Refined and mixed models for free vibration analysis of functionally graded material plates and shells

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The volume fraction of the constituents of a functionally graded material (FGM) changes gradually in a preferred direction (usually the thickness direction z) and consequently the elastic properties depend on the considered coordinate. FGMs have been presented as an alternative to laminated composite materials that show a mismatch in properties at the material interfaces. This material discontinuity in laminated composite materials leads to large interlaminar stresses and the possibility of the initiation and propagation of cracks [1].

A free vibrations analysis is conducted for both one-layered and multi-layered plate and shell geometries. A typical multilayered structure is given in Figure 1, two external layers with constant properties are linked with an internal FGM layer where the elastic coefficients C and the density mass ρ vary with continuity in the thickness direction z.

The refined and advanced/mixed two-dimensional models are obtained in the framework of Carrera's Unified Formulation (CUF) [2], this permits to obtain in a unified manner several two-dimensional models. They can differ for the order of expansion in the thickness direction for primary variables (N=1,...,4) and for the multilayer approach: Equivalent Single Layer (ESL) or Layer Wise (LW).

Refined models based on the Principle of Virtual Displacements (PVD) have the displacement $\mathbf{u}$ as primary variable [1]:

$$\delta \mathbf{u_s^k} : \mathbf{K_u^{k\tau s}} \mathbf{u_\tau^k} = \mathbf{M^{k\tau s}} \ddot{\mathbf{u}}_\tau^\mathbf{k}, \tag{1}$$

where $\mathbf{K_u^{k\tau s}}$ is the so-called fundamental nucleus which permits to obtain the global stiffness matrix by expanding via indexes τ, s for the order of expansion in the thickness direction z, and via k for the multilayer assembling procedure. $\mathbf{M^{k\tau s}}$ is the fundamental nucleus from which the inertial matrix of the structure is obtained. Classical theories such as Classical Lamination Theory (CLT) and First order Shear Deformation Theory (FSDT) are obtained as particular cases of ESL models and they are included in the proposed results for comparison purposes.

Mixed models based on Reissner's Mixed Variational Theorem (RMVT) [3] have both displacement $\mathbf{u}$ and transverse shear/normal stresses $\sigma_\mathbf{n}$ as primary variables [4]:

$$\begin{aligned} \delta \mathbf{u_s^k} &: \mathbf{K_{uu}^{k\tau s}} \mathbf{u_\tau^k} + \mathbf{K_{u\sigma}^{k\tau s}} \sigma_{\mathbf{n}\tau}^\mathbf{k} = \mathbf{M^{k\tau s}} \ddot{\mathbf{u}}_\tau^\mathbf{k} \\ \delta \sigma_\mathbf{ns}^\mathbf{k} &: \mathbf{K_{\sigma u}^{k\tau s}} \mathbf{u_\tau^k} + \mathbf{K_{\sigma\sigma}^{k\tau s}} \sigma_{\mathbf{n}\tau}^\mathbf{k} = \mathbf{0}, \end{aligned} \tag{2}$$

four different fundamental nuclei are given in Equation 2 and they are assembled via indexes τ, s, k.

Governing equations 1 and 2 are solved in analytical way by considering harmonic forms for displacements and transverse shear/normal stresses and simply supported boundary conditions. Chosen the waves number in the in-plane directions, several frequencies are obtained by depending on degrees of freedom of the considered 2D model. The presence of an FGM layer does not modify this procedure: the FGM layer is included in the model simply considering the elastic coefficients C and the mass density ρ depending on the thickness coordinate z and including them in the integrals in z direction made for the thickness functions employed for the axiomatic 2D approximation.

In the proposed results the importance of refined and mixed models is remarked in the case of multilayered configurations and/or moderately thick plates and shells. Several thickness laws for elastic properties and mass density are investigated, in particular their effects on the frequencies for the free vibration problem. Higher order modes are investigated by considering further frequencies over the fundamental one, and/or by imposing higher values of waves number. The use of refined and advanced models results mandatory in the case of higher modes investigation [5].

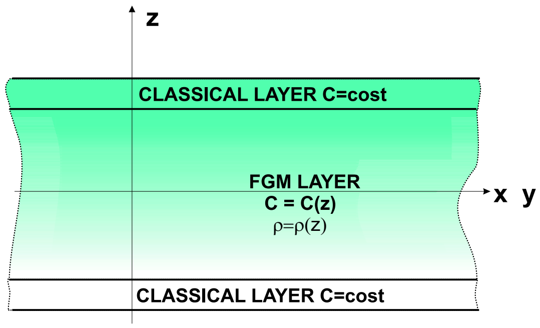

Figure 1: Multilayered plate with an internal functionally graded layer.

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