

## REFINED BEAM ELEMENTS FOR THE MULTISCALE ANALYSIS OF FIBER-REINFORCED COMPOSITE STRUCTURES

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**Key words:** Higher-order beam, Carrera Unified Formulation, Composite structures, Finite Element Method.

### ABSTRACT

Beam models are widely used to investigate the mechanical behavior of slender bodies such as aircraft wings, wind turbines, and slender bridges. Classical beam theories are those by Euler-Bernoulli and Timoshenko [1] which are particularly effective in analyzing the bending of compact slender structure made of isotropic materials. As the structure becomes thin-walled, short, or composite materials are used, classical model validity ceases since a number of non-classical effects has to be taken into account: warping, cross-section distortion, shear effects, etc.. The overcome of these limitations can be obtained by adopting refined beam models. Interesting papers on this topic are those by Kapania and Raciti [2, 3], and Yu and Hodges [4].

This paper is embedded in the framework of the Carrera Unified Formulation, CUF, for higher-order plate/shell [4] and beam [5] models. CUF permits us to deal with any-order theories in a hierarchical manner with no need of *ad hoc* implementations since the order of

the model is set as an input of the analysis. Recent papers on beams [6, 7] showed the enhanced capabilities offered by CUF in dealing with thin-walled structures and arbitrary cross-section geometries, that is, CUF beam models are able to detect shell-like solutions with a considerable reduction of the computational costs.

The beam cross-section displacement field is described by an expansion of certain classes of polynomials and the finite element matrices are formulated in terms of a few fundamental nuclei whose form is independent of the order of the expansion. Taylor-like and Lagrange expansions are herein exploited. The former lead to an Equivalent Single Layer, ESL, description of the beam, the latter enrich the formulation by permitting a Layer Wise, LW, approach. The Lagrange based beam models offer further important advantages: 1. they have only displacement degrees of freedom; 2. the refinement process can be localized to certain portions of the cross-section; 3. arbitrary constraint distributions above the cross-section are allowed.

A particularly interesting application of the present beam models is represented by the multiscale analysis of fiber-matrix structures. CUF Lagrange based beams appear to be perfect candidates to conduct this kind of analysis since they are particularly effective to study slender bodies such as fibers and, moreover, they offer a wide number of options to model the interface phenomena. A number of examples are carried out in this work to show the various analysis approaches that CUF beam models offer. Comparisons with classical approaches are made to highlight the accuracy and computational cost performances of the present formulation. Particular attention is given to possible extensions to the failure analysis of fiber-reinforced composite structures.

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