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MITC-TYPE SHELL ELEMENTS BASED IN RMVT UNIFIED FORMULATION

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Summary. *This paper deals with the formulation of finite mixed shell elements in the framework of the Reissner Mixed Variational Theorem (RMVT). The Mixed Interpolation of Tensorial Components (MITC) technique is applied in order to overcome the locking.*

1 INTRODUCTION

Multilayered plates or shells are widely employed by now to devise most of next generation fly and ship and advanced aerospace, automotive vehicles.

Referring to the two dimensional modelling of multilayered plates and shells, the main interest is to study the mechanical and physical properties, that may change in the thickness direction and to take account of the higher transverse shear and normal stresses. Moreover, for equilibrium reasons, the transverse stresses are required to be continuous in each layer interface. Finally, the fulfilment of the interlaminae continuity (IC) is a crucial point of the two dimensional modelling of multilayered structures.

The solution of these advanced structures subjected to static or dynamic loads is hard to find in closed-form and the use of approximated solutions is especially useful. Among the several computational techniques proposed, the finite element method showed successful performances and appears one of the most popular procedure for the analysis of plates and shells. In this regard, two variational formulations are available to reach the stiffness matrices, the *principle of virtual displacement* (PVD) and the *Reissner mixed variational theorem* (RMVT). Unfortunately, the PVD, being formulated with only displacement unknowns, cannot describe a priori IC for transverse and normal stresses in both the ESLM (*equivalent single-layer models)* and the LWM (*layer-wise models)* approach. On the contrary, in-plane and transverse stresses can be assumed in the framework of RMVT, which consists of a mixed principle for multilayered structures. Carrera showed [3] that RMVT leads to an effective description of the in-plane and out-plane response, with *a priori* excellent accuracy of transverse stresses. Finite elements based on PVD and RMVT are referred as *classical* and *advanced* respectively.

It is known that when a finite element method is used to discretize a physical model, the phenomenon of numerical locking may arise, because some hidden constrains that are not well represented in the finite element approximation. In the analysis of plates, when standard low-order finite elements are used, the solution degenerates very rapidly for small thickness and the so called *shear locking* phenomenon happens. Furthermore, handling the shell-structures, the *shear locking* is still present, but another and more severe type, referred as *membrane locking,* appears.

In the PVD approach, the most common techniques to overcome the locking, are the use of higher-order finite elements. However, in the case of very small thickness and when the element is not of degree as high as need, they are unable of prevent a loss in the rate of convergence. A well known remedy for the locking is the use of reduced integration or mixed interpolation techniques [2], [4], [5], [6].

In this paper, we introduce a strategy similar to MITC approach [1] in the RMVT formulation in order to construct an *advanced* locking-free finite element to treat the multilayered plates and shells, assuming the in-plane and transverse stresses. It is known that unless the combination of finite element spaces for displacement and stresses is chosen carefully, the problem of locking is likely to occur. Following this suggestion, we propose a finite element scheme that exhibits both properties of convergence and robustness.

REFERENCES

[1] K.J. Bathe and E.N. Dvorkin, “A formulation of general shell elements-The use of Mixed Interpolation of Tensorial Components”, *Int. J. Num. Meth. Engng,* 22, 697-722 (1986).

[2] E. Carrera, M. Cinefra and P. Nali, “MITC Tecniques Extended to Variable Kinematic Multilayered Plate Elements”, *Composite Structures,* 92, 1888-1895 (2010).

[3] E. Carrera and L. Demasi, “Classical and advanced multilayered plate elements based upon PVD and RMVT. Part. 1: Derivation of finite element matrices ”, *Int. J. Num. Meth. Engng*, 55, 191-231 (2002).

[4] C. Chinosi and L. Della Croce, “Mixed-interpolated elements for thin shells”, *Com. Num. Meth. Engng*, 14, 1155-1170 (1998).

[5] M. Cinefra, C. Chinosi and L. Della Croce, “ MITC9 shell elements based on refined theories for the analysis of isotropic cylindrical structures”, to appear *in Mechanics of Advanced Materials and Structures* (2011).

[6] R.S. Falk, “Finite Elements for the Reissner-Mindlin Plate ”, *Lct. Notes Math*, 1939, 195-232 (2008).