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THERMO-MECHANICAL ANALYSIS OF FUNCTIONALLY GRADED STRUCTURES VIA REFINED SHELL FINITE ELEMENTS

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Summary. *Finite shell elements based on the Unified Formulation by Carrera (CUF) are here extended to the thermo-mechanical analysis of functionally graded plates and shells. The elements have cylindrical geometry and the Mixed Interpolation of Tensorial Components (MITC) is used to overcome the membrane and shear locking phenomenon. Unlike the classical models, the refined models contained in the CUF permit to account for the grading material variation in the thickness direction. The governing thermo-dynamical equations are derived from the Principle of Virtual Displacements. The temperature profile is not assumed linear in the thickness direction, but it is calculated by solving the Fourier's heat conduction equation. The results are presented in terms of temperature, displacements and stresses along the thickness and they are compared with quasi-3D solution obtained analytically.*

INTRODUCTION

The concept of Functionally Graded Materials (FGMs) was first proposed in Japan in the late 1980s, as a thermal barrier material [1]. The severe temperature loads involved in many engineering applications require materials that are resistant to high temperatures. As the laminated composite materials, FGMs combine the desirable properties of the constituent phases to obtain a superior performance and avoid the problem of interfacial stresses.

Over the last decade, extensive research has been carried out on the modeling of plates and shells made of FGM. Some analytical methods for the thermo-elastic response of functionally graded structures have been proposed in literature [2], but they are only suitable for the simply supported boundary conditions. Some researchers have instead treated the problem with finite element approaches, such as in [3]. They have formulated a finite element based on First-order Shear Deformation Theory (FSDT) for the thermo-mechanical analysis of FGMs. This model takes into account the non-linear distribution of both material properties and temperature along the thickness, but it is not able to accurately describe the non-linear thermoelastic response of FGMs.

For these reasons, the shell finite element based on the Carrera's Unified Formulation (CUF) [4,5], has been extended to the analysis of FGM plates and shells subjected to both mechanical and thermal loads. The main feature of the CUF, is the unified manner in which all the variables (displacements u and temperature θ) are expressed:

$$u(x,y,z) = F_\tau(z) u_\tau(x,y) \,, \quad \theta(x,y,z) = F_\tau(z)\theta_\tau(x,y), \quad \text{with } \tau = 0, \dots, N \,,$$

where F_τ are the so called thickness functions. Different refined theories can be obtained by changing the order of expansion N in the thickness direction (usually from 1 to 4).

The shell finite element considered has cylindrical geometry (the plate is seen as a particular case) and it employs the Mixed Interpolation of Tensorial Components (MITC) method to overcome the membrane and shear locking phenomenon. The governing equations of the thermo-mechanical problem are derived from the Principle of Virtual Displacement (PVD). The temperature is seen as an external load and it is calculated by solving the Fourier's heat conduction equation. The Mori-Tanaka model is used to describe the variation of material properties along the thickness.

The results are provided in terms of displacements, stresses and temperature along the thickness of the plate/shell for different thickness ratios as well as different material indexes. The comparison with the quasi-3D solution shows that the CUF shell element works very well for the thermo-mechanical analysis of functionally graded materials.

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