Cotutelle Ph.D. student at the Politecnico di Torino and University of Lille

Homogenization of linear and nonlinear highly heterogeneous plates and shells and edge effect evaluation

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Duration: 36 Months. Starting date: September 2019.

Status: Full-Time. Salary (after taxes): approx. 1400 Euro per month

Application Documents Applicants should send the following documents to erasmo.carrera@polito.it anderick.pruchnicki@gmail.com:

- A motivation letter.
- CV.
- Copies of education certificates and transcripts of records with GPA.
- Copies of English certificates.
- List of publications (if any).
- 2-3 letters of references.

Background and objectives

The theory of homogenization of highly heterogeneous structures is well established. One of the best models is the well-known Caillerie-Kohn-Vogelius that is mathematically elegant and rigorous but appliesonly to simple engineering models, such as the Kirchhoff plate. Pruchnicki (2017a) proposed arigorous mathematical extension to higher-order plate models but this approach requires the proper scaling of the dimensions of the representative volume element (RVE), as discussed previously by Lewinski (1992). The most important problem results from the a-priori scaling that assumes applied loads, or deformations, concerning the shell thickness. In engineering applications, the applied loads are external and, often, cannot relate to the thickness. Such a shortcoming affects the quality ofresults for homogeneous thin structures. Recently, Pruchnicki 2018 (a, b) has proposed an approach to improve the framework via a multiscale finite-strain shell theory for simulating the mechanical response of highly heterogeneous shells with varying thickness for linear and nonlinear cases. The method exploits a higher-order stress-resultant shell formulation based on the multiscale homogenization. At the macroscopic scale, the displacement field isa fourth-order Taylor-Young expansion along the thickness. A boundary value problem over the 3D RVE accounts for the microscale fluctuations. For the sake of simplicity, the microstructure is periodic and has curvilinear coordinates. The RVE has body forces and stress vectors acting on the upper and lower faces. The geometry of the RVE complies with the representative heterogenous microstructure andthe in-plane homogenization is combined with a through-the-thickness stress integration. The macroscopic stress resultants stem from the microscopic stress via the macro-micro Hill-Mandel condition expressing the equivalence between the internal macroscopic and microscopic energies. All microstructural constituents are 3D first-order continua described by the standard equilibrium and the constitutive equations. The main shortcoming of this method is the lack ofa double scale displacement field making difficult to considerthe well-known edge effects. According to the work of Lee, et al. (2014), the double scale displacement field is the sum of a macroscopic displacement field depending only on the global variable and a microscopic displacement field depending on local (microscopic) and global (macroscopic) scales. The macroscopic displacement field can be the one proposed by Song and Dai (2016) and Pruchnicki 2018 (a, b). Then, as in Pruchnicki (2006), a double scale variational formulation is used to show that the microscopic part of the displacement field satisfies the local variational formulation. The two problems are coupled, and the solution is numerical. The most important advantage of this formulation is that it enables the treatment of edge effects. Examples of theoretical developments of this type are in Berdichevsky (1979) and Pruchnicki 1998 (a, b). The latest developments are the topic of a presentation of Pruchnicki at the fourth congress of Mechanicsin Krakow - September 2019 - and Gaeta - May 2019, www.math.uzh.ch/gaeta2019.

The thesis aims to numerically implement and assess this new theory. The implementation focuses, first, on the local macroscopic constitutive laws, then, on the structural computation with and without the boundary layer on the lateral boundary. Finally, the candidate student can implement numerically the new theoretical model for heterogeneous plates as proposed by Pruchnicki (2017b) as an extension of a new type of bidimensional models for homogeneous plates (Schneider et al. 2014).

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Pruchnicki, E. (1998 a). Overall Properties of thin hyperelastic plate at finite strain with edge effects using asymptotic method. International Journal of Engineering Science, 36(9), pp. 973-1000.

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Lee, CY, Yu, W, and Hodges, DH. (2014). Refined modeling of composite plates with in-plane heterogeneity. Z. Angew. Math. Mech. 2014; 94(1-2): 85-100.

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