(z_k + z_{k+1})\right) v_Z \\ w = w_0 + z w_1 + z^2 w_2 \end{cases}$$

sinusZZ0:

$$\begin{cases} u = u_0 + z u_1 + \sin\left(\frac{\pi z}{h}\right) u_s + (-1)^k \frac{2}{h_k}\left(z - \frac{1}{2}(z_k + z_{k+1})\right) u_Z \\ v = v_0 + z v_1 + \sin\left(\frac{\pi z}{h}\right) v_s + (-1)^k \frac{2}{h_k}\left(z - \frac{1}{2}(z_k + z_{k+1})\right) v_Z \\ w = w_0 \end{cases}$$

Material properties

The power-law function is used to describe the volume fraction of the metal (V_m) and ceramic (V_c) phases and the material homogeneization technique adopted to the Young's modulus is the law of mixtures:

$$E(z) = E_m V_m + E_c V_c \quad \text{with} \quad V_c = \begin{cases} \left(\frac{z-h_0}{h_1-h_0}\right)^p, & z \in [h_0, h_1] \\ 1, & z \in [h_1, h_2] \\ \left(\frac{z-h_3}{h_2-h_3}\right)^p, & z \in [h_2, h_3] \end{cases}$$

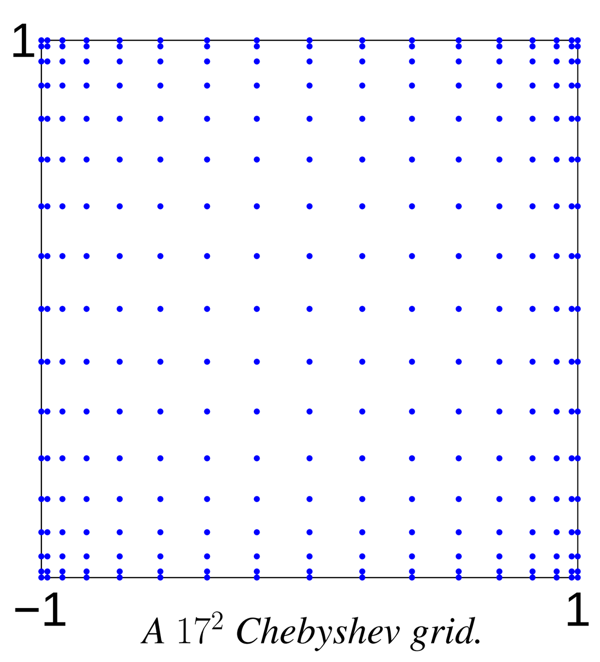
Carrera's Unified Formulation

The governing equations and boundary conditions are derived under a generalization of Carrera's Unified Formulation (CUF) [1] based on the principle of virtual displacements. Although the sandwiches present 3 physical layers, we consider $N_l = 91$ virtual (mathematical) layers of constant thickness.

Meshless method

The governing equations are interpolated by global collocation with radial basis functions [2]. We consider the compact-support Wendland function defined as $\phi(r) = (1 - c\,r)_+^8 \left(32(c\,r)^3 + 25(c\,r)^2 + 8c\,r + 1\right)$.

The shape parameter (c) is optimized [3] and a 17^2 Chebyshev grid is used.



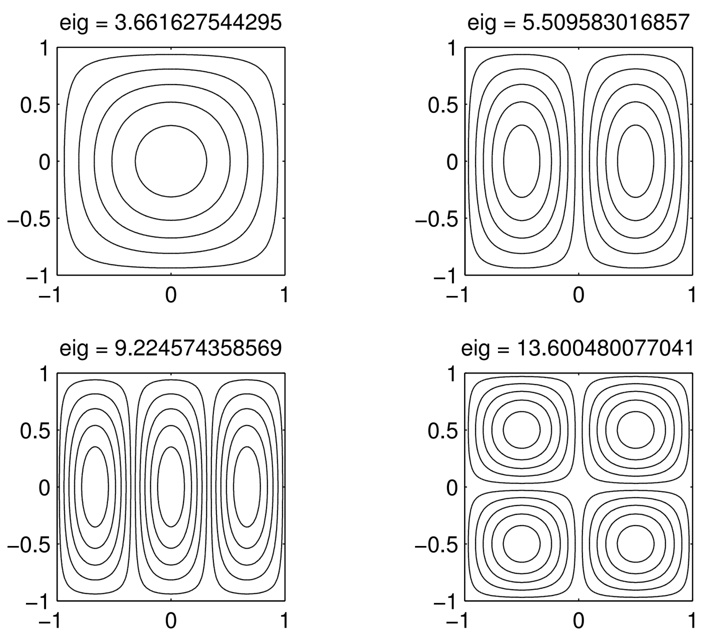
A 17^2 Chebyshev grid.

Results

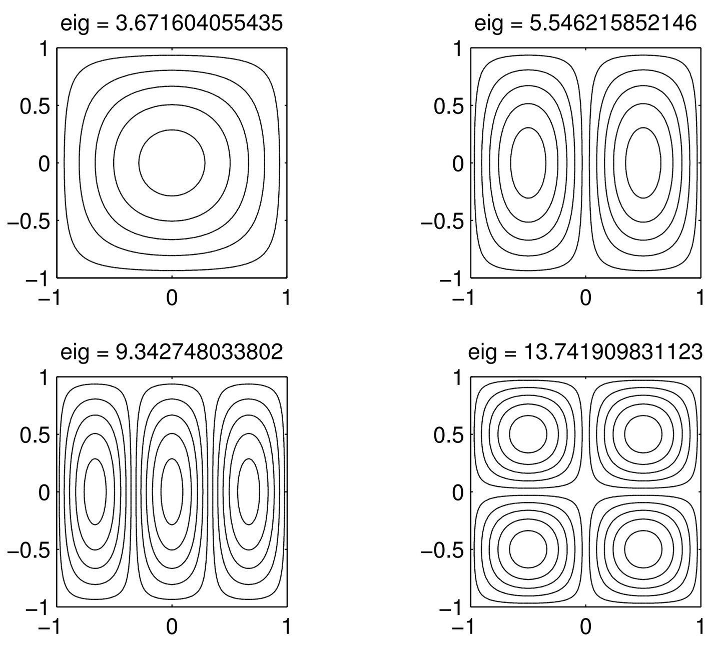
Simply supported square ($a = b = 2$) plates with thickness h and $a/h = 10$ are analysed. The material properties are $E_m = 70E_0$, $E_c = 380E_0$, with $E_0 = 1$GPa, and $\nu_m = \nu_c = \nu = 0.3$. The non-dimensional parameter used is $\bar{P} = \frac{Pa^2}{100h^3E_0}$.

Uni-axial buckling load: First four buckling modes, p=10, 2-2-1

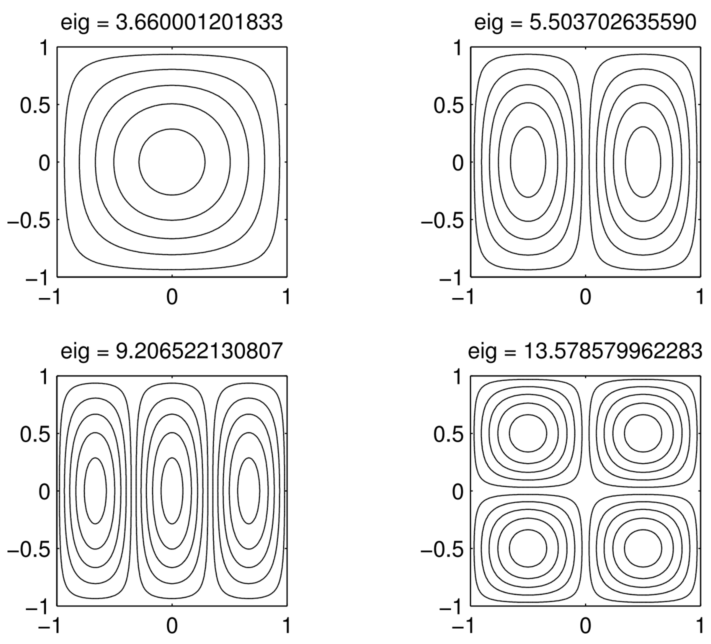
sinus theory



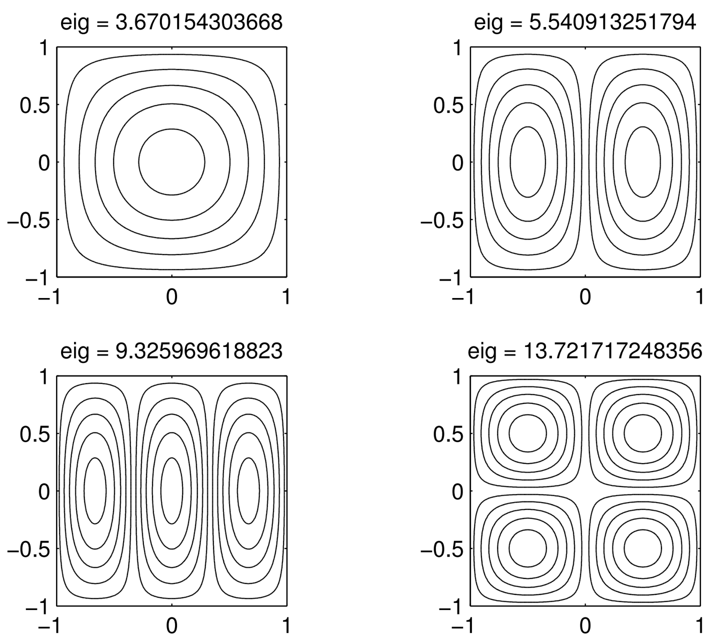
sinus0 theory



sinusZZ theory



sinusZZ0 theory



Bi-axial buckling load: Fundamental bi-axial buckling load

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	$sinus$	6.47656	6.47656	6.47656	6.47656	6.47656	6.47656
	$sinus0$	6.50272	6.50272	6.50272	6.50272	6.50272	6.50272
	$sinusZZ$	6.47650	6.47598	6.47641	6.47601	6.47595	6.47655
	$sinusZZ0$	6.50266	6.50219	6.50258	6.50224	6.50214	6.50272
0.5	$sinus$	3.58115	3.85821	3.99480	4.09640	4.27584	4.47083
	$sinus0$	3.59380	3.87175	4.00855	4.11069	4.29064	4.48642
	$sinusZZ$	3.58112	3.85799	3.99480	4.09592	4.27541	4.47075
	$sinusZZ0$	3.59377	3.87155	4.00855	4.11026	4.29020	4.48636
1	$sinus$	2.53076	2.85573	3.02734	3.15750	3.39199	3.65983
	$sinus0$	2.53937	2.86520	3.03681	3.16779	3.40271	3.67165
	$sinusZZ$	2.53073	2.85562	3.02735	3.15707	3.39169	3.65975
	$sinusZZ0$	2.53935	2.86511	3.03683	3.16740	3.40238	3.67158
5	$sinus$	1.31820	1.50377	1.68126	1.76496	2.02535	2.32346
	$sinus0$	1.32348	1.50927	1.68601	1.77075	2.03084	2.33022
	$sinusZZ$	1.31816	1.50349	1.67983	1.76497	2.02528	2.32344
	$sinusZZ0$	1.32344	1.50897	1.68469	1.77076	2.03080	2.33019
10	$sinus$	1.23615	1.35996	1.53030	1.57865	1.83081	2.10273
	$sinus0$	1.24130	1.36529	1.53475	1.58414	1.83579	2.10893
	$sinusZZ$	1.23606	1.35840	1.52613	1.57829	1.83000	2.10224
	$sinusZZ0$	1.24121	1.36367	1.53075	1.58374	1.83508	2.10843

Conclusions

The zig-zag effects have influence on the buckling loads of simply supported square sandwich plates with functionally graded skins. By comparing $sinus$ and $sinusZZ$ theories we see that the first one (without ZZ effect) gives higher buckling loads than the other (with ZZ effects). Same happens to $sinus0$ and $sinusZZ0$ theories. The influence of the warping effects is stronger than the ZZ effects.

References

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[2] Kansa, E. J. Multiquadrics- A scattered data approximation scheme with applications to computational fluid dynamics. I: Surface approximations and partial derivative estimates. *Computers and Mathematics with Applications* 19(8/9):127-145, 1990.

[3] Ferreira, A. J. M., and Fasshauer, G. E. Computation of natural frequencies of shear deformable beams and plates by a RBF-Pseudospectral method. *Computer Methods in Applied Mechanics and Engineering*, 196:134-146, 2006.

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