ECCM 2010
IV European Conference on Computational Mechanics
Palais des Congrès, Paris, France, May 16-21, 2010

A mixed axiomatic/asympthotic approach for the evaluation of refined plate theories

E. Carrera[1], M. Petrolo[2,3]

[1] *Department of Aeronautic and Space Engineering, Politecnico di Torino, Italy, erasmo.carrera@polito.it*
[2] *Department of Aeronautic and Space Engineering, Politecnico di Torino, Italy, marco.petrolo@polito.it*
[3] *Institut Jean Le Rond d'Alembert, UMR7190 CNRS, Paris06, France*

The axiomatic and the asymptotic methods are the most important techniques to construct structural models, such as beam, plate, and shell theories. The axiomatic approach exploits the intuition of an Eminent Scientist to build a model. Well-known examples are the Kirchhoff and the Reissner-Mindlin theories [1–3] which are the classical models for plates. Reviews on refined theories based on an axiomatic approach can be found in [4–7]. The asymptotic method [8–10] uses a perturbation parameter to evaluate which terms of a theory have to be retained. A typical perturbation parameter is the thickness-to-length in the case of plates/shells. All the retained terms are those which have the same order of magnitude as the introduced perturbation parameter when the latter vanishes. Both methods present some drawbacks. While the error estimation and the convergence properties are difficult to be considered in the axiomatic approach, the asymptotic one requires *ad hoc* analyses to deal with different structural problems.

This work presents a mixed axiomatic/asympthotic approach to construct refined plate theories. This approach furnishes asymptotic-like results starting from a preliminary axiomatic choice of the plate model. This technique is based on the Carrera Unified Formulation (CUF) [11] which is used to implement any-order plate theories and obtain closed-form solutions. Taylor-type polynomials are adopted to model the displacement field along the thickness direction of the plate. The role of each displacement variable in the solution is investigated by measuring the loss of accuracy due to its neglect, if the loss is null, a term will be considered ineffective, i.e. negligible. 3D solutions are considered as references. Reduced kinematics models, based on a set of retained displacement variables, are then obtained. Different configurations are considered to evaluate the sensitivity of various plate theories with respect to variations in geometry, orthotropic ratio, and stacking sequence. In this work additional results are presented on the base of those in [12].

A simply supported plate has been considered. Four different types of material have been addressed: isotropic, orthotropic, composite, and bi-metallic. A bi-sinusoidal distributed load has been applied to the top surface of the plate. In the case of isotropic material, four different length-to-thickness ratios are considered: 100, 10, 5 and 2, that is, thin, moderately thick, thick and very thick plates are considered, respectively. In the orthotropic material case, four different orthotropic ratios, E_L/E_T , have been assumed: 1, 5, 10, 50 and 100. E_L stands for Young's modulus along the longitudinal direction, E_T indicates the Young's modulus along the transverse direction. The composite plate have been analyzed considering three stacking sequences: two symmetrical $0^\circ/90^\circ/0^\circ$ and $0^\circ/0^\circ/0^\circ$, and one asymmetrical $0^\circ/0^\circ/90^\circ$. The bi-metallic plate has been modeled using an aluminium layer together with a titanium one.

The reduced plate models are presented using a graphical scheme based on Table 1 where each term of the full fourth-order model is placed. z is the thickness coordinate of the plate. White and black triangles are used to indicate the status of a displacement variable: white color means that a term is non-active, black color indicates that a term is activated. Table 2 is referred to the plate model which has the displacement variable u_{y_3} non-activated.

An example of typical result which is obtainable with the mixed approach is shown in Table 3: the kinematics models which are able to furnish the same accuracy as the 3D model for thin and very thick plates

in case of isotropic material. M_e states the number of effective displacement variables, that is, the computational cost of a plate model. It is evident the increase of M_e as the plate become thicker. Moreover, each output variable needs its own plate model to accomplish the accuracy requirement.
It has been found that the terms that have to be used according to a given accuracy varies from problem to problem, but they also vary when the variable that has to be evaluated (displacement, stress components) is changed. Features as the asymmetrical lamination, anisotropy, and a small length-to-thickness ratio, require a large number of displacement variables. Thus, in these cases, a full implementation of CUF is recommendable.

Tables

$N=0$	$N=1$	$N=2$	$N=3$	$N=4$
u_{x_1}	$u_{x_2}\ z$	$u_{x_3}\ z^2$	$u_{x_4}\ z^3$	$u_{x_5}\ z^4$
u_{y_1}	$u_{y_2}\ z$	$u_{y_3}\ z^2$	$u_{y_4}\ z^3$	$u_{y_5}\ z^4$
u_{z_1}	$u_{z_2}\ z$	$u_{z_3}\ z^2$	$u_{z_4}\ z^3$	$u_{z_5}\ z^4$

Table 1: Locations of the displacement variables within the tables layout

▲	▲	▲	▲	▲
▲	▲	△	▲	▲
▲	▲	▲	▲	▲

Table 2: Symbolic representation of the reduced kinematics model with u_{y_3} deactivated

u_z	σ_{xx}	σ_{xz}	σ_{zz}	COMBINED
		$a/h=100$		
$M_e=4$	$M_e=4$	$M_e=5$	$M_e=4$	$M_e=7$
△▲△△△ △▲△△△ ▲△▲△△	△▲△△△ △▲△△△ ▲△▲△△	△▲△▲△ △▲△△△ ▲△▲△△	△△△△△ △△△△△ ▲▲▲△▲	△▲△▲△ △▲△△△ ▲▲▲△▲
		$a/h=2$		
$M_e=13$	$M_e=14$	$M_e=7$	$M_e=13$	$M_e=15$
▲▲▲▲△ ▲▲▲▲△ ▲▲▲▲▲	▲▲△▲▲ ▲▲▲▲▲ ▲▲▲▲▲	△▲△▲△ △▲△▲△ ▲△▲△▲	▲△▲▲▲ ▲△▲▲▲ ▲▲▲▲▲	▲▲▲▲▲ ▲▲▲▲▲ ▲▲▲▲▲

Table 3: Comparison of the sets of effective terms for isotropic plates with different a/h

References

[1] Kirchhoff, G., "Uber das gleichgewicht und die bewegung einer elastischen scheibe", *J. Angew. Math.*, Vol. 40, 1850, pp. 51–88.

[2] Reissner, E., "The effect of transverse shear deformation on the bending of elastic plates", *ASME J. Appl. Mech.*, Vol. 12, 1945, pp. 69–76.

[3] Mindlin, R. D., "Influence of rotatory inertia and shear in flexural motions of isotropic elastic plates", *ASME J. Appl. Mech.*, Vol. 18, 1950, pp. 1031–1036.

[4] Ambartsumian, S. A., "Contributions to the theory of anisotropic layered shells", *Appl. Mech. Rev.*, Vol. 15, 1962, pp. 245-249.

[5] Librescu, L., and Reddy, J. N., "A critical review and generalization of transverse shear deformable anisotropic plates, euromech colloquium 219, kassel", *Refined Dynamical Theories of Beams, Plates and Shells and Their Applications*, September 1986, pp. 32–43, I Elishakoff and Irretier (eds), Springer Verlag, Berlin.

[6] Carrera, E., "Developments, ideas and evaluations based upon the Reissner's mixed variational theorem in the modeling of multilayered plates and shells", *Applied Mechanics Review*, Vol. 54, 2001, pp. 301–329.

[7] Carrera, E., "Theories and finite elements for multilayered plates and shells", *Archives of Computational Methods in Engineering*, Vol. 9, No. 2, 2002, pp. 87–140.

[8] Cicala, P., "Sulla teoria elastica della parete sottile", *Giornale del Genio Civile*, Vol. 4, 6 and 9, 1959.

[9] Fettahlioglu, O. A., and Steele, C. R., "Asymptotic solutions for orthotropic non-homogeneous shells of revolution", *ASME J. Appl. Mech.*, Vol. 44, 1974, pp. 753–758.

[10] Widera, D. E. O., and Fan, H., "On the derivation of a refined theory for non-homogeneous anisotropic shells of revolution", *ASME J. Appl. Mech.*, Vol. 110, 1988, pp. 102–105.

[11] Carrera, E., "Theories and finite elements for multilayered plates and shells: a unified compact formulation with numerical assessment and benchmarking", *Archives of Computational Methods in Engineering*, Vol. 10, No. 3, 2003, pp. 216–296.

[12] Carrera, E., and Petrolo, M., "Guidelines and Recommendations to Construct Refinements of Classical Theories for Metallic and Composite Plates", Submitted.