Non-linear transient dynamic analysis of sandwich plate with composite face-sheets embedded with shape memory alloy wires and flexible core- based on the mixed LW (layer-wise)/ESL (equivalent single layer) models

M. Botshekanan Dehkordi a, S.M.R. Khalili b,*, E. Carrera c

a Shahrekord University, Faculty of Engineering, P.O. Box 115, Shahrekord, Iran
b Centre of Excellence for Research in Advanced Materials and Structures, Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Tehran, Iran
c Department of Mechanics and Aerospace Engineering, Politecnico di Torino, Torino, Italy

Article history:
Received 20 December 2013
Received in revised form 7 September 2015
Accepted 15 October 2015
Available online 26 October 2015

Keywords:
Shape memory alloys
Material nonlinearity
Sandwich plate
Mixed LW/ESL models
Flexible core

Abstract
In this study, a nonlinear dynamic analysis of sandwich plate with flexible core and laminated composite face sheets embedded with shape memory alloy (SMA) wires is investigated using the mixed LW/ESL models in the framework of Carrera’s Unified Formulation. The instantaneous phase transformation effects are considered for every point on the face sheets. Since, this research deals with the transient nonlinear problem and the structure is thick plate with flexible core, employing a model with high accuracy and low computational cost is vital in this work. For this aim, the new mixed LW/ESL models proposed are employed in this study. The constitutive equation proposed by Brinson is used for modeling the nonlinear behavior of SMA wires. The governing equations are derived employing the Reissner Mixed Variational Theorem (RMVT) in order to satisfy the interlaminar continuity of transverse stresses between the layers. The nonlinear governing equations are solved based on the transient finite element along with the iterative incremental method. Some parametric studies such as intensity of impulsive pressure, location of SMA wires, plate aspect ratio, ratio of face sheet’s thickness to the total thickness, volume fraction of the SMA wires and also boundary conditions upon the loss factor are investigated.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

Over the past two decades, due to the increasing technological request for the passive control of the undesirable vibrations, there has been a growing attention for developing high damping materials. Shape memory alloys possess the inherent ability to change their material properties, in particular their Young’s modulus and damping capacity due to the pseudoelastic effect of this materials which lead to their wide use in the mechanical engineering components. However, due to the nonlinear behavior of this material, it is difficult to present a comprehensive mathematical model for these structures. In this regard, there are many simplifications to model these structures. For example, in the research by Hashemi & Khadem [1], Jafari & Ghiasvand [2] and Zabiciak [3], they modeled the SMA beam based on the many assumptions. In the recent years, many attentions have been focused on the use of SMA wires in the fiber reinforced polymer materials to improve the properties of composite structures. Various applications of SMA hybrid composite components have been studied by many researchers. For example, in the research done by Rogers and Barker [4] SMA wires were employed for controlling the frequency of a graphite/epoxy laminated beam. Baz et al. [5] studied the effect of SMA wires on the controlling of the buckling and frequency of composite beam. They showed that the buckling load of a composite beam can be increased up to three times the uncontrolled beam. Epps and Chandra [6] performed an experimental—analytical work on the natural frequency of SMA hybrid composite beams. However, there are no details on the pseudoelastic effect of SMA wires. Ostachowicz et al. [7] studied dynamic and buckling analysis of composite plates embedded with SMA wires.
Lee and Lee [8] studied the buckling and post-buckling analysis of a SMA hybrid composite plate. They found that, by activation of the pre-strained SMA wires the critical load of the composite plate is increased. Chen and Levy [9] investigated the effect of temperature on frequency, loss factor and control of a constrained viscoelastic layer and shape memory alloy layer. They found that the temperature in the SMA layer is very important in constrained viscoelastic layer and shape memory alloy layer. Baz et al. [10] controlled the shape of composite beams by sets of flat strips of a shape memory nickel—titanium alloy (NITINOL). Roh and Kim [11] analyzed a hybrid smart composite plate under low velocity impact. They optimize the distribution of volume fraction of SMA fibers for reducing the deflection of the plate. Park et al. [12] investigated the composite plate embedded with SMA fibers subject to the aerodynamic and thermal loading in the supersonic region. The nonlinear finite element equations based on the first-order shear deformation plate theory were used for modeling the laminated composite plate embedded with SMA fibers. They studied the effect of the SMA on the critical temperature, thermal post-buckling deflection, natural frequency and critical dynamic pressure of the SMA composite plate. Ghosh et al. [13] studied the dynamic response of the thick beam embedded with SMA ribbon. They used the shape memory effect of the SMAs for reducing a deflection of the beam. In their work, a finite element model is developed to predict both passive and active performance of the beam. Cho and Rhee [14] formulated the nonlinear finite element model for static analysis of SMA hybrid composite shells. They employed the constitutive equation proposed by Brinson based on the iterative technique for modeling the behavior of SMA wires. Khalili et al. [15] studied a non-linear dynamic analysis of a multilayered composite beam embedded with SMA wires. They employed the one dimensional constitutive equation proposed by Brinson for modeling the pseudoelastic behavior of the SMA wires. Qiao et al. [16] analyzed the post buckling of the shape memory polymer composite laminate bonded with alloy film. The finite element model is also implemented for validation of the theoretical method. The relation between the recovery force and the material geometry parameters were also investigated. Bodaghi et al. [17] studied an active shape/stress control of laminated beams subjected to the static loading with integrated/embedded shape memory alloy layer. Euler—Bernoulli beam theory and von Karman geometrically non-linearity are utilized to describe displacement and strain fields of laminated beams consist of SMA and elastic layers. Khalili et al. [18] presented the nonlinear finite element formulation for dynamic analysis of multilayered composite plate embedded with SMA wire based on the Carrera’s Unified Formulation (CUF). They considered the effect of phase transformation for all points on the plate. The CUF [19] has the potential to unify many theories in a unified manner which can be differed by the order of expansion and description of the variables in the thickness direction. This can be equivalent single layer (ESL), if the unknown variables are used for the whole plate, or layer-wise (LW), if the unknown variables are used for each layer, independently.

Sandwich panels with stiff face sheets and flexible low strength cores have been extensively employed in construction of many structures such as automotive, aerospace, ship and building structures. Most of the recent applications have employed the face sheets constituted by multilayered composite materials. In this regard, many studies have implemented by researchers to model the sandwich panels. Carrera and Brischetto [20] have investigated the accuracy of the available theories proposed for the sandwich panels. Noor et al. [21] reviewed more than 800 papers on the modeling of sandwich panels. Khalili et al. [22] investigated the non-linear dynamic analysis of sandwich beam with composite face sheets embedded with SMA wires. In their study a new high order element was proposed based on the high order sandwich panel theory. They studied the effect of the SMA wires on the damped vibration of sandwich beam, taking in to account the phase transformation effects. According to the best knowledge of the authors, there is no study regarding the dynamic analysis of sandwich plate with flexible core and composite face sheets embedded with SMA wires. In this regard, in the present work a nonlinear dynamic analysis of sandwich plate with flexible core and multilayered composite face sheets embedded with SMA wires is investigated for the first time. The instantaneous phase transformation effect is considered for all points on the face sheets of the plate. The Brinson’s constitutive equation [23] is used to model the nonlinear behavior of SMA wires. Coupled equations of the motion are solved based on the transient finite element along with the iterative incremental method. Since, this research deals with a transient nonlinear problem and also the structure is thick plate with flexible core, employing a model with high accuracy and low computational cost is vital in this study. For this aim, the mixed LW/ESL models proposed by the authors [24] are employed in this research. One of the advantages of the ESL/LW models is that, the total number of freedom is independent on the number of layers. This advantage of the mixed LW/ESL models is evidence in this study. Parametric studies like intensity of impulsive pressure, location of SMA wires, plate aspect ratio, ratio of face sheets’ thickness to the total thickness, volume fraction of the SMA wires and also boundary conditions upon the loss factor are investigated in this work.

2. Unified formulation and FEM analysis

The Carrera’s Unified Formulation (CUF) allows to obtain several two dimensional theories for plates and shells, using the separation of the unknown variables into a set of thickness functions depending only on the thickness coordinate z, and the ones coincide with the in-plane coordinates (x, y) [19,25]. In the framework of Carrera’s unified formulation the generic variable \( a(x,y,z) \) and its variation \( \delta a(x,y,z) \) can be expressed as follows [24]:

\[
a(x,y,z) = F_r(z) a_r(x,y), \quad \delta a(x,y,z) = F_r(z) \delta a_r(x,y) \quad \text{with} \quad r,s = t,b,r \quad \text{and} \quad r = 2,\ldots,N
\]

(1)

Bold letters imply arrays and the summing convention with repeated indices \( r \) and \( s \) is considered. The indices \( r \) and \( s \) mean the number of desired points along the thickness for definition of thickness functions \( F \) based on the Legendre polynomials as explained later. Subscripts \( t \) and \( b \) are the top and the bottom values. In addition \( b \) stands for the higher order terms of the expansion. The order of expansion \( N \) can be defined from first to fourth order. The thickness functions \( F_r(z_k) \) are defined for the \( k \)th layer. The Legendre polynomials \( P_r(z_k) \) are expressed as follows:

\[
\begin{align*}
P_0 & = 1, & P_1 & = z_k, & P_2 & = \frac{3z_k^2 - 1}{2}, & P_3 & = \frac{5z_k^3 - 3z_k}{2}, \\
P_4 & = \frac{35z_k^4 - 15z_k^2 + 3}{4}, & P_5 & = \frac{63z_k^5 - 45z_k^3 + 15z_k}{8}
\end{align*}
\]

(2)

where \( z_k = 2z_k/h_k \), while \( z_k \) and \( h_k \) are the local coordinate and the thickness, both are referred to \( k \)th layer, therefore \(-1 \leq z_k \leq 1 \), in this regard see Fig. 1. The thickness functions are obtained by combination the Legendre polynomials as follows [19]:

\[
F_r = \frac{P_0 + P_1}{2}, \quad F_1 = \frac{P_0 - P_1}{2}, \quad F_2 = P_r - P_{r-2}, \quad r = 2,3,\ldots,N
\]

(3)

Therefore, these functions have the following properties:
3. Theoretical formulation and methods

By using the principle of Hamilton, it can be written:

$$\delta L_{\text{int}} - \delta L_{F_n} - \delta L_{\text{ext}} = 0$$

(8)

where, \(L_{\text{int}}\) is the internal work, \(L_{F_n}\) is the work of the inertial force and \(L_{\text{ext}}\) is the work of the external force. The total internal work can be divided to the mechanical work and the work done by the phase transformation.

$$L_{\text{int}} = L_{\text{mech}} + L_{\text{trans}}$$

(9)

where, \(L_{\text{mech}}\) is the work due to the mechanical stresses and \(L_{\text{trans}}\) is the work done by the phase transformation of SMA wires.

In this study, the governing equations are derived using the (RMVT) [26,27] in order to satisfy the interlaminar continuity of transverse stresses between the layers. For the study of SMA hybrid multilayered sandwich plate, the RMVT is explained as follows:

$$\delta L_{\text{int}} = \sum_{k=1}^{N_i} \int_{a_k}^{b_k} \left( \delta \epsilon_{\text{pc}}^k(\xi) \sigma_{\text{pc}}^k(\xi) + \delta \epsilon_{\text{nc}}^k(\xi) \sigma_{\text{nc}}^k(\xi) \right) d\Omega_{zk}$$

(10)

The term \(\delta \sigma_{\text{nc}}^k(\xi) (e_{\text{nc}}^k(\xi) - e_{\text{nc}}^k(\xi))\) is Lagrange’s Multiplier which permits the compatibility of the transverse strains \(\epsilon_{\text{nc}}^k\), \(\Omega\) and \(A_k\) stand for the in-plane and the thickness domains of the lamina, respectively. Subscript \(C\) implies that the corresponding parameters and are calculated by the geometrical relations, while \(C\) indicates the corresponding parameters and are obtained using the constitutive equations. Also, subscript \(M\) means that the stress components are assumed a priori. In the framework of CUF, the strain components are written by the following form:

$$\epsilon_{\text{pc}}^k(\xi) = \left\{ \epsilon_{xx}(\xi), \epsilon_{yy}(\xi), \epsilon_{xy}(\xi) \right\}^T = D_p \, u^k(\xi)$$

(11a)

$$\epsilon_{\text{nc}}^k(\xi) = \left\{ \gamma_{xz}(\xi), \gamma_{yz}(\xi), \epsilon_{zz}(\xi) \right\}^T = (D_{np} + D_{nz}) \, u^k(\xi)$$

(11b)

where the subscripts \(p\) and \(n\) indicate the in-plane and normal components, respectively. The differential matrices are defined by the following relations:

$$D_p = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ 0 & 0 & \partial_z \end{bmatrix}, \quad D_{np} = \begin{bmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_y \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{nz} = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_z & 0 \\ 0 & 0 & \partial_z \end{bmatrix}$$

(12)
Also, the stress components are expressed in the following form:

\[ \sigma_{p}^k(\xi) = C_{pp}^k(\xi)E_{p}^k(\xi) + C_{pn}^k(\xi)E_{n}^k(\xi) \]

\[ \sigma_{n}^k(\xi) = C_{np}^k(\xi)E_{p}^k(\xi) + C_{nn}^k(\xi)E_{n}^k(\xi) \]

where \( C_{pp}^k, C_{np}^k, C_{pn}^k, \text{ and } C_{nn}^k \) are expressed as follows:

\[ C_{pp}^k(\xi) = \begin{bmatrix} C_{11}^k(\xi) & C_{12}^k(\xi) & C_{16}^k(\xi) \\ C_{12}^k(\xi) & C_{22}^k(\xi) & C_{26}^k(\xi) \\ C_{16}^k(\xi) & C_{26}^k(\xi) & C_{66}^k(\xi) \end{bmatrix}, \quad C_{pn}^k(\xi) = \begin{bmatrix} 0 & 0 & C_{13}^k(\xi) \\ 0 & 0 & C_{23}^k(\xi) \\ 0 & 0 & C_{36}^k(\xi) \end{bmatrix} \]

\[ C_{np}^k(\xi) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_{nn}^k(\xi) = \begin{bmatrix} C_{33}^k(\xi) \\ 0 \\ 0 \end{bmatrix} \]

In the above relations the \( C_{kk}^k \) implies the stiffness of the SMA hybrid composite for the \( k \)th lamina of the plate. The material properties of a SMA hybrid composite lamina are calculated as follows [28]:

\[ E_{i}(\xi) = E_{k}^k c_{i} + E_{i}(\xi)k_{s} \]

\[ E_{i}(\xi) = E_{k}^k \left[ 1 - \sqrt{k_{c}} \left( 1 - E_{i}(\xi) \right) \right] \]

\[ G_{lt}(\xi) = \frac{G_{lt}G_{s}(\xi)}{(k_{c}G_{s}(\xi) + k_{c}G_{l})} \]

\[ v_{lt} = v_{lt}^k c_{k} + v_{lt}k_{s} \]

where indices ‘s’ and ‘c’ stand for the SMA and the composite medium material, respectively. When RMVT is used to derive the governing equations, the displacements and transverse stresses are unknowns; therefore the constitutive equations must be rewritten in the following form:

\[ \sigma_{p}^k(\xi) = C_{pp}^k(\xi)E_{p}^k(\xi) + C_{pn}^k(\xi)E_{n}^k(\xi) \]

\[ \sigma_{n}^k(\xi) = C_{np}^k(\xi)E_{p}^k(\xi) + C_{nn}^k(\xi)E_{n}^k(\xi) \]

The new coefficients are expressed:

\[ \tilde{C}_{pp}^k(\xi) = C_{pp}^k(\xi) - C_{pn}^k(\xi)C_{nn}^{-1}(\xi)C_{pn}^k(\xi) \quad \tilde{C}_{np}^k(\xi) = C_{np}^k(\xi)C_{nn}^{-1}(\xi) \]

\[ \tilde{C}_{pn}^k(\xi) = -C_{mn}^{-1}(\xi)C_{np}^k(\xi) \quad \tilde{C}_{nn}^k(\xi) = -C_{nn}^{-1}(\xi) \]

By substituting Eqs. (17), (18), (22) and (27) in Eq. (21), the following expression is obtained:

\[ \delta L_{int}^k = -\left\{ \delta q_{k}^{T}(\xi) D_{1}(N_{1}) Z_{pp}^{krs}(\xi) D_{1}^{T}(N_{1}) q_{k}^{*}(\xi) \right\} \triangleright \Omega + \left\{ \delta q_{l}^{T}(\xi) D_{1}(N_{1}) Z_{pn}^{krs}(\xi) N_{j} q_{l}^{*}(\xi) \right\} \triangleright \Omega + \left\{ \delta q_{l}^{T}(\xi) D_{1}(N_{1}) Z_{np}^{krs}(\xi) N_{j} q_{l}^{*}(\xi) \right\} \triangleright \Omega \]

\[ \left\{ \delta q_{l}^{T}(\xi) Z_{np}^{k}(\xi) N_{j} q_{l}^{*}(\xi) \right\} \triangleright \Omega \]

where the layer’s stiffness and compliance are defined as follows:

\[ (Z_{pp}^{krs}(\xi), Z_{pn}^{krs}(\xi), Z_{np}^{krs}(\xi)) = (C_{pp}^{k}(\xi), C_{pn}^{k}(\xi), C_{np}^{k}(\xi), E_{k}^{k}) \]

\[ k_{k}^{k} E_{k}^{k} \]

\[ \left\{ \delta q_{l}^{T}(\xi) Z_{im}^{krs}(\xi) N_{j} q_{l}^{*}(\xi) \right\} \triangleright \Omega \]

The symbols ‘<…>’ indicate the integrals on \( \Omega \). The integration in the thickness direction can be obtained a priori as follows:

\[ E_{k}^{k}, E_{s}, E_{s}, \int_{0}^{1} (F_{r}F_{r}, F_{s}F_{s}, F_{s}F_{s}) \right\} \triangleright \Omega \]

And therefore the \( \delta L_{int}^k \) can be written as follows:

\[ \delta L_{int}^k = \delta q_{l}^{T}(\xi) \left[ K_{k}^{k}(\xi) q_{k}^{*}(\xi) + K_{k}^{k}(\xi) q_{k}^{*}(\xi) \right] + \delta q_{l}^{T}(\xi) \left[ K_{k}^{k}(\xi) q_{l}^{*}(\xi) + K_{k}^{k}(\xi) q_{l}^{*}(\xi) \right] \]

where

\[ K_{k}^{k}(\xi) = \left\{ D_{1}(N_{1}) Z_{pp}^{krs}(\xi) D_{1}^{T}(N_{1}) \right\} > \Omega \]

\[ K_{k}^{k}(\xi) = \left\{ D_{1}(N_{1}) Z_{pn}^{krs}(\xi) N_{j} + D_{1}^{T}(N_{1}) E_{s} N_{j} + E_{s} N_{j} \right\} > \Omega \]

\[ K_{k}^{k}(\xi) = \left\{ N_{j} E_{s} D_{1}(N_{1}) + E_{s} N_{j} N_{j} - N_{j} Z_{np}^{krs}(\xi) D_{1}(N_{1}) \right\} > \Omega \]

\[ K_{k}^{k}(\xi) = \left\{ -N_{j} Z_{im}^{krs}(\xi) N_{j} \right\} > \Omega \]

Above components are [3x3] ‘fundamental nuclei’ which the stiffness matrices of the whole structure can be obtained by assembling them through the indices \( k, r, s, i, j \). The explicit expression of this nucleus are expressed as follows:
reduced integration is employed at layer-level in order to overcome therefore, they must be remain in the integral domain. Both the $K_k$

\[ K_{ij}^{k} \]

\[ K_{ij}^{k} \]

$K_{ij}^{k}$

\[ K_{ij}^{k} \]

\[ K_{ij}^{k} \]

\[ K_{ij}^{k} \]

\[ K_{ij}^{k} \]

\[ K_{ij}^{k} \]

\[ K_{ij}^{k} \]

Since, the coefficients $Z_{ij}^{k}$ (i, j = p, n) are location depended; therefore, they must be remain in the integral domain. Both the integrals in the surface and in the thickness direction are obtained numerically by the Gaussian quadrature method. The selective reduced integration is employed at layer-level in order to overcome the shear locking phenomenon [29]. $\delta L_{\text{intSMA}}^{k}$ is obtained by:

\[ \delta L_{\text{intSMA}}^{k} = \int_{\Omega_s} \int_{\Omega_t} \left( \delta \epsilon_{\Sigma}^{k} \right) \delta \epsilon_{\Sigma}^{k} \left( \xi \right) + \delta \epsilon_{\Sigma}^{y} \delta \epsilon_{\Sigma}^{y} \left( \xi \right) d\Omega d\Omega \]

(24)

where $\sigma_{\Sigma}^{k}$ and $\sigma_{\Sigma}^{y}$ are the stresses due to the phase transformation in the x and y directions respectively, that can be defined by the following form:

\[ \sigma_{\Sigma}^{k} = k_{\Sigma}^{k} \epsilon_{\Sigma}^{k} \]

(25a)

\[ \sigma_{\Sigma}^{y} = k_{\Sigma}^{y} \epsilon_{\Sigma}^{y} \]

(25b)

where $k_{\Sigma}^{k}$ and $k_{\Sigma}^{y}$ indicate the volume fraction of the SMA wires in the $k^{th}$ layer in the x and y directions, respectively. The entry of $\epsilon_{\Sigma}^{k} (i = x, y)$, which is because of the phase transformation in the SMA wires is explained as follows:

\[ \epsilon_{\Sigma}^{k} (\xi) = -\epsilon_{S}^{k} \left( E_{k}^{S} \right) \frac{d}{dx} \]

(26a)

\[ \epsilon_{\Sigma}^{y} (\xi) = -\epsilon_{S}^{y} \left( E_{k}^{S} \right) \frac{d}{dx} \]

(26b)

By substituting Eqs. (36)–(38) in Eq. (35), the following expression is obtained:

\[ \delta L_{\text{intSMA}}^{k} = \left[ \delta q_{ij}^{k} \right] P_{\text{sma, rl}}^{k} \]

(28)

where

\[ P_{\text{sma, rl}}^{k} = \left[ p_{\text{sma, rl}}^{k} \right] \]

(29a)

\[ p_{\text{sma, rl}}^{k} = \left[ \int_{\Omega_s} N_{i} \phi_{k}^{\Sigma} (\xi) d\Omega \right] \]

(29b)

and

\[ E_{r} = \int_{\Omega_t} d\Omega \]

(31)

According to the CUF, $\delta L_{F_{n}}^{k}$ is obtained as follows:

\[ \delta L_{F_{n}}^{k} = \left[ \delta q_{ij}^{k} \right] M_{ij}^{k} q_{ij}^{k} \]

(32)
where
\[
\mathbf{M}^{ksi} = \begin{bmatrix}
\epsilon_{ksi} & [N_iN_j] & 0 & 0 \\
0 & [N_iN_j] & 0 & 0 \\
0 & 0 & [N_iN_j] & 0
\end{bmatrix}
\]
(33)
and
\[
m_{ri}^k = \int_{\gamma_{ri}} \rho_k F_r F_s dz
\]
(34)

The method utilized to derive finite element stiffness/compliance matrices is employed to obtain the work done by the external loads. For example it is assumed that a distribution of pressure is applied on the layer \( k \), with the distant \( \zeta_k = \zeta_k^1 \) from the reference surface. The external work done by these pressure is obtained as follows:
\[
\delta W_{ext} = \int_{\Omega_k} \delta \mathbf{u}^T (x,y,\zeta_k^1) \mathbf{P}(x,y,\zeta_k^1) d\Omega
\]
(35)
where
\[
\mathbf{u}^k = F_i N_i \mathbf{q}_{ri}^k \quad (i = 1, 2, ..., N_i)
\]
(36)
and \( \mathbf{P}(x,y,\zeta_k^1) \) is the pressure and can be written by the following relation:
\[
\mathbf{P}^k = F_i N_i \mathbf{p}^k_{ri} \quad (i = 1, 2, ..., N_i)
\]
(37)
where
\[
\mathbf{p}^k_{ri} = \begin{bmatrix}
p_{xri}^k & p_{yri}^k & p_{zri}^k
\end{bmatrix}^T
\]
(38)

Therefore:
\[
\mathbf{P}^k = F_i N_i \mathbf{p}^k_{ri}
\]
(39)

By substituting Eq. (48) and Eq. (52) in Eq. (47), the following relation is obtained:
\[
\delta W_{ext} = \int_{\Omega_k} \delta \mathbf{q}_{ri}^T (\xi) \left( F_i F_s^T \right) (N_iN_j) \mathbf{p}_{ri}^k d\Omega = \delta \mathbf{q}_{ri}^T (\xi) \mathbf{P}_{ri}^k
\]
(40)

where:
\[
\mathbf{P}_{ri}^k = \begin{bmatrix}
\epsilon_{ksi} & [N_iN_j] & 0 & 0 \\
0 & [N_iN_j] & 0 & 0 \\
0 & 0 & [N_iN_j] & 0
\end{bmatrix}
\]
(41)

3.1. Mixed LW/ESL models

In the recent works done by Carrera [30,31], the transverse shear/shear stresses are always modeled by a LW approach, because it is necessary to enforce the interlaminar continuity of theses stresses. Meanwhile, the displacements can be described as ESL or LW. If the ESL models are used, the description of displacements doesn’t considered the zig-zag effects as discussed in Ref. [19] and therefore the accuracy of the solution is to be less. On the other hand, if the LW approach is employed, the accurate displacements are obtained but the computational cost of the problem is very high. In fact, the LW approach is particularly required when the layers’ stiffness of the laminate are very different from each other. The aim of mixed LW/ESL models is that, in the case of sandwich structures with multilayer face sheets, it is convenient to establish the ESL and LW models such that the ESL models is used for the face sheet level and LW models are employed at sandwich level. For example, as shown in Fig. 2, one sandwich plate is considered, which has multilayer faces consists of two layers. If a first-order theory (\( N = 1 \)) is utilized for the analysis, in the Figs. 3–6 the assembling scheme are presented for different fundamental nuclei of equation [21]. It must be mentioned that the nodal stress variables are eliminated using the ‘static condensation’ technique [32]. Therefore, the equation (15) are re-written by the following form:
\[
\mathbf{K}(\xi) \mathbf{q}(\xi) + \mathbf{M}(\xi) \dot{\mathbf{q}}(\xi) = \mathbf{P}(\xi)
\]
(42)

where
\[
\mathbf{K}(\xi) = \left[ \mathbf{K}_{uu}(\xi) - \mathbf{K}_{uw}(\xi) (\mathbf{K}_{uw}(\xi))^{-1} \mathbf{K}_{uw}(\xi) \right] \quad \mathbf{P}(\xi) = \mathbf{P} - \mathbf{P}_{\text{smaw}}(\xi)
\]
(43)

\[
\delta \mathbf{q}_{ri}^T (\xi) : \quad \mathbf{K}_{wri}^k (\xi) \mathbf{q}_{ri}^k (\xi) + \mathbf{K}_{wri}^k (\xi) \mathbf{g}_{ri}^k (\xi) + \mathbf{M}^{ksi} \mathbf{q}_{ri}^k (\xi) = \mathbf{P}_{ri}^k - \mathbf{P}_{\text{smaw}}^k (\xi)
\]
(43a)

According to these equations, it can be observed that stiffness/compliance matrices and also force vector are dependent on the martensite volume fraction and so, these stiffness/compliance matrices and force vector are dependent on the time and the position of any point on the plate. Meanwhile, the martensite volume fraction is dependent on the stress and subsequently the displacement values, hence it can be said that these stiffness/compliance matrices and force vector are unknown. Consequently, in this research not only the material properties are variable with time and position, but they are also unknown. Therefore, the governing equations of motion and the kinetic relations of phase transformation are coupled together, which makes the problem physically non-linear.
The nodal variables in LW/ESL models are:

\[
\begin{align*}
\mathbf{u}_{iT} &= \{ u_{x1}^{face1}, u_{y1}^{face1}, u_{z1}^{face1}, u_{x1}^{core}, u_{y1}^{core}, u_{z1}^{core}, u_{x1}^{face2}, u_{y1}^{face2}, u_{z1}^{face2} \}^T \\
&= \{ u_{x1}, u_{y1}, u_{z1}, u_{x1}^{face1}, u_{y1}^{face1}, u_{z1}^{face1}, u_{x1}^{core}, u_{y1}^{core}, u_{z1}^{core} \}^T \\
&= \{ u_{x1}^{face2}, u_{y1}^{face2}, u_{z1}^{face2}, u_{x1}^{face2}, u_{y1}^{face2}, u_{z1}^{face2} \}^T
\end{align*}
\]

while, in the case of fully layer-wise nodal variables are:

\[
\mathbf{u}_{IT} = \{ u_{x1}^{1}, u_{y1}^{1}, u_{z1}^{1}, u_{x1}^{2}, u_{y1}^{2}, u_{z1}^{2}, u_{x1}^{3}, u_{y1}^{3}, u_{z1}^{3}, u_{x1}^{4}, u_{y1}^{4}, u_{z1}^{4}, u_{x1}^{5}, u_{y1}^{5}, u_{z1}^{5} \}^T
\]

(46)

Therefore, it can be seen that, ESL/LW models allow to save degrees of freedom (12 d.o.f.) in comparison to fully layer-wise models (18 d.o.f.), while the results show the same accuracy of fully LW models. The advantages of mixed LW/ESL models are more remarkable, if higher-order theories with more layers are considered. For example, when the faces are consists of five layers and the third order theory is employed, the degrees of freedom for each node in the case of fully layer-wise are 102, while in the case of LW/ESL model are 30. In general, it can be demonstrated that, in the LW/ESL models the total number of freedom is \((9N + 3)\) which is independent on the number of layers, while in the LW models is \((3N/(N+1) - N + 1)\) which is dependent on the number of layers. This advantage of mixed LW/ESL models is evident in this research. In other words, since, the problem of this research is nonlinear and time depended, the run time of the program is very high. Therefore, employing the model with high accuracy and low computational cost same as the mixed LW/ESL models, is very vital in this study. In order to abbreviate the mixed LW/ESL models, a new acronym “LEMN” is defined. The letter L means LW and also
An approximation of the initial acceleration can be calculated by:
\[ \ddot{q}_0 = M^{-1}(P_0 - Kq_0) \]  
(52)

where the index 0 indicates the initial value of the corresponding vector. At the end of every time increment, the new velocity and acceleration vectors are calculated as follows:
\[ \ddot{q}_{m+1} = a_3(a_2q_m + a_1q_m - a_5q_m) \]  
(53a)
\[ q_{m+1} = q_m + a_2q_m + a_1q_{m+1} \]  
(53b)

where \( a_1 = \alpha \Delta t \) and \( a_2 = (1 - \alpha)\Delta t \).

3.3. The proposed numerical solution procedure

The following step-by-step procedure is developed to solve the highly non-linear equations:

1. At the first step initial values are adopted for \( q_0 \), \( \dot{q}_0 \) and \( \{\xi^{s}_m\}_0 \) vectors (they are usually assumed zero values) and then solving Eq. (52) for initial acceleration \( \ddot{q}_0 \).
2. Defining the new time \( t_{m+1} = t_m + \Delta t_{m+1} \) and adopting \( \{\xi^{s}_m\}_m \) and \( \{\xi^{s}_m\}_k \).
3. Calculating the material properties and \( \chi_m(\xi) \) using the \( \{\xi^{s}_m\}_m \).
4. Solving Eq. (49) for \( q_{m+1} \) based on the \( \{\xi^{s}_m\}_m \).
5. Calculating \( \{\sigma\}_m \) using the \( \{\xi^{s}_m\}_m \) and \( q_{m+1} \).
6. Computing \( \{\xi^{s}_m\}_m \) using the phase transformation kinetic equations, based on the \( \{\sigma\}_m \).
7. The iterative technique is continued until the convergence is obtained. The following criterion can be employed in this regard:
\[ \max \left( \frac{|\xi^{s}_m - \xi^{s}_{m+1}|}{|\xi^{s}_{m+1}|} \right) < \delta \]  
(54)

where the index \( p \) implies the predictor, \( k \) implies \( k \)th layer of the plate and \( \xi^{s}_{m+1} \) indicates the value of martensite volume fraction of the \( i \)th gauss quadrature point in the \( x \) and \( y \) directions respectively for each element. Also, \( \delta \) is a sufficiently small number. If the convergence criterion is obtained, \( q_{m+1} \) and \( \dot{q}_{m+1} \) are calculated using Eq. (48) and then, increment the time and the mentioned steps are repeated from step 2 to step 7. If the convergence criterion is not obtained, the new estimates of \( \{\xi^{s}_m\}_m \) is computed using the relaxation method, by the following form:

In step (6), to identify the state of phase transformation at each gauss quadrature point on the plate, the algorithm shown in Fig. 7 is utilized at every time increment.

4. Numerical results and discussion

A new program code in MATLAB software is developed to derive the results based on the procedure mentioned in the previous sections. At first, in order to verification the present mixed LW/ESL model, a particular example is studied and compared with the available results reported in the literature. Some examples regarding the nonlinear dynamic response of the sandwich plate with isotropic and also composite face sheets embedded with SMA wires are investigated as new results. Some parametric studies such
as thickness of face sheets -to-total thickness ratio, location of SMA wires, plate aspect ratio, effect of load intensity, and also the effect of boundary conditions, on the loss factors are investigated.

Example 1. In order to validate the proposed mixed LW/ESL models, a static analysis of a simply supported sandwich plate with flexible core and composite face sheets, is investigated. The face sheets are consists of ten layers with $0/90/0/90/0/90/0/90/0/0$ scheme lamination.

The geometrical and material properties of the face sheets and the core are presented in the Tables 1 and 2, respectively [20].

The sandwich plate is under bi-sinusoidal load over the top surface of the plate in $z$ direction as follows:

$$P_z = p_z \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi y}{b} \right)$$

The results of this example are normalized by the following relations:

$$U(x, y, z) = \frac{u_2 100h^3 E_2}{p_z a^4}, \quad \sigma_{zz}(x, y, z) = \frac{\sigma_{zz}}{p_z}, \quad \sigma_{zz}(x, y, z) = \frac{\sigma_{zz}}{p_z}$$

The normalized deflections at the center of the sandwich plate are listed in Table 3 for different $a/h$ ratios. It can be seen that a very good agreement is observed between the mixed LW/ESL results and 3D results [20]. In other words, in this example the degrees of freedom are 39 for each node in the LEM4 model, while in the case of full layerwise are 255. Therefore, it can be seen that the mixed

---

**Table 1**

| Geometrical and material properties of the face sheets [20]. |
|-----------------|-----------------|
| $E_i$(GPa)      | 50              |
| $E_f = E_d$(GPa) | 10              |
| $\rho$          | 0.25            |
| $G$(GPa)        | 5               |
| $h$ for each layer of face sheets (m) | 0.02 |
| $a = b$(m)      | 4, 10, 100, 1000 |
| Scheme lamination | 0/90/0/90/0/90/0/90/0/90 and 90/0/90/0/90/0/90/0/90/0 |

---

**Table 2**

| Geometrical and material properties of the core [20]. |
|-----------------|-----------------|
| $E_i$(GPa)      | 0.01            |
| $E_d$(GPa)      | 75.85           |
| $\rho$          | 0.01            |
| $G$(GPa)        | 22.5            |
| $h$(m)          | 0.6             |
| $a = b$(m)      | 4, 10, 100, 1000 |

---

**Table 3**

| Normalized deflections at the center of the sandwich plate for different $a/h$ ratios. |
|-----------------|-----------------|
| $a/h$           | 3D [20]         |
| 4               | 10              |
| 10              | 100             |
| 100             | 1000            |
| LEM1            | 55.2528         |
| LEM2            | 55.2498         |
| LEM3            | 55.3473         |
| LEM4            | 55.3475         |

---

**Fig. 7.** Proposed algorithm for dynamic phase transformation.
LW/ESL models reproduce fully layerwise results with saving the computational cost, which is very important in this transient nonlinear research. According to Table 3, it can be observed that the LEM2 model has a good accuracy with low computational cost. In other words, the error of LEM2 model for $a/h = 10$ is less than 0.07% and the degrees of freedom for each node in this model are 21, while in the LEM4 model are 39. Therefore, for the next analysis of this research the LEM2 model is employed.

Fig. 8 shows the distribution of the normalized deflection through the thickness for the thickness ratio $a/h = 4$. It is observed that the LEM4 model presents very well the 3D results taking into account the zig-zag form. Figs. 9 and 10 show the distribution of transverse shear stresses $\tau_{xz}$ and $\tau_{zz}$ through the thickness on the points $(0, b/2)$ and $(a/2, b/2)$, respectively. Also here, it can be seen that the LEM4 model allows obtaining the 3D results and also the interlaminar continuity of transverse stresses is satisfied, because of using the mixed formulation with the RMVT.

Example 2. In this example for further validation of the present model, especially in the case of dynamical response, the present model is verified at two parts. At the first part, in order to verify the dynamical phase transformation scheme, a dynamic response of a full SMA beam is investigated and the results are compared with the available results. Zbiciak [3] studied the dynamic response of a SMA beam based on the rheological model of SMA material with the assumption that the material properties are constant. Therefore, to enable comparison, in the present model the material properties are considered to be constant and the present results are compared with the results obtained by Zbiciak. The material properties and geometrical parameters are presented in Table 4. The SMA beam is subjected to a step impulse load at the center of the beam, having the intensity expressed as follows:

\[ f(t) = 100 \text{ kN if } t \leq 0.05 \quad \& \quad 0 \text{ kN if } t > 0.05. \]

A uniform 20 elements mesh is employed along the length of the beam and the cross-section of the beam is divided into 30 layers. The deflection at the center of the beam is shown in Fig. 11. As can be seen, the results of the present model are reasonably in good agreement with the results obtained by Ref. [3]. The maximum discrepancy is less than 8%. The discrepancies are mainly due to the different solution procedures. Specially, in Ref. [3], the finite difference method (FDM) along with Runge–Kutta method is used for solving the problem. Therefore, the deviations of the

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>x = 0 clamped</th>
<th>x = L Simply supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1 m, $h = 0.08$ m, $b = 0.05$ m</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>50 GPa, $\sigma^{\text{M}} = 350$ MPa, $\sigma^{\text{K}} = 150$ MPa</td>
<td></td>
</tr>
</tbody>
</table>

---

**Table 4**

Geometrical parameters and material properties of the SMA beam [3].

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>x = 0 clamped</th>
<th>x = L Simply supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1 m, $h = 0.08$ m, $b = 0.05$ m</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>50 GPa, $\sigma^{\text{M}} = 350$ MPa, $\sigma^{\text{K}} = 150$ MPa</td>
<td></td>
</tr>
</tbody>
</table>

---

**Fig. 8.** Variation of normalized deflection through the thickness.

**Fig. 9.** Variation of normalized transverse shear stresses through the thickness.

**Fig. 10.** Variation of normalized transverse normal stresses through the thickness.

**Fig. 11.** Time history of deflection at the center of the SMA beam.
predicted structural stiffness with respect to the exact stiffness may be remarkable for FDM. For this reason, the response and also the period time resulted by FDM is different in comparison to the present model. It must be noted that, in Ref. [3], only the flexural equation has been solved, while in the present model, the membrane equation is also considered.

At the second part, in order to verify the dynamic response of the sandwich panel, the present model is simplified to the sandwich beam with composite faces and flexible core, as the effect of SMA wires is eliminated by neglecting the volume fraction of SMA wire. Lam and Chun [34] analyzed the dynamic response of sandwich beam subjected to impulse loading using the superposition of normal modes. The sandwich beam consists of isotropic face sheets and orthotropic core and its material and geometrical properties are:

\[ L = 91.44 \text{ cm}, \quad c = 1.27 \text{ cm}, \quad h_t = h_b = 0.045 \text{ cm}, \quad b = 2.54 \text{ cm} \]

\[ E_c = 201.74 \text{ MPa}, \quad G_c = 82.68 \text{ MPa}, \quad \rho_c = 32.8 \text{ kg/m}^3 \]

\[ E_i = E_s = 6980 \text{ MPa}, \quad \nu_s = 0.25, \quad \rho_t = \rho_b = 2680 \text{ kg/m}^3 \]

The sandwich beam is subjected to impulse load over the top surface of the beam. The load is assumed to be uniform pressure having the intensity expressed as follows:

\[ P(t) = 68.9 \left( 1 - \frac{t}{0.004} \right) \exp \left( -\frac{1.98}{0.004} t \right) \text{(kPa)} \]

The deflection at the center of the top face sheet is shown in Fig. 12. As can be seen, the results of the present model are in good agreement with the results obtained analytically by Lam and Chun [34]. It must be noticed that the theory that is used by Lam and Chun [34] underestimates the global stiffness of the sandwich beam. For example, the fundamental natural frequency obtained by this model is 57.28 Hz [34], which is less than the frequency obtained by 3D model (57.724 Hz, result of ABAQUS software done by the authors).

**Example 3.** In this example, the nonlinear dynamic response of the sandwich plate with flexible core and elastomer face sheets embedded with SMA wires is analyzed. Geometrical and material properties of the sandwich are as follow:

\[ E_e = 2.866 \text{ GPa}, \quad \nu_e = 0.3 \]

\[ \rho_t = \rho_b = 1500 \text{ kg/m}^3, \quad a = 0.6 \text{ m}, \quad b = 1 \text{ m}, \quad h_t = h_b = 0.03 \text{ m}, \quad h = 0.16 \text{ m} \]

\[ E_c = 75 \text{ MPa}, \quad G_c = 30 \text{ MPa}, \quad \nu_c = 0.25, \quad \rho_c = 100 \text{ kg/m}^3 \]

It is assumed that the face sheets are consists of 8 layers and SMA wires are embedded in the layers 1, 2, 7 and 8 of both face sheets along the x direction. The volume fraction of SMA wires is 40% for each layer. The material properties of the SMA wire are given in Table 5.

The boundary conditions are simply supported and a uniform step impulse pressure with the intensity of \( P = 3 \text{ MPa} \) is applied over the upper surface of the sandwich plate in the z direction. Fig. 13 shows the stress-strain diagram history at the midpoint \((a/2, b/2)\) of the upper layer of the top face sheet considering the pseudoelastic behavior of the SMA wires. It can be observed that, the stress-strain history shows the hysteresis loops. In Fig. 14, the time history of the deflection at the center point of the upper surface of the plate \(u_s(a/2, b/2, h/2)\) is presented. From this figure, it can be seen that, the response of the plate is damped gradually. This is because of the hysteresis loops which dissipate the energy.

**Table 5**

<table>
<thead>
<tr>
<th>Material properties of shape memory alloys [23].</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_e = 67 \text{ GPa} )</td>
</tr>
<tr>
<td>( E_s = 26.3 \text{ GPa} )</td>
</tr>
<tr>
<td>( \sigma_y = 100 \text{ MPa} )</td>
</tr>
<tr>
<td>( \sigma_y = 170 \text{ MPa} )</td>
</tr>
<tr>
<td>( c_t = 0.067 )</td>
</tr>
</tbody>
</table>

![Stress-strain diagram at the center of the upper layer of the top face sheets.](image)

**Fig. 13.** Stress-strain diagram at the center of the upper layer of the top face sheets.

![Time history of deflection at the center of the top face sheet.](image)

**Fig. 12.** Time history of deflection at the center of the top face sheet.

![Time history of deflection at the center of the upper surface of plate.](image)

**Fig. 14.** Time history of deflection at the center of the upper surface of plate.
3.1. Effect of intensity of the pressure:

In Fig. 15, the response of the sandwich plate is studied for different intensity of the pressure (IP). Fig. 16 shows the variation of corresponding loss factor with different IP. From this figure it can be seen that, as the IP increases, the loss factor increases. In other words, as the IP increases, the structure shows higher capability for dissipating the energy. Loss factor of the structure is evaluated by the measurement of the vibration amplitude and using the following relation:

$$\zeta = \frac{1}{2\pi} \ln \left( \frac{x_1 - x_{\text{mean}}}{x_{n-1} - x_{\text{mean}}} \right)$$

3.2. Effect of location of the SMA wires:

In order to investigate the effect of through thickness location of the SMA wire inside the face sheets, the dynamic response of sandwich plate is studied for 4 cases as shown in Fig. 17. In this regard, the SMA wires are embedded in the two layers of each face sheets symmetrically. In the case 1, the SMA wires are embedded in the layers 1 and 8 for both face sheets. In the case 2, they are embedded in the layers 2 and 7; in the case 3, the SMA wires are embedded in the layers 3, 6 and in the last case, they are embedded in the layers 4 and 5. According to Fig. 17, it can be seen that, when the SMA wire are embedded in the layers 1 and 8, they show more capability for damping the response. In other words, as the SMA wires are embedded further to the mid surface of the face sheets, the dissipation of energy by the SMA wires increases. The reason is that, as the SMA wires are embedded in the outer layers, the
magnitude of the stress in the SMA wires reaches faster to the critical stress for transformation.

**Example 4.** This example deals with the nonlinear dynamic analysis of sandwich plate with flexible core and composite face sheets embedded with SMA wires. The sandwich plate has a $[(0/\text{C}14)_{8}/(0/\text{C}14)_{8}]$ lamination scheme. Geometrical and material properties of the sandwich are as follow:

- $E_1 = 50$ GPa, $E_2 = E_3 = 10$ GPa, $G_{12} = G_{13} = G_{23} = 5$ GPa,
- $r_{12} = r_{13} = r_{23} = 0.25$
- $\rho_t = \rho_b = 1600$ kg/m$^3$, $a = 0.6m$, $b = h_t = h_b = 0.03m$,
- $h = 0.16m$, $E_c = 75$ MPa, $G_c = 30$ MPa, $\nu_c = 0.25$, $\rho_c = 100$ kg/m$^3$.

The SMA wires with volume fraction 40% are embedded in the layers 1, 2, 7 and 8 in the x direction. The step impulsive pressure with intensity $P = 10$ MPa is applied on top surface of the sandwich plate. The response is shown in Fig. 16 for two models LEM2 and EM4 (more details about the EM4 model can be found in Ref [18]). According to Fig. 18, it can be observed that the ESL models are not appropriate for studying the sandwich plate with flexible core since the error of EM4 is more than 70%.

### E 4.1. Effect of volume fraction of the SMA wires:

In Fig. 19, the effect of volume fraction on the nonlinear dynamic response of sandwich plate is studied. It can be seen that as the volume fraction of SMA wires is increased, due to the large energy dissipation of the SMA wires, the deflection history damped faster. Fig. 20, shows the variation of loss factor with volume fraction of the SMA wires. As can be seen, with increasing the volume fraction of SMA wires, the loss factor decreases.

### E 4.2. Effect of aspect ratio of the sandwich plate:

Fig. 21, shows the nonlinear dynamic response of the sandwich plate for different aspect ratio $a/b$. The intensity of pressure (IP) is such that the maximum deflection at first pick of the sandwich plate is in the same range for different aspect ratios. The IPs for different $a/b$ ratios are presented in Table 6. Fig. 21

---

**Table 6**

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>0.75</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP (MPa)</td>
<td>12</td>
<td>10</td>
<td>9</td>
<td>8.5</td>
</tr>
</tbody>
</table>

---

**Fig. 20.** Variation of the loss factor with volume fraction of SMA wires.

**Fig. 21.** Time history of deflection at the center of the upper surface of plate for different $a/b$ ratios.

**Fig. 22.** Variation of the loss factor with $a/b$ ratio.

**Fig. 23.** Time history of deflection at the center of the upper surface of plate for $h/f$ ratios.
shows that, as the a/b ratio decreases, more reduction in the amplitude of vibration can be observed in such a way that for example in the case of a/b = 0.4, after 6 cycles the amplitude of vibration reaches to the value which is about 75% of its value obtained at the first pick. Fig. 22, shows the effect of aspect ratio on the loss factor. It can be seen that with increasing the a/b ratio, the percentage of loss factor decreases. In other words, when the SMA wires are embedded in the x direction as the a/b increases, the SMA wires show higher capability in damping the response of structure.

E 4.3. Effect of the face sheet thickness to the total thickness ratio:

In Figs. 23 and 24, the effect of the face sheet thickness to the total thickness ratio is studied for a/b = 0.6. The IPs for different thickness ratios are presented in Table 7. Fig. 24, shows that as the thickness ratios increases, the loss factor of structure increases.

E 4.4. Effect of boundary conditions:

This part deals with the effect of boundary conditions on nonlinear dynamic response of the sandwich plate. In this regard, a square sandwich plate (a = b = 1m) with non-symmetric lamination scheme $[[90^\circ, 0^\circ, 90^\circ, -90^\circ], [0^\circ, 90^\circ, 0^\circ, -90^\circ]]_s$ is considered. It is assumed that the SMA wires with volume fraction 50% are embedded in the layers 2 and 7 in the x direction and also layers 1, 3, 6 and 8 in the y direction for the bottom face sheet. Also, the SMA wires with 50% are embedded in the layers 1, 3, 6 and 8 in the x direction and also layers 2 and 7 in the y direction for the top face sheet, simultaneously. The step impulsive pressure with intensity $P = 7$ MPa is applied on top surface of the plate. The dynamic response of the sandwich plate is investigated for the boundary conditions SSSS, CCCC and CSCS, where S and C mean the simply supported and clamped boundary conditions, respectively. The results are shown in Fig. 25. It can be seen that as the rigidity of the boundary conditions increases, the SMA wires show more capability in damping the response such that the loss factor is 0.65, 1.32 and 1.83 for SSSS, CSCS and CCCC, respectively. According to Figs. 26 and 27, the reason is that in the case of CCCC not only the points of the center of the plate take part in the phase transformation, also the phase transformation take place on the points near the edge, while in the case of SSSS only the center points of the plate take part in the phase transformation.

<table>
<thead>
<tr>
<th>$h_f/h$</th>
<th>0.0938</th>
<th>0.125</th>
<th>0.1563</th>
<th>0.1875</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP (MPa)</td>
<td>6</td>
<td>6.5</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 7

Intensity of pressure (IP) for different $h_f/h$ ratios.

![Figure 24. Variation of the loss factor with $h_f/h$ ratios.](image1)

![Table 7](image2)

![Figure 25. Time history of deflection at the center of the upper surface of plate for different boundary conditions.](image3)

![Figure 26. Distribution of martensite volume fraction on the upper layer of the top face sheet in the case of CCCC boundary conditions at time $t = 0.0028$ s.](image4)
5. Conclusion

This study deals with the nonlinear dynamic analysis of the sandwich plate with flexible core and composite face sheets embedded with SMA wires. The material nonlinearities due to the phase transformation of SMA wires are considered for all of the points on the face sheets. Since, the problem of this research is transient nonlinear and also the structure is thick plate with flexible core, employing a model with high accuracy and low computational cost is necessary in this study. For this reason, the mixed LW/ESL models proposed by the authors are used in this research. One of the advantages of the ESL/LW models is that, the total number of freedom is independent on the number of layers. This advantage of the mixed LW/ESL models is evidence in this study. The Brinson’s SMA constitutive equation is employed for modeling the pseudoelastic effect of the SMA wires. The governing equations are derived using the Reissner Mixed Variational Theorem (RMVT) in order to satisfy the interlaminar continuity of transverse stresses between the two adjacent layers. The governing equations of motion and the kinetic relations of phase transformation are coupled together which make the problem nonlinear and more complicated. The nonlinear equations of motions are solved based on the transient finite element along with the iterative incremental method. Results show that as the intensity of impulsive pressure, ratio of face sheet thickness to the total thickness and the volume fraction of the SMA wires increases, the loss factor is also increases. Also, when the SMA wires are embedded in the outer layers of the face sheets and also the rigidity of the edges increases, the loss factor increases. Furthermore, with decreasing the aspect ratio of the plate, the loss factor increases.

References

[34] Lam KY, Chun L. Dynamics response of a simply supported sandwich beam subjected to impulsive loading. Compos Struct 1994;27:331–7.