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Review

Stress, vibration and buckling analyses of FGM plates—A state-of-the-art review

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ABSTRACT

This paper presents a comprehensive review of the various methods employed to study the static, dynamic and stability behavior of Functionally Graded Material (FGM) plates. Both analytical and numerical methods are considered. The review is carried out with an emphasis to present stress, vibration and buckling characteristics of FGM plates predicted using different theories proposed by several researchers without considering the detailed mathematical implication of various methodologies. The effect of variation of material properties through the thickness, type of load case, boundary conditions, edge ratio, side-to-thickness ratio and the effect of nonlinearity on the behavior of FGM plates are discussed. The main objective of this paper is to serve the interests of researchers and engineers already involved in the analysis and design of FGM structures.

Contents

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1. Introduction

It has long been that the laminated composites are being extensively used in aircrafts, spacecrafts, shipbuilding, automotive and various other industries because of their flexibility in design to have desired strength and stiffness. Since, in laminated composites two dissimilar materials are bonded together, the laminates may tend to de-bond. Cracks are likely to initiate at interfaces and propagate into the weaker material section. Further, the difference in thermal coefficients of the materials may result in residual stresses. These problems can be mitigated by replacing conventionally used laminated composites with functionally graded materials where the material properties are gradually varied at microscopic scale in the thickness direction.

Functionally Graded Materials (FGMs) are the heterogeneous composite materials in which the material properties are gradually varied between two points in a predetermined manner. FGM can also be defined as a composite in which the material properties are gradually varied along a certain direction as a function of the position coordinates to achieve desired strength and stiffness. Thus, mitigating problems induced due to the sudden change of thermo-mechanical properties as in the case of laminated composites. Discontinuous changes such as a stepwise gradation of the material constituents can also be regarded as FGMs.

The concept of FGM was proposed in 1984 by materials scientists in the Sendai area as a means of preparing thermal barrier materials. However, most of the research works have been published in the last fifteen years. Tanigawa [1] presented a brief review on the method of analytical development for the thermoelastic analysis of nonhomogeneous materials, such as FGMs. It was found that the governing equations for thermoelastic problems are highly nonlinear and very difficult to solve. Therefore, such problems are addressed by introducing certain linearization techniques. Markworth et al. [2] discussed the modeling techniques involved for the evaluation of microstructure-dependent thermophysical properties as well as models for the design, processing, and performance of FGMs. Also, the authors gave recommendations relative to areas in which additional work is needed. Noda [3] presented a brief review of various types of thermally induced cracks in FGM plates and their treatments using analytical and numerical methods. The progress in the characterization, modeling and analysis of FGM structures during the year of 2000–2007 was reviewed by Birman and Byrd [4]. Liew et al. [5] discussed the developments in element-free or meshless methods and their applications for the analysis of laminated and FGM plates and shells. A critical review on the studies reported in the area of thermoelastic and vibration analyses of FGM plates was presented by Jha et al. [6]. But none of the above discussed various theories and solution methodologies employed for the stress, vibration and buckling analyses of FGM plates under single platform with distinct separate classification based on the method of analysis (analytical/numerical) and applied load case (mechanical/thermal). In this paper an attempt has been made to discuss the various methods used for analysis of FGM plates with separate subclasses based on the method of analysis and applied load case under each main class of the stress, vibration and buckling analyses.

2. Stress analysis

In this section, various theories based on three-dimensional elasticity theory and two-dimensional elasticity theory are discussed separately under the broad heading of analytical methods whereas the methods based on finite element method and meshless method are discussed under the broad heading of numerical methods employed.

2.1. Analytical methods

In this section several analytical methods used for linear and nonlinear stress analyses of FGM single-layer plates and FGM sandwich plates are discussed under two separate classes depending on the applied load: (i) Mechanical load case; (ii) Thermo-mechanical load case.

2.1.1. Three-dimensional (3D) elasticity theory

In order to obtain an exact 3D stress field and nature of stress concentration in FGM plates that have edge boundaries, a 3D elasticity boundary value problem must be solved analytically. Various solution methods employed by different authors to obtain analytical solution for 3D stress analysis based on 3D elasticity equations for FGM plates under various loading conditions and boundary conditions as well are discussed in the subsequent section.

2.1.1.1. Mechanical load case. Though the concept of FGM was proposed in the year of 1984, the first exact solution using 3D elasticity equations was presented by Pan [7] who extended Pagano's solution to derive a 3D exact solution for exponentially graded rectangular FGM plates with simply-supported boundary condition under a surface load, and expressed the solutions in terms of an elegant Stroh like formalism. It was clearly shown that the tensile stress at the top or the compressive stress at the bottom in homogeneous plate can be reduced by bonding a suitable FGM layer to it. Kashtalyan [8] used Plevako method to develop 3D elasticity solution for simply-supported FGM plates subjected to transverse load and showed that the variation of the out-of-plane normal stress in FGM plates is similar to that of homogenous isotropic plates. Later, Kashtalyan and Menshykova [9] extended the same method of solution to an FGM coating/substrate system to show that the continuity achieved in the in-plane normal stress across the coating/substrate system is at the cost of increase in the stress magnitude at the top surface of the plate. In both the above works, it was assumed that the elastic moduli were exponentially varied through the thickness for which exact solution is available with constant coefficients. However, the exact solution is hard to find for FGM plates where the material properties are varied in other non-exponential pattern. Zhong and Shang [10] also used the Plevako method for FGM plates with specific variation of material properties such as exponential, linear and reciprocal variation of material properties through the thickness of the plate. It was observed that the linear model has highest rigidity and the reciprocal model is the softest one. Li et al. [11] obtained an elastic solution for simply-supported transversely isotropic circular FGM plates subjected to pure bending. Again, Li et al. [12] used the same approach to derive a set of solutions for the axisymmetric problem of transversely isotropic FGM plates under simply-supported and clamped boundary conditions subjected to transverse load in the form of qr^k. In both the above cases, elastic compliance coefficients are considered as arbitrary functions of the thickness coordinate. Further it was emphasized that the theoretical study is strongly needed when k is a finite odd number, which would make it possible to obtain analytical solutions for FGM circular plates subjected to any type of axisymmetric load. A more general procedure was presented by Zenkour [13] who considered the effect of both transverse shear and normal deformations to solve the 3D elasticity equations for the bending analysis of simply-supported FGM plates with exponentially graded material properties through the thickness. The numerical and graphical results were presented for the displacements and stresses for various values of aspect ratio, side-to-thickness ratio and exponential law parameter. Chang and Tarn [14] developed a state space formalism for the analyses of anisotropic composite laminates and FGM plates under extension, torsion and bending, and represented in a neat explicit form so that mathematical methods such as separation of variables, eigen function expansion, transfer matrix, and matrix algebra can be used to find the analytical solution for the problem. Li et al. [15] developed a set of axisymmetric elasticity solutions for uniformly loaded annular FGM plates with various boundary conditions by using governing equations for axisymmetric problems and direct displacement method. The important feature of this method is that the material properties can be varied arbitrarily and independently along the thickness direction. Yang et al. [16] extended the FGM plate theory suggested by Mian and Spencer [17] to obtain the elasticity solution for FGM plates with material properties varied arbitrarily under different boundary conditions. It was concluded that the higher gradient in material inhomogeneity causes the stress concentration near the stiffer locations. Liu and Zhong [18] used the Peano–Baker solution method for 3D analysis of simply-supported orthotropic rectangular FGM plates based on the state space method with an arbitrary distributed material properties in the thickness direction. Zheng and Zhong [19] expanded displacement function as Fourier–Bessel series based on basic equations and presented an exact analysis for axisymmetric bending of exponentially graded FGM circular plates under two different boundary conditions namely, elastically supported and rigid slipping. An exact solution was obtained by Yun et al. [20] using the direct displacement method to study the axisymmetric bending of transversely isotropic FGM circular plates subjected to Bessel function-type transverse load under two specific boundary conditions namely elastic simply support and rigid slipping support. In addition, the analytical solutions for simply-supported and three types of boundary conditions with clamped edges were also presented. Woodward and Kashtalyan [21] derived a 3D elasticity solution for simply-supported transversely isotropic FGM plates with exponentially graded material properties using displacement functions. It was observed that an increase in the degree of inhomogeneity and anisotropy results in the increase of most of the stress components in the upper half of the plate. England's solution was extended by Yang et al. [22] to derive an elasticity solution for FGM plates with an arbitrary variation of material properties. Although the method provides an exact solution, approximation has to be made for satisfying the boundary conditions around the circumferential edge of the plate. Woodward and Kashtalyan [23] used both analytical and finite element methods to study the bending of isotropic FGM plates under localized transverse load. Comparison of the results were presented for stress and displacement fields in FGM and homogeneous plates subjected to uniformly distributed load and patch loadings obtained using both analytical and finite element methods.

2.1.1.2. Thermo-mechanical load case. Mian and Spencer [17] presented a large class of exact solutions for 3D elasticity equations for isotropic linearly elastic, inhomogeneous materials generalized from solutions for stretching and bending of symmetrically inhomogeneous plates. Yang et al. [24] extended the same method for axisymmetric bending analysis of annular FGM plates under uniform loading. The major drawback of this method of solution is that, it is valid only for traction-free boundary condition, and it cannot deal with transverse stresses. Again, Yang et al. [25] extended the same theory to derive simplified Two-Dimensional (2D) governing equations for transversely isotropic rectangular FGM plates with two opposite edges simply-supported and subjected to uniformly distributed load at the top and bottom. The influence of boundary conditions and material inhomogeneity on the transverse deflection and normal stresses were studied for FGM plates with power law variation of material properties through the thickness. Further, Reddy and Cheng [26] extended the asymptotic expansion method for the analysis of thermo-mechanical deformation of simply-supported FGM rectangular plates. In both the above cases, it was found that the assumption of the constant deflection holds good when the mechanical load case is considered, but the same does not valid in the case of thermal load. Vel and Batra [27] obtained an exact solution for 3D deformation of simply-supported FGM plates subjected to mechanical and thermal loads. The obtained displacements and stresses were used to assess the accuracy of the classical plate theory, the first-order shear deformation theory and Third-order Shear Deformation Theory for FGM plates. For thick plates, significant differences were observed between the results of exact solution and that obtained using any of the plate theories mentioned above, even when the transverse normal and the transverse shear stresses were computed by integrating the 3D elasticity equations. Chen et al. [28] investigated the thermal behavior of thick transversely isotropic FGM rectangular plates within the scope of 3D elasticity by using the state space formulation and laminate approximate model. In spite of 3D elasticity theory is being used, the accuracy of the solution depends on the discretization of given thickness into the number of layers. A 3D analytical solution based on uncoupled, quasi-static linear thermo-elasticity theory for thermo-mechanical deformations of simply-supported FGM rectangular plates subjected to time-dependent thermal loads on its top and/or bottom surfaces was presented by Vel and Batra [29]. The authors found that the longitudinal and transverse shear stresses change their sign with the passage of time. Ootao and Tanigawa [30–32] presented exact solutions for FGM plates subjected to transient thermal loading. Peng and Li [33] proposed a new approach to solve the thermo-elastic problem of an FGM circular annulus by transforming it to a Fredholm integral surface. The plane stress state condition is considered to study the effects of varying the material gradient on thermal stresses and radial displacement. It was clearly demonstrated that the thermal stresses can be relaxed for some specific gradients of material property. Alibeigloo [34] used Fourier series and state space-method to study the 3D thermoelastic deformation of simply-supported FGM rectangular plates subjected to thermo-mechanical loads. It was found from the results that the influence of inhomogeneity in the thermal load case is greater than that of mechanical load case.

2.1.2. Two-dimensional (2D) plate theories

It is not always possible to get the analytical solution for 3D elasticity equations. Particularly, the solution becomes difficult and tedious when a power law is used for the gradation of material properties. In addition, boundary value and eigenvalue problems of 3D elasticity equations are hard to solve. Thus, 3D elasticity theory is simplified into 2D problems by making certain assumption based on the kinematics of deformation and constitutive behavior.

2.1.2.1. Mechanical load. Reddy et al. [35] used a first-order shear deformation theory for bending and stretching analyses of axisymmetric FGM annular and circular plates and expressed the solutions in terms of the classical solutions based on the Kirchhoff plate theory. Again, Reddy [36] presented both theoretical formulation and finite-element model based on the Third-order Shear Deformation Theory (TSDT). Cheng and Batra [37] established an exact relationship between the deflection of an FGM plate and that of an equivalent homogeneous Kirchhoff plate. The authors have also verified the accuracy of the 2D-elasticity theory and Classical Plate Theory (CPT) using finite element method. Elishakoff et al. [38] and Elishakoff and Gentilini [39] used the Ritz energy method to study the 3D static response of clamped FGM rectangular plates subjected to uniformly distributed load at the top surface. The effects of varying the volume fraction and the thickness-to-side ratio on the non-dimensional displacements and in-plane normal stresses were presented. However, the effect of varying the volume fraction on transverse normal stresses and shear stresses (in-plane and transverse) was not reported. It was concluded that the deflection of the plates that correspond to properties intermediate to that of the metal and ceramic necessarily lies in between that of the ceramic and the metal irrespective of boundary conditions. A generalized refined self-consistent theory was developed by Bian et al. [40], who extended the Soldatos plate theory to investigate the cylindrical bending behavior of single and multi-span FGM plates. A transfer matrix method was employed to determine the shape function which will not increase the order of the set of equations to be solved irrespective of the number of layers. Zenkour [41] used sinusoidal shear deformation plate theory to develop a general formulation for FGM sandwich plates made of three layers with isotropic ceramic/metal FGM facings and homogeneous ceramic as core. The stretching and bending coupling may be avoided by choosing an FGM sandwich plate with symmetric material properties. Further using the sinusoidal shear deformation theory himself, Zenkour [42] presented Navier's analytical solution to study the static response of an FGM rectangular plate. It has been found from the results that the deflection of the plates that correspond to properties intermediate to that of the metal and ceramic necessarily lies in between that of the ceramic and the metal irrespective of boundary conditions. Chi and Chung [43,44] presented a closed-form solution for Power law FGM (P-FGM), Sigmoid FGM (S-FGM) and Exponential FGM (E-FGM) plates based on the CPT. Nguyen et al. [45] obtained shear correction coefficients for rectangular FGM plates and sandwich plates modeled using FSDT. It was found from the results that the shear correction factor is a function of the ratio between elastic moduli of constituents and material inhomogeneity. Transverse shear stresses were calculated by integrating the 3D elasticity equations and the obtained results were compared with those of TSDT and Sinusoidal Shear Deformation Theory (SSDT) calculated using constitutive relations instead of 3D elasticity, which is an inappropriate method. Fares et al. [46] presented a refined 2D theory for FGM plates using a modified version of mixed variational approach. The proposed theory accounts for transverse shear and normal strains, as well as the stresses which are completely in consistent with the top and bottom surface boundary conditions. The theory also does not require any shear correction factor. Saidi et al. [47] studied axisymmetric bending and buckling of FGM circular plates using the Unconstrained Third-order Shear Deformation Theory (UTST). In addition, relationships between the UTST solutions for thick FGM circular plates and the CPT solutions of homogeneous thin circular plates were also presented for axisymmetric bending and buckling. It was observed that the UTST results were much closer to TSDT results than FSDT ones when bending analysis is considered. Sahraee [48] presented the bending analysis of thick circular FGM sector plates using the Levinson Plate Theory (LPT). It was observed from the results that the LPT solution for deflection of homogenous plates is higher than those of FGM plates. Sahraee and Saidi [49] used the Fourth-order Shear Deformation Theory (FOST) for axisymmetric bending and stretching analyses of thick FGM circular plates and the solutions were expressed in terms of the CPT solution of homogeneous circular plates. Also, the obtained results were compared with those of FSDT, TSDT as well as an exact 3D elasticity solution. It was concluded that through the thickness variation of shear stress is more accurately predicted by FOST than by TSDT. A generalized refined theory was presented by Lu et al. [50] using the approximate laminate theory and state space method to incorporate the surface effects for FGM ultra-thin films. It was established that the surface effect on the elastic behavior of FGM films rely significantly on the volume fraction profile of the component phases. Because of the coupling between bending and stretching, it is difficult to get the analytical solution for the static analysis of FGM plates, except when the plate is simply-supported. Thus, governing equations of FGM plates have to be decoupled. Therefore, Saidi and Jomehzadeh [51] developed an analytical method to decouple the governing equations for bending–stretching analysis of FGM rectangular plates based on FSDT/Mindlin plate theory. Once stretching and bending equations are decoupled, Levy method can be effectively used for the static analysis of FGM plates with different boundary conditions. Jomehzadeh et al. [52] presented an analytical solution for stress analysis of FGM annular sector plates under various boundary conditions based on FSDT using a boundary layer function in polar coordinates. Mechab et al. [53] extended the two variable refined plate theory proposed for laminated composite plates by Kim et al. [54] for the analysis of FGM plates. Exact relationships between the bending solution of the classical (Kirchhoff) thin plate theory and the sinusoidal shear deformation thick plate theory for homogeneous and FGM plates were derived by Zenkour [55] using the mathematical similarity of governing equations of the two theories, and the basis of load equivalence. The two edges of the FGM plate were assumed to be simply supported while the remaining two edges were considered as simply supported or clamped or free. Wu and Li [56] developed the Reissner Mixed Variational Theorem (RMVT) based TSDT for the static analysis of simply-supported, multi-layered FGM plates. The obtained results were compared with those obtained using the Principle of Virtual Displacement (PVD) based TSDT. It was observed that the RMVT based TSDT is more accurate than PVD based TSDT. Further, it was suggested that the accuracy of the proposed method can be improved by replacing the present displacement model with zig-zag higher-order displacement model by taking account of the effect of transverse normal stress. A variable kinematic model was obtained by Brischetto and Carrera [57] who extended the Unified Formulation (UF) to FGM plates using RMVT. Better values of transverse normal/shear stresses were obtained with the use of RMVT as they were considered as primary variables and obtained directly from the model. Carrera et al. [58] developed several refined and advanced hierarchical models by using Carrera's Unified Formulation (CUF) for the analysis of FGM plates and shells. It was found that the required thickness of the FGM layer and the elastic property law is governed by the variable displacement, in-plane or transverse stresses) which is to be modified and on its position in the thickness direction. Again, Carrera et al. [59] used the CUF to study the effect of transverse normal strain in single-layered and multilayered plates and shells with embedded FGM layers based on classical and 2D higher-order theories. It was concluded that the increase in the order of expansion for in-plane displacements is meaningless if the transverse normal strains were not considered. The effect of normal strain is significant in thick and moderately thick plates and shells. A new higher-order theory proposed by Mantari et al. [60] for sandwich and composite laminated plates was extended for the analysis of static response of FGM plates by Mantari et al. [61]. The obtained results were compared with the other higher-order theories available in literature and the quasi-3D analysis. Though the obtained results were in excellent agreement with quasi-3D theory, they differ considerably with other higher-order theories. Daouadji et al. [62] used a higher-order theory which involves only four degrees of freedom for static and dynamic analysis of FGM plates. The Navier's solution technique was used to solve the governing equations. The obtained results were validated by comparing them with those available in the literature and concluded that the higher-order shear deformation theory gives better results compared to FSDT and TSDT. A semi-analytical Maclaurin-type power-series solution was used for the first time by Alipour and Shariyat [63] for axisymmetric bending and stress analysis of circular FGM sandwich plates based on the elasticity-equilibrium based zig-zag theory. The obtained results were compared with those obtained using ABAQUS software to demonstrate the accuracy of the theory in predicting the results even for thick sandwich plates with soft cores. Since the elasticity equilibrium equation is used rather than the constitutive equations, the transverse shear and the normal stresses do not show jumps at the layer interfaces unlike the majority of the global, layerwise and zig-zag theories proposed based on the constitutive equations. Mantari and Soares [64] used a higher-order shear deformation theory in which the displacements of the middle surface were expanded as trigonometric functions of the thickness coordinate and the transverse displacement is taken as uniform through the thickness. The Navier's solution technique is employed to obtain the solution of governing equations. The obtained results were validated by comparing with the results obtained using 3D elasticity solutions and SSDT. It was observed that the higher-order shear deformation theory gives better results compared to SSDT. A solution methodology based on boundary-discontinuous generalized double Fourier series approach was used by Oktem et al. [65] for the static analysis of FGM plates and doubly-curved shells based on the Reddy's TSDT. The obtained results were validated by comparing them with those obtained using commercially available software (ANSYS). Mantari and Soars [66] used a new trigonometric higher-order theory which includes stretching effect for static analysis of FGM plates. Governing equations were derived using the principle of virtual work and solution was obtained using the Navier's technique. The obtained results were compared with those obtained using 3D exact solution and SSDT. It was concluded that the trigonometric higher-order theory gives better results compared to SSDT. Mechab et al. [67] employed a new function for static and dynamic analyses of FGM plates using the four-variable refined plate theory. The validity of the theory was established by comparing the obtained results with those obtained using CPT, FSDT and TSDT. A quasi-3D hyperbolic shear deformation theory for bending and vibration analysis of FGM plates was developed by Thai et al. [68] and Hebali et al. [69]. The number of unknowns was reduced to five by splitting the transverse displacement into the bending shear parts. Thai and Choi [70] refined FSDT into more simple and efficient form by reducing the number of unknowns and governing equations to four instead of five for bending and free vibration analyses of FGM plates. Thai et al. [71] extended the same methodology for bending, buckling and free vibration analyses of FGM sandwich plates made of FGM face sheets and isotropic homogeneous core. A refined trigonometric plate theory with four-unknowns was proposed by Zenkour [72,73] for bending analysis of FGM plates and FGM sandwich plates. The accuracy of the obtained results was demonstrated by comparing with those already available in the literature. Zenkour [74] presented the bending response of FGM plates using a simplified shear and normal deformations theory having only four unknowns. Belabed et al. [75] presented an efficient and simple higher-order shear and normal deformation theory for bending and free vibration analyses of FGM plates. The number of unknowns and governing equations were reduced to five instead of six by splitting the transverse displacement into bending, shear and thickness stretching parts. The sinusoidal higher-order shear deformation theory was optimized by Mantari and Soars [76] for the bending analysis of FGM plates and shells with improved results compared to the five and six unknown quasi-3D trigonometric plate theories. A new, attractive, and very accurate shear strain shape function was used which potentially can be used in other numerical methods and analytical higher-order shear deformation formulations.

A new set of field equations was presented and reformulated in terms of displacement and stress potential function for the nonlinear bending analysis of inhomogeneous plates by Cheng [77] using the first-order von-Kármán nonlinear theory and mixed Fourier series solution. Ghannadpour and Alinia [78] studied the large deflection behavior of FGM plates under uniform pressure using the CPT. Effects of material properties on the deflection and stress field were presented. An inverse problem of an FGM elliptical plate with large deflection and disturbed boundary under uniform load is solved by Hsieh and Lee [79] using perturbation method. The governing equations were derived for a thin plate with large deflection based on the classical von-Kármán nonlinear plate theory. Navazi et al. [80] used a Levy type method for the nonlinear analysis of FGM plates based on the classical von-Kármán nonlinear plate theory and showed the inadequacy of linear plate theories for analysis of FGM plates even for small deflection range. An analytical solution for FGM thin plates based on physical neutral surface was presented by Zhang and Zhou [81]. The method was based on the concept given by Morimoto et al. [82] and Abrate [83] that there is no stretching and bending coupling in constitutive equations if the reference surface is properly selected. Navazi and Haddadpour [84] presented an analytical method for the cylindrical bending analysis of FGM plates using FSDT. It was observed from the results that the bending–stretching coupling makes the stress field more critical than homogeneous plates under in-plane tensile loading. It was observed that there is a significant difference in the results of nonlinear FSDT and nonlinear CPT. Alinia and Ghannadpour [85] studied the nonlinear behavior of rectangular FGM plates using the classical von-Kármán nonlinear plate theory. The solution was obtained using the minimization of total potential energy. The energy concept was combined with FSDT and TSDT by Khabbaz et al. [86] for the large deflection and through the thickness stress analysis of FGM plates. Significant differences were found in the stress values of CPT for thick plates and the compressive stress on the upper surface as predicted by CPT changes to tensile stress when FSDT and TSDT were used. Nosier and Fallah [87] used the first-order von-Kármán nonlinear theory to study the large deflection behavior of FGM circular plates. They reformulated the system of five nonlinear coupled equations into three equations viz., one linear second-order differential equation defining edge-zone problem in terms of a potential stress function and two nonlinear fourth order equations in terms of transverse deflection and a stress function. Although the variation of transverse deflection and radial stresses through the thickness were plotted, the variation of in-plane deformation, angular and shear stresses were not presented. It was observed from the results that the linear analysis is inadequate for the analysis of simply-supported FGM plates restrained in radial direction even in the small deflection range. In addition the response of FGM plates under a positive load is different than that of the plates under the negative load of the same magnitude. Sadr-Lahidjani [88] studied the large deflection behavior of relatively thick FGM plates subjected to uniform pressure using TSDT. The stress distribution was found to be linear in aluminum and alumina plates whereas FGM plates were found to behave nonlinearly. Reddy and Berry [89] used the modified coupled stress theory of Yang et al. [90] along with the von-Kármán nonlinearity effect to develop micro-structure dependent nonlinear theory for axisymmetric bending of circular plates based on CPT and FSDT, which can account for through-the-thickness power-law variation of two-constituent material with temperature dependent material properties. The governing equations were derived using Hamilton's principle. Kaci et al. [91] presented a mathematical solution for the nonlinear cylindrical bending of sigmoid FGM plates based on CPT and von-Kármán nonlinearity. It was concluded that Navier's equations for plates according to the large deflection theory can be expressed as linear equations with nonlinear boundary conditions.

2.1.2.2. Thermo-mechanical load. Obata and Noda [92] discussed the difficulty of using analytical method of solution for analyzing transient thermal stresses in FGM plates and proposed a new analytical method using a Laplace transforms and the perturbation method to study the one-dimensional transient stress problem in FGM plates. Aboudi et al. [93] developed a new higher-order micromechanical theory for FGMs (HOTFGM-1D) that explicitly couples the heterogeneous microstructure of the material with the global effects. A thermo-mechanical analysis has been carried out for linear, quadratic and cubic fiber spacing. It was observed from the results that desired stress distributions can be obtained with suitable grading or tailoring of the microstructure of the composite. The HOTFGM-1D was extended by Aboudi et al. [94] to incorporate inelastic and temperature-dependent parameters for through the thickness analysis of functionally graded metal matrix composites subjected to a thermal gradient. Aboudi et al. [95] further extended the HOTFGM-1D by relaxing the constraint on in-plane deformation through a partial homogenization procedure in the non-functionally graded (periodic) directions, which combines the elements of coupled higher-order theory in the direction in which the micro–macrostructural coupling is essential, with the elements of the standard micromechanics treatment applied to those directions in which coupling is not necessary. Aboudi and Arnold [96] again extended the HOTFGM to analyze the thermo-elastic response of composites functionally graded with nonuniform fibers in two directions (HOTFGM-2D). Finally, Aboudi et al. [97] presented a complete generalization of HOTFGM that enabled the thermo-inelastic analysis of materials with spatially varying microstructures in all the three orthogonal directions. Pitakthapanaphong and Busso [98] obtained analytical and semi-analytical solutions for thermo-elastic and thermo-elasto-plastic analysis of three layered system consisting of metal and ceramic layer joined together by an FGM layer with an arbitrary composition profile. A thermo-elastic framework, developed by Freund [99] was generalized to incorporate various forms of nonlinear FGM compositional profiles (linear, quadratic, inverse-quadratic and cubic) and then a self-consistent constitutive approach was used to describe thermo-elasto-plastic behavior of FGM plates. It was concluded by the authors that by choosing an appropriate FGM compositional gradation, the stress distribution within the system can be controlled so that undesirable stresses at critical locations are minimized or avoided. A more accurate thermo-visco-elastic analysis of FGM thin plates was presented by Zhang and Wang [100], who considered the effect of mid-plane strains induced by inhomogeneous material properties across the thickness to reduce the thermo-visco-elastic constitutive equation of FGM thin plates based on Kirchhoff's hypothesis of CPT. Results have been plotted only for deflection against various parameters. It was observed that for mechanical load, an increase in the graded parameter resulted in enhanced deflection whereas for the nonlinear temperature field, the absolute value of deflection is directly proportional to the graded parameter. Nosier and Fallah [101] reformulated the governing equations of FSDT for FGM circular plates into interior and edge zone equations and then developed analytical solutions using the boundary layer function to study the axisymmetric and asymmetric behavior of FGM circular plates with various clamped and simply-supported boundary conditions. An exact solution too was presented for 1D heat conduction equation with variable heat conductivity coefficient. It was concluded based on the results that the response of solid circular plates having various clamped supports is identical under mechanical load case and the bending does not occur for clamped plates under the thermal load case. Chiba and Sugano [102] used the Vodicka's method along with the first-order perturbation method for stochastic thermo-elastic analysis of FGM plates subjected to thermal loading by assuming FGM plate as multi-layered plate with distinct, random thermal conductivity and linear coefficient of thermal expansion in each layer. Brischetto et al. [103] extended the CUF for the deformation analysis of simply-supported FGM rectangular plates subjected to thermo-mechanical loadings. It was concluded that the assumption of constant transverse deflection through the thickness is not valid in the thermal load case, but still a justified assumption in the mechanical load case. Further, Cinefra et al. [104] extended the CUF to FGM shells subjected to thermal and mechanical loads. It was found by the authors that the temperature profile cannot be assumed linear in FGM layers even when a very thin shell is considered. Matsunaga [105] presented a 2D higher-order theory for the stress analysis of FGM plates. Convergence properties of displacements and stresses were studied through numerical examples to show that the 2D higher-order theory can provide accurate results for the distribution of displacements and stresses in FGM plates. A semi-analytical method which combines the state-space method and differential quadrature method was employed by Ying et al. [106] for 3D thermo-elastic analysis of FGM thick plates. The employment of differential quadrature technique enabled the application of the method to treat non-simply-supported plates too. Zenkour [107] presented a thermomechanical bending analysis for a simply supported, rectangular FGM sandwich plate subjected to a transverse mechanical load and a through-the-thickness thermal load using the refined sinusoidal shear deformation plate theory which can consider the effect of both shear and normal deformations. The theory was further simplified by enforcing traction-free boundary conditions at the plate faces. Zenkour and Alghamdi [108] used SSDT to study the thermo-mechanical bending response of nonsymmetric FGM sandwich plates with FGM (ceramic–metal) face sheets and homogeneous and isotropic core. The obtained results were compared with those computed using FSDT, TSDT and CPT. The deflection calculated using SSDT were found to be closer to those calculated using TSDT and intermediate to those obtained using FSDT and CPT especially for $a/b \geqslant 1$. The magnitude of shear stresses obtained by applying SSDT and TSDT were smaller than those obtained using FSDT. Hamidi et al. [109] used a new four variable refined plate theory to investigate the bending response of FGM sandwich plates under thermo-mechanical loading. The obtained results were validated with those obtained using CPT, FSDT, SSDT, ESDT and TSDT. It was found that the results obtained have a close agreement with those of TSDT compared to other theories. The Reddy's TSDT was used by Akbarzadeh et al. [110] for coupled thermo-elastic analysis of FGM plates. The solutions were obtained using a double Fourier series and Laplace inverse method. It was concluded that there is no significant difference between coupled and uncoupled solutions but the effect of coupling is in the form of damping which decreases the amplitude of vibration and increases the frequency of the vibration with the increase of time. Houari et al. [111] developed a new higher-order shear and normal deformation theory for thermo-elastic bending analysis of sandwich plates consisting of FGM face sheets and homogeneous core made of isotropic ceramic material.

Shen [112] used a two-step Galerkin perturbation method for nonlinear bending analysis of FGM plates with or without piezoelectric actuators subjected to combined action of thermal and electrical loads due to heat conduction based on Reddy's higher-order shear deformation theory that includes thermo-piezoelectric effects. Woo and Meguid [113] used the classical von-Kármán theory of nonlinearity to study the large deflection behavior of FGM plates and shallow shells subjected to thermo-mechanical loading. The solution for deflection and stress function was expressed in double Fourier series form. It was demonstrated that the stress distribution through the thickness, which is linear in isotropic metal or ceramic plates becomes nonlinear in FGM plates. Fallah and Nosier [114] studied the cylindrical bending of FGM plates subjected to mechanical, thermal, and combined thermo-mechanical loadings based on the von-Kármán nonlinear FSDT. An exact solution was also developed for the one-dimensional heat conduction equation with variable heat conductivity coefficients. Due to the bending–extension coupling, simply-supported FGM plates were found to respond differently under the same magnitude of positive and negative mechanical loading. The bending–extension coupling was not observed in clamped FGM plates. Finally it was concluded that linear analysis is inadequate for simply-supported FGM plates even in the small deflection range especially under thermal loading. A layer wise theory was used by Tahani and Mirzababaee [115] to show the significance of nonlinear analysis of FGM plates subjected to thermo-mechanical loading. It was observed that the nonlinearity reduces the magnitude of transverse deflection in mechanical loading case and increases in thermal loading case. Reddy and Kim [116] developed a general third-order theory with microstructure-dependent length scale parameter and the von-Kármán nonlinearity. The theory contains 11 generalized displacements which allows C^0-approximation and accounts for temperature dependent properties of constituents of FGM. Houri et al. [117] verified the accuracy of the two variable refined plate theory developed by Mechab et al. [53] for the thermo-elastic bending analysis of FGM sandwich plates. Fahsi et al. [118] used a four variable refined plate theory for the nonlinear cylindrical bending analysis of FGM plates under thermo-mechanical loadings. The solution is obtained by minimizing the total potential energy. The authors emphasized that when only thermal load is present, the use of linear analysis may result in great errors.

2.2. Numerical methods

Because of the fact that high mathematical complexity is involved in solving initial and boundary value problems, analytical methods of solutions are limited to the problems with relatively simple geometry and boundary condition. The nonlinear variation of material properties through the thickness of FGM plates makes the solution even more tedious. Therefore, numerical methods are being extensively used for complex engineering problems.

2.2.1. Finite element method

Finite Element Method (FEM) is the most extensively used computational techniques for solving variety of engineering problems. In FEM, the continuum is divided into a finite number of non-overlapping regions called elements. The equilibrium requirements of each element are specified in terms of a finite number of state variables. The final solution of the entire system is obtained by assembling the results of the individual elements.

2.2.1.1. Mechanical load case. A discrete layer theory in combination with the Ritz method was presented by Ramirez et al. [119] for 3D static analysis of FGM plates. A transition function was directly incorporated into the governing equations which allow any continuous function for describing the variation of material properties through the thickness. The numerical results of FGM plates showed 25% reduction in vertical displacement and 20% increase of in-plane normal stresses over homogeneous plate. Zhen and Wanji [120] developed a higher-order shear deformation theory for the analysis of simply-supported FGM plates, which includes global and local displacement components. It was concluded by the authors that the higher-order shear deformation theory is computationally more efficient than 3D theories. A refined three-node triangular element was also presented based on the higher-order shear deformation theory to satisfy C^1 weak-continuity condition and is very ideal for analysis of inter-laminar stresses of FGM laminated plates. Kubair and Bhanu-Chandar [121] used a multiple-isoparametric finite element formulation to study the effect of material inhomogeneity on the stress concentration around a circular hole in an FGM rectangular plate. It was found that the magnitude of stress and its angular variation is affected by material inhomogeneity while the angular position of the maximum tensile stress on the surface of the hole is independent of material inhomogeneity. Talha and Singh [122] used a higher-order shear deformation theory with special modification in the transverse displacement along with a C^0 isoparametric Lagrangian finite element with 117 degrees of freedom per element to study the static and free vibration analysis of FGM plates under various boundary conditions. A 3D analysis of single and multi-layered FGM plates was performed by Wu and Li [123] using the RMVT and PVD based Finite Layer Methods (FLM). It was observed that the RMVT-based FLMs perform better than PVD-based FLMs. Orakdogen et al. [124] used a layer wise, isoparametric mixed finite element to study the effect of extension–bending coupling on FGM plates based on Kirchhoff–Love theory subjected to transverse load under four different types of simply-supported boundary conditions. The extension–bending coupling was found to be significant for some simply-supported plates compared to some others. Wang and Qin [125] used a boundary integral based graded formulation for the analysis of FGM plates. An HFS-FEM model was proposed where the fundamental solution of graded materials was used to interpolate the intra-element displacements and stress fields, and the shape functions used in conventional FEM were employed to approximate the frame field defined only on the element boundary. Though through the thickness variation of transverse and in-plane displacements is presented, the variation of in-plane and transverse stresses were not presented. A nine-nodded shell element with exact cylindrical geometry was used by Cinefra et al. [126] for the static analysis of FGM shells based on the CUF. The Mixed Interpolation of Tensorial Components (MTC) technique was extended to the CUF in order to withstand the membrane and shear locking phenomenon. Tran et al. [127] presented an Iso Geometric Approach (IGA) based on he Nonuniform Rational B-Spline (NURBS) functions which can ensure C^1 continuity to study the behavior of FGM plates based on Reddy's TSDT. A C^1 continuous four-node quadrilateral plate element with ten degrees of freedom per node was employed by Thai and Choi [128] to develop a displacement based finite element model using various four unknown shear deformation theories for the bending and free vibration analyses of FGM plates. Though the thickness stretching effect is ignored, the same methodology can be extended to higher-order shear deformation theories which are capable of considering the through the thickness variation of transverse displacement. Castellazzi et al. [129] presented the Nodal Integrated Plate Element (NIPE) formulation is for the static analysis of FGM plates based on FSDT.

Two new Mindlin-type plate bending elements were derived by Oyekoya et al. [130] to show that the structural integrity and strength of the composites can be improved by grading the material properties over the structure. Han et al. [131] used 9-node Lagrangian shell element to show the significance of nonlinearity in the analysis of S-FGM structures based on higher-order shear deformation theory. Singha et al. [132] used a four-node high precision plate bending element based on the exact neutral surface and FSDT for the analysis of FGM plates based on FSDT. A shear correction factor was obtained based on the energy equivalence principle.

2.2.1.2. Thermo-mechanical load case. Fuchiyama and Noda [133] developed a computer program using an eight-node plane strain quadrilateral isoparametric element for the transient thermal analysis of FGM plates. A layerwise model was used with uniform and homogeneous material properties in each sub layer. Pindara and Dunn [134] evaluated the accuracy of HOTFGM-1D using FEM. The comparison between the results of the FEM and the higher-order theory presented by the authors establishes the accuracy of the theory in predicting the thermal and stress fields within composites with a finite number of fibers in the thickness direction subjected to a through-the-thickness thermal gradient. A discrete finite element method along with the Crank–Nicolson–Galerkin scheme was employed by Cho and Oden [135] to study the effects of material gradation through the thickness and size of the FGM layer inserted between ceramic and metal layers in a layered composite material. The material is graded using two parameter family of curves. The transient and steady-state behaviors were found to be different for some combination of parameters and hence advised to select an optimum combination of parameters. Biner [136] investigated the thermo-elastic behavior of FGMs using Voronoi finite elements, which do not require any assumption in either physical or mechanical properties of the layer. Nemat-alla [137] used a 2D eight-node thermal solid element for transient thermal analysis of 2D-FGM plates and showed that the 2D-FGM is highly capable of reducing thermal stresses. Croce and Venini [138] presented a hierarchic family of finite elements capable of handling locking phenomenon for the analysis of Reissner–Mindlin FGM plates using the variational approach. The performance of a simple locking-free discontinuous Galerkin finite element of nonconforming type was assessed by Chinosi and Groce [139] for approximation of Reissner–Mindlin FGM plates. It was observed that as the temperature effect reduces with increasing mechanical loads, the deflections become negative and tend towards a behavior similar to the case of pure pressure loads. Pai and Palazotto [140] presented a 2D sublamination plate theory with varying degrees of freedom for the analysis FGM plates that can account for layerwise higher-order transverse shear deformation, interlaminar shear stress continuity and boundary effect caused by free edges and warping restraint at boundaries. Bagri and Eslami [141] investigated the thermal and mechanical wave propagation using the Galerkin FEM in an annular FGM disk based on the Lord–Shulman theory. It was observed that an increase in the power law index resulted in the reduction of amplitude of temperature variation, displacement and corresponding stresses. Nguyen et al. [142] extended an edge based smoothed finite element method for the analysis of Reissner/Mindlin FGM plates. It was demonstrated by the authors that their method can provide a simple and effective tool for the computation and simulation of FGM plate structures and can be extended to FGM shells. Valizadeh et al. [143] used NURBS based iso-geometric finite element method to study the static and dynamic response of FGM plates in thermal environment based on the FSDT. The formulation found to suffer from shear locking effect when it is applied to thin plates which can be mitigated by using the modified shear correction factor given by Kikuchi and Ishii. [144]. Natarajan and Manickam [145] presented a higher-order shear deformation theory with 13 degrees-of-freedom for the bending and vibration analyses of FGM sandwich plates. They used an 8 nodded quadrilateral plate element for the analysis. The supremacy of the theory was demonstrated by comparing the results with other higher-order theories and FSDT available in the literature. A C^0 continuous isoparametric Lagrangian finite element with seven degrees of freedom for each node was employed by Gulshan Taj et al. [146] for the static analysis of rectangular and skewed FGM plates based on TSDT. Saidi et al. [147] used the new hyperbolic shear deformation theory in which the stretching effect is included to investigate the thermo-mechanical bending response of sandwich plates consisting of FGM face sheets and homogeneous core made of isotropic ceramic material.

Reddy [36], also studied the nonlinear behavior of FGM plates using a four-nodded isoparametric rectangular element based on the FSDT. An elasto-plastic deformation analysis of $Ni–Al_2O_3$ graded composites with different packing arrangements of constituents was presented by Weissenbek et al. [148] using detailed finite element models. Square-packing arrangements were found to provide the best possible bounds for the thermal strains and coefficient of thermal expansion although, no particular unique bounds were identified for mechanical load. Praveen and Reddy [149] investigated the nonlinear thermo-elastic response of functionally graded ceramic–metal plates using a plate finite element that accounts for the transverse shear strains, rotary inertia and moderately large rotations in the von-Kármán sense using the Reddy's TSDT. The effects of variation of volume fraction and temperature field on the static and dynamic behavior of FGM plates were discussed. Based on the results, it was concluded that the basic response of FGM plates which corresponds to properties intermediate to that of the metal and ceramic does not necessarily lie in between that of the ceramic and metal. It was also concluded that in the absence of thermal loading, the dynamic response of graded plates is intermediate to that of the metal and ceramic. The same is not the case when both thermal and mechanical loadings are applied together. Reddy and Chin [150] carried out the thermo-mechanical analysis of FGM plates and cylinders using FSDT that accounts for transverse shear strains and rotations, coupled with 3D heat conduction equation. The coupling effect on temperature was found to vanish after a certain period of time. Aliaga and Reddy [151] used a four-node rectangular element based on the TSDT to study the linear and nonlinear thermo-mechanical response of FGM plates subjected to static and dynamic loads. Although the variation of transverse displacement in the thickness direction was presented, the variation of in-plane displacement and stresses (in-plane and transverse stresses) were not presented. A 3D degenerated shell element was used by Naghdabadi and Kordkheili [152] for the thermo-elastic analysis of FGM plates and shells. Na and Kim [153] studied the nonlinear bending response of Al_2O_3–Ni FGM plates using a 3D solid element. Three types of thermal loads were considered across the thickness; uniform, linear and sinusoidal temperature rise. It was observed that the central deflection is largest for uniform temperature rise and smallest for sinusoidal temperature rise. Na and Kim [154] also studied the volume fraction optimization of FGM plates considering stress and critical temperature using 18-nodded 3D solid element. Phung-Van et al. [155] employed a cell-based smoothed three-node Mindlin plate element (CS-MIN3) using FSDT for nonlinear analysis of FGM plates based on C^0-type higher-order shear deformation theory. An MLPG approach using moving Kriging interpolation technique capable of possessing Kronecker delta function property was developed by Zhu et al. [156] for geometrically nonlinear thermo-elastic analysis of FGM plates based on FSDT.

2.2.2. Meshless methods

Even though FEM is being extensively used in most of the engineering problems, there are some drawbacks because of its mesh-based interpolation method. Distortion of the mesh during the large deformation of the structure and requirement of intensive remeshing in case of structures with complex geometries and discontinuities are the major drawbacks of FEM. Therefore, a new type of method called meshless method based on a set of scattered nodes rather than meshes has been developed and successfully applied in various engineering problems. Some of the drawbacks of FEM such as mesh distortion and remeshing can be circumvented by using meshless methods because of their flexibility in mesh requirements.

2.2.2.1. Mechanical load case. Qian et al. [157] used the Higher-Order Shear and Normal Deformation Theory HOSNDPT) proposed by Batra and Vedoli [158] along with the Meshless Local Petrov–Galerkin (MLPG) method for the static deformation and free vibration analyses of thick FGM plates with simply-supported edges. It was demonstrated that the results given by compatible HOSNDPT and MLPG method for thick FGM plates are very close to the results obtained using analytical methods than those obtained using TSDT. Furthermore, it was concluded that the HOSNDPT is well suited if the in-plane side to thickness ratio $\leqslant 5$. Ferreira et al. [159] employed the meshless collocation method and multiquadric radial basis functions for the analysis of FGM plates based on the TSDT and showed that the proposed method is computationally less intensive than MLPG method of solution. However, the computational effort required varies with the choice of shape parameters. Later, Ferreira et al. [160] optimized the shape parameter for the same formulation using the cross-validation technique suggested by Rippa [161]. A 3D analysis of FGM plates was presented by Chun and Zheng [162] based on the Haar-wavelet method. It was observed from the results that the total stiffness and deflection of the plate changes with the choice of the material gradient. However, when the ratio of Young's modulus at the top and bottom surfaces is relatively small, the effect of gradation of material properties on stress distribution is small and can be neglected. Sladek et al. [163] developed a local integral equation method using the MLPG method along with Moving Least Squares (MLS) approximation for the analysis of FGM plates defined by Reissner–Mindlin plate theory. It was observed from the results that the bending moment in homogeneous plates and isotropic FGM plates are almost same while, the bending moment at the plate center and at the center of the clamped edge of an orthotropic FGM plate is always greater than that of an isotropic FGM plate. Gilhooley et al. [164] showed that the MLPG method along with radial basis function is computationally less expensive than MLS scheme for the analysis of FGM plates based on the HOSNDPT. Miers and Telles [165] developed a boundary element free method for the 2D elasto-static analysis FGM plates. Cheng and Cao [166] proposed a new microelement method for the analysis of FGM net structures with medium components. A 3D static solution was developed by Vaghefi et al. [167] using the MLPG method along with the MLS approximation for the analysis of thick FGM plates with various boundary conditions. A DQM based on the state-space formalism was used by Lü et al. [168] to obtain 3D elastic solution of orthotropic multi-dimensional FGM plates. Aghdam et al. [169] used the extended Kantorovich method for the static analysis of moderately thick FGM annular sector plates with clamped edges based on the FSDT. It was shown that the presented method converges fast and provides accurate predictions for annular, solid sector plates and rectangular plates. A meshless collocation and an Element-Free Galerkin (EFG) method were developed by Wu et al. [170] using the Differential Reproducing Kernel (DRK) interpolation for the quasi-3D analyses of single and multilayered FGM plates. The shape function at each sampling node was separated into a primitive function possessing the Kronecker delta properties and enrichment function constituting the reproducing function. Neves et al. [171] used a collocation method with radial basis function for the quasi-3D analysis of FGM plates based on the CUF. It was found that the obtained results are closer to those theories in which thickness stretching effect is considered and clearly deviates from those theories in which transverse normal strain is neglected. Therefore it was suggested by the authors that the transverse normal stress should not be neglected even for thin plates. Mojdehi et al. [172] presented the 3D elastic analysis of FGM plates based on the MLPG method and MLS approximation. The centroidal deflection of FGM plates was found to be between those of a pure ceramic and a pure metallic plate for both static and dynamic loads. Fereidoon et al. [173] successfully applied the polynomial differential quadrature and Harmonic differential quadrature methods for the bending analysis of FGM plates and FGM coated plates subjected to transverse loading based on the CPT. The variation of Murakami's zig-zag function was proposed by Neves et al. [174] to study the static deformations of thin FGM sandwich plates which includes the hyperbolic sine term for the in-plane displacement expansion and allows through the thickness variation of transverse deformation. The governing equations and boundary conditions were obtained by generalization of the CUF and further interpolated by the radial basis function collocation method. Xiang and Kang [175] used the meshless global collocation method based on the thin plate spline radial basis function for the bending analysis of FGM plates based on the *n*th-order shear deformation theory. The Reddy's TSDT was considered as a special case to demonstrate the *n*th-order theory. Xiang and Kang [176] used a meshless method based on thin plate spline radial basis function for the static analysis of FGM plates based on various higher-order shear deformation theories of Levinson, Touratier, Karama, Aydogdu and Mantari. Neves et al. [177] used the quasi 3D higher-order shear deformation theory along with radial basis functions for the static, free vibration and buckling analysis of FGM sandwich plates. It was found that the effect of thickness stretching has a significant influence on FGM plates even for relatively thin plates.

Zhao and Liew [178] used the element-free kp-Ritz method to study the nonlinear response of ceramic–metal FGM plates based on the von-Kármán nonlinear FSDT. For FGM plates under thermal loading only, the linear displacement is less than the corresponding nonlinear displacement because the thermal force resultants counteract the in-plane forces and thus reduce the overall plate stiffness. Axisymmetric nonlinear bending of annular FGM plates based on the FSDT and TSDT was studied by Golmakani and Kadkhodayan [179] who employed dynamic relaxation method combined with finite difference technique. Based on their study, the authors concluded that though the difference between results obtained using TSDT and FSDT becomes greater with an increase in the thickness ratio, the effects of different types of boundary conditions is also of equal importance. Suresh Kumar et al. [180,181] used the Navier's method along with the Newton–Raphson iterative technique for the nonlinear analysis of FGM plates based on higher-order refined theories with twelve and nine degrees of freedom. Jam et al. [182] using generalized differential quadrature method along with the Newton–Raphson iterative scheme for non-linear bending analysis of moderately thick FGM plates based on the FSDT and von-Kármán type of non-linearity. The multi-quadratic radial basis function method was used by Singh and Shukla [183] to study the flexural response of FGM plates subjected to different types of transverse loads under various boundary conditions based on the Levinson plate theory and von-Kármán nonlinearity. It was observed that the effect of nonlinearity is more pronounced for SS boundary condition compared to CC boundary conditions that too at the center of the plate.

2.2.2.2. Thermo-mechanical load case. A Green's function approach was developed by Kim and Noda [184] based on the Kirchhoff's classical laminate theory along with the Galerkin method for the analysis of the 2D transient heat conduction equation and the deflection of FGM plates. Tsukamoto [185] combined micromechanical approach with the macromechanical approach for analyzing the transient thermal stresses in a ceramic–metal FGM plates subjected to through the thickness heat flow. However, they did not consider the continuous variation of material properties and interaction between particles at microscopic scale. Qian et al. [186] used the MLPG method for the analysis of plane strain static thermo-elastic deformation of FGM rectangular plates with material properties smoothly varied in the thickness direction. Qian and Ching [187] also used the MLPG method for the transient thermo-elastic deformation analysis of thick FGM plates. Based on the results, it was concluded that the inertial forces have a negligible influence on the displacements and stresses induced in FGM plates. Qian and Batra [188] used the HOSNDPT along with the MLPG method to study the transient thermo-elastic deformation of thick FGM plates with edges being subjected to uniform temperature. It was concluded that the centroidal deflection and axial stress at the top and bottom surfaces of the plates are significantly influenced by the boundary conditions. The authors have also concluded that the inertia forces have a negligible effect on the displacement and stresses induced due to transient thermal loads. Dai et al. [189] used the MLS method with C^1 consistency for the analysis of FGM plates based on the FSDT and showed that the relation between the deflection and volume fraction is different for thermal and mechanical load cases. Yang and Huang [190] studied the nonlinear transient response of simply-supported imperfect FGM plates in thermal environments based on the Reddy's TSDT and von-Kármán-type nonlinear kinematics. Three types of initial geometric imperfections were considered viz., sine type, global type and localized type imperfections. A semi-analytical approach comprising of an improved perturbation approach, Galerkin technique and Runge–Kutta iteration process were used to obtain the asymptotic solution. The imperfect FGM plates were found to show much higher deflection and bending moment compared to perfect FGM plates under the same intensity transverse dynamic load. Wang and Qin [191] developed a meshless algorithm based on the analog equation theory and method of fundamental solutions coupled with the radial basis function to study the static thermal stress distribution in 2D FGM plates. Saidi et al. [192] decoupled the five highly coupled partial differential equations based on the FSDT under static load using a boundary layer function. They also studied the effect of stress singularity and boundary layer function using the Differential Quadrature Method (DQM). Reid and Paskaramoorthy [193] extended the CPT for the analysis of FGM plates which can accommodate any through the thickness variation of elastic, hygrothermal and piezoelectric loading. Zhou et al. [194] considered the coupling, non-Fourier effect and motion inertia to study the transient thermoelastic response of FGM plates using the generalized coupled thermoelasticity based on the Lord–Shulman theory.

A mixed Galerkin-perturbation technique was employed by Shen [195] for nonlinear bending analysis of simply-supported FGM rectangular plates based on the Reddy's higher-order shear deformation plate theory subjected to thermal and mechanical loads. It was found that the differential equations of FGM plates are identical to those of unsymmetric cross-ply laminated plates under thermo-mechanical loading and the stretching-bending coupling gives rise to bending curvature when in-plane compressive edge load is applied. Later, Yang and Shen [196] extended the same solution for the nonlinear bending analysis of shear deformable FGM plates subjected to thermo-mechanical loads and under various other boundary conditions using a semi-numerical method comprising of perturbation technique, one-dimensional differential quadrature approximation and Galerkin method. Ma and Wang [197] used a shooting method to study the axisymmetric nonlinear bending and thermal post-buckling of FGM circular plates based on the classical nonlinear von-Kármán plate theory considering both simply-supported and clamped boundary conditions. Zhao and Liew [198] used the element-free kp-Ritz method along with the Newton–Raphson approach to study the nonlinear response of FGM cylindrical shells based on the Sander's modified nonlinear shell theory restricted to moderate rotations under thermal and mechanical loads. A dynamic relaxation method along with the finite difference method was employed by Golmakani and Kadkhodayan [199] for the deflection analysis of circular and annular FGM plates based on the FSDT and von-Kármán large deflection equations. For plates having higher thickness-to-radius-ratio, it was concluded that the thermal load has less influence on the central deflection.

3. Vibration analysis

Various methods have been employed so far for the vibration analysis of FGM plates and FGM sandwich plates and are discussed in this section under two broad headings as analytical methods and numerical methods. Theories based on three-dimensional elasticity theory and two-dimensional plate theories are discussed under the heading of analytical methods whereas finite element method and meshless methods are discussed under the heading of numerical methods.

3.1. Analytical methods

Vibration analysis of FGM plates mainly involves the solution of eigenvalue problems. Solution of eigenvalue problems using 3D elasticity theories is hard to obtain particularly when the material properties are graded using the power law parameter. Thus, 2D plate theories based on displacement field and stress function are developed and effectively being used for the analysis of plates.

3.1.1. Three-dimensional (3D) elasticity theory

A 3D asymptotic theory was formulated in terms of transfer matrix by Reddy and Cheng [200] to study the harmonic vibration problem of FGM plates. Instead of using multiple time scales expansion, they refined asymptotic formulation by expanding the frequency parameter in a rather simpler way which enables the asymptotic approach to find any higher-order solution with the help of the solvability condition. Vel and Batra [201] presented an exact solution using the power series expansion method for the free and forced vibration of simply-supported FGM rectangular plates. The obtained results were compared with those of CPT, FSDT and TSDT. It was found that the FSDT performs better than TSDT. The results and solutions obtained by the CPT differed considerably from those obtained using the exact method even when the transverse shear and normal stresses were computed by integrating the 3D elasticity equations. A 3D analysis was carried out by Nie and Zhong [202] for the free and forced vibration analysis of FGM circular plates under various boundary conditions using a semi-analytical approach which makes use of state space method and 1D differential quadrature method. It was found by the authors that the semi-analytical method requires much less computational effort and it has great advantage in the computational efficiency. Later, Nie and Zhong [203] extended the same procedure for the free and forced vibration analyses of FGM annular sector plates with simply-supported radial edges. Dong [204] studied the 3D free vibration behavior of FGM circular plates with various boundary conditions using the Chebyshev–Ritz method. Two types of material gradients were considered viz., power law and exponential variation of material properties. A 3D elasticity solution using the Ritz method was developed by Li et al. [205] for the free vibration analysis of symmetric and unsymmetric FGM sandwich plates with simply-supported and clamped boundary conditions. The authors conducted a convergence study and concluded that the number of thickness expansion terms required for convergence mainly depend on the thickness, length and width of the plate. It was also concluded that the convergence is independent of material gradient parameter. Tsai and Wu [206] presented the 3D free vibration of simply-supported, doubly curved FGM magneto-electro-elastic shells using an asymptotic expansion method. It was illustrated that the material property gradient has a significant influence on the through-the-distribution electric and magnetic field. The displacement-based formulation in Pagano method was replaced by mixed formulation so that the lateral boundary conditions on the outer surfaces and continuity condition at the interface between adjacent layers can be directly applied. Li et al. [205] used Chebyshev polynomials along with the Ritz energy method for the 3D vibration analysis of FGM plates in thermal environment. Simply-supported and clamped FGM plates subjected to uniform, linear and nonlinear temperature rise through the thickness were considered. Hosseini-Hashemi et al. [207] presented an exact closed-form solutions using the 3D elasticity theory to study both in-plane and out-of-plane free vibration of thick simply-supported FGM plates. The displacement fields were described in Levinson's representation form. The material properties were assumed to be exponentially graded in the thickness direction. The solution was validated by comparing the results with corresponding results available in the literature as well as with 3D FEM from ABAQUS package.

3.1.2. Two-dimensional (2D) plate theories

Ng et al. [208] used the Hamilton's principle and assumed mode technique to study the parametric resonance of FGM rectangular plates based on the CPT under harmonic in-plane loading. It was concluded by the authors that the parametric resonance can be controlled especially with regard to the points of origin of the instability regions by appropriately varying the power law exponent. Huang and Shen [209] used an improved perturbation technique to study the nonlinear vibration and dynamic response of FGM plates in thermal environment based on Reddy's TSDT. It was found from the results that the temperature distribution and volume fraction distribution have a significant influence on the nonlinear vibration and dynamic response of initially stressed FGM plates in thermal environment. The Rayleigh–Ritz procedure was employed by Kim [210] to develop a theoretical method to investigate vibrational characteristics of initially stressed FGM plates in thermal environment based on the TSDT. Two types of thermal loads were considered viz., in the first type the temperature was imposed on the upper surface and the other value on the lower surface and in the second type heat flows from upper surface to the lower one held at a prescribed temperature. Zenkour [211] investigated vibration natural frequencies of simply supported FGM thick plates using a trigonometric shear deformation plate theory which consider the effect of rotary inertia. Abrate [212] used the extensive results available in the literature to show that the behavior of FGM plates is proportional to that of homogeneous plates and can be assessed from the results of homogeneous plates. Woo et al. [213] presented a solution in terms of mixed Fourier series to study the nonlinear free vibration behavior of FGM plates based on the classical plate theory and von-Kármán nonlinearity for large transverse deflection. It was demonstrated that the nonlinear coupling effects play a major role in dictating the fundamental frequency of FGM plates. Efraim and Eisenberger [214] presented an exact solution using the exact element method and the dynamic stiffness method for the vibration analysis of thick annular plates with variable thickness based on the FSDT. Batra [215] derived a higher-order shear and normal deformation plate theory for linearly elastic incompressible FGM plates. It was found by the authors that the frequencies of simply-supported rectangular FGM plates computed from the zeroth-order plate theory agree with those found from the analytical solution but the corresponding two mode shapes need not match with each other. The method of power series expansion of displacement components along with the Hamilton's principle were employed by Matsunaga [216,217] to study the free vibration and stability behavior of rectangular FGM plates and shallow shells based on a 2D higher-order theory which takes complete account of transverse shear, normal deformation, thickness changes and rotary inertia. The governing equations were solved using the Navier's solution technique. Abrate [83] used the CPT to show that FGM plates can be idealized as homogeneous plates by properly selecting the reference surface so that no special tool is required to analyze their behavior. By making a suitable choice of the reference surface, the bending-stretching coupling can be eliminated so that the bending of the plate is governed by the same equation of motion as that of homogeneous plates. Allahverdizadeh et al. [218,219] presented a semi-analytical method using assumed-time-mode method and Kantorovich time averaging technique to study the axisymmetric free and forced vibration behavior of thin circular plates based on the CPT in thermal environment. The authors have found that the vibration frequencies were dependent on large vibration amplitudes and were independent of temperature. Fares et al. [46] presented a refined equivalent single layer theory using the mixed variational approach for the bending and free vibration analyses of FGM plates. The theory does not require any shear correction factor since it accounts for both the transverse shear and normal strain to be in complete consistence with boundary conditions at top and bottom of the plate. Cinefra et al. [220] extended the RMVT in combination with the CUF for the free vibration analysis of multi-layered FGM shells based on various shell theories using the Navier's solution technique. Classical theories were compared with higher-order, layer-wise and mixed shell theories for accuracy. It was concluded based on numerical results that the accuracy of various theories depends largely on various geometrical parameters as well as on the dynamic modes. A Levy-type solution was obtained by Liu et al. [221] for the free vibration analysis of FGM plates based on the CPT when material is graded along the in-plane direction rather than conventional thickness direction. It was observed from the numerical results that the in-plane material gradient parameter has a certain effect on the natural frequencies of the plate. However, the material inhomogeneity may be adjusted to meet the special requirements. Hosseini-Hashemi et al. [222] presented an exact closed-form frequency equation for the free vibration analysis of FGM circular and annular plates based on the Mindlin's FSDT. A Levi type solution was employed by Hosseini-Hashemi et al. [223] for the free vibration analysis of FGM rectangular plates based on the FSDT, where two opposite edges were simply-supported and other two edges were under various boundary conditions. Hosseini-Hashemi et al. [224] once again used the same method of solution for the free vibration analysis of thick rectangular plates based on the Reddy's TSDT. An n-order shear deformation theory was proposed by Xiang et al. [225] for the free vibration analysis of FGM and composite plates. The authors considered the Reddy's TSDT as a special case to present the *n*-order theory. Benachour et al. [226] presented the free vibration analysis of FGM plates with arbitrary gradient based on a four variable refined plate theory using the Navier's solution technique and Ritz method. The theory can take account of transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the plate, hence it does not need shear correction factors unlike FSDT. The obtained results were validated by comparing with those obtained using FSDT and TSDT. Hadji et al. [227] used Shimpi's four variable refined plate theory for the free vibration analysis of FGM sandwich plates. The accuracy of the solution was established by comparing the results with those of 3D elasticity solution, CPT, FSDT, TSDT and SSDT. It was concluded that though the refined plate theory is simple and predicts accurate results. However, it needs improvement especially when it is applied to laminated structures to satisfy interlayer shear stress continuity. Alijani et al. [228] used the Galerkin procedure and multiple scale method to study the nonlinear forced vibration of FGM doubly curved shallow shells with rectangular based on the Donnell's nonlinear shallow shell theory. Uymaz et al. [229] used the Ritz method and assumed displacement functions in the form of the Chebyshev polynomials for the free vibration analysis of FGM plates with in-plane material inhomogeneity based on the higher-order shear deformation theory with five degrees-of-freedom. Zhang [230] used the Ritz energy method to study the nonlinear post-buckling, nonlinear bending and vibration of FGM plates based on physical neutral surface and Reddy's TSDT. An efficient shear deformation theory having only four unknowns was proposed by Thai et al. [231] for the free vibration analysis of FGM plates based on the assumption that the in-plane and transverse displacements consists of bending and shear components, in which the bending components do not contribute towards shear forces and, likewise, the shear components do not contribute toward bending moments. The theory was found to account for the quadratic variation of the transverse shear strains across the thickness and also satisfied the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. Ungbhakorn and Wattanasakulpong [232] investigated the thermo-elastic vibration response of FGM plates carrying distributed patch mass based on the TSDT. Three types of temperature distribution through plate thickness were considered viz., uniform temperature rise, linear temperature rise and nonlinear temperature rise. The significant effects of magnitude, size and location of the patch mass on vibration frequency were investigated. The observed frequencies were lowest when the distributed patch mass was placed at the center and the corresponding frequencies were maximum when the patch mass was moved closer to the edge support. A unified solution method for the free vibrations of FGM cylindrical, conical shells and annular plates with general boundary conditions based on the FSDT was presented by Su et al. [233]. Each of the displacements and rotations irrespective of boundary conditions was expressed as a modified Fourier series which is capable of representing any function including the exact solutions.

3.2. Numerical methods

To overcome the difficulty in obtaining analytical solution for problems with geometrical and material complexity, numerical methods are being employed. In this section, various methods based on the finite element methods and meshless methods for vibration analysis of FGM plates are discussed separately.

3.2.1. Finite element method

Sundararajan et al. [234] used an eight-nodded shear flexible quadrilateral plate element based on consistency approach to analyze the large amplitude free flexural vibration behavior of FGM plates based on the FSDT. It was found that the effect of skew angle is to increase the ratio of nonlinear frequency to linear frequency compared to rectangular case. Pradyumna and Bandopadhyay [235] used an eight-nodded C^0 continuity element for the free vibration analysis of simply-supported FGM rectangular curved panels based on the higher-order formulation proposed by Kant and Khare [236]. The authors observed that though the higher-order shear deformation theory is computationally expensive, it shows good performance for both thin as well as thick panels and hence recommended for the free vibration analysis of both thin and thick FGM plates and shell panels. A p-version of FEM in conjunction with a blending function method was adopted by Alijani et al. [228] for the non-linear free vibration analysis of FGM doubly-curved shallow shell with an elliptical plan based on the FSDT. It was shown that FGM plates exhibit hardening behavior which depends on the volume fraction exponent and thickness ratio. Talha and Singh [122] used a C^0 continuous element with 13 dof at each node to present a higher-order shear deformation theory with a special modification in the transverse displacement which contributes additional freedom to the displacements through the thickness and fundamentally eradicates the over-correction for the static and free vibration analyses of FGM plates using the higher-order shear deformation theory. Talha and Singh [237] studied the large amplitude free flexural vibration analysis of shear deformable FGM plates based on the higher-order shear deformation theory using a C^0 continuous element with 13 dof at each node. Malekzadeh and Shojaee [238] used an eight-nodded solid element along with the Newmark's time integration scheme to investigate the response of FGM plates based on the FSDT under arbitrary boundary conditions and subjected to moving heat source.

3.2.2. Meshless methods

Yang and Shen [239] employed a one-dimensional differential quadrature method along with the Galerkin procedure to study the dynamic response of initially stressed FGM thin plates based on the CPT. The two opposite edges were clamped and remaining two edges were simply-supported and subjected to impulsive lateral load. Yang and Shen [240] employed the Galerkin approach and the modal superposition method to study the free and forced vibration behavior of initially stressed FGM plates in thermal environment based on the Reddy's higher-order shear deformation plate theory. Based on the results, the authors concluded that the plates with intermediate material properties under thermal environment do not necessarily have an intermediate dynamic response. Although the effect of geometric nonlinearity had been considered, the heat conduction and temperature dependent material properties were neglected. Further Kitipornchai et al. [241,242] used the similar methodology to study the effect of initial imperfection on the nonlinear vibration and random free vibration of laminated FGM plates in thermal environments. The MLPG method was employed by Qian et al. [243] to study the free and forced vibration response of FGM plates based on the higher-order shear deformation plate theory in which both the transverse shear deformation and transverse normal deformation were considered. The obtained results were compared with those obtained using the Galerkin FEM and analytical methods as well. It was observed that the frequencies computed by the Galerkin FEM were found to be higher than analytical values. However, frequencies computed using MLPG do not exhibit the same attribute. Dai et al. [244] extended a Meshfree radial point interpolation method for static and dynamic analysis of FGM plates based on the FSDT. Despite the capability of the model, the authors found two major setbacks viz., background cells were still required to be integrated and sometimes the computational cost is 3–8 times more expensive than the FEM. Chen [245] presented the nonlinear partial differential equations for the nonlinear vibration of FGM plates subjected to non-uniform initial stress based on the nonlinear von-Kármán assumptions. The Galerkin's method was used to transform governing partial differential equations into ordinary differential equations which are then solved by using the Runge–Kutta method. Chen et al. [246] derived nonlinear partial differential equations for the vibration motion of initially stressed FGM plates based on the CPT. It was found by the authors that the pure metal plates under compressive stress had the lowest frequency while the pure ceramic plates under tensile stress had the highest frequency. Ferreira et al. [247] presented solution for the free vibration analysis of FGM plates based on FSDT and TSDT using the global collocation method along with multi-quadratic radial basis function. The obtained results were validated using the 3D elasticity solution provided by Vel and Batra [201] and MLPG formulation provided by Qian et al. [157]. Finally, it was concluded by the authors that meshless method was computationally more efficient than the MLPG method since it neither requires nodal connectivity nor evaluation of any integral over a subdomain of the plate's midsurface. Fung and Chen [248] used a perturbation technique, Galerkin method and Runge–Kutta method for the nonlinear vibration analysis of initially stressed FGM plates with geometric imperfections based on the FSDT and von Karmen's nonlinearity. A multiquadratic radial basis function was employed by Roque et al. [249] for the free vibration analysis of FGM plates under various boundary conditions based on the third-order theory of Pandya and Kant [250]. Chen and Tan [251] used a perturbation technique, the Galerkin method and the Runge–Kutta method to study the effect of Imperfection on the nonlinear vibration of initially stressed FGM plates using the CPT. The authors demonstrated that the initial stress, geometric imperfection and volume fraction index greatly change the behavior of nonlinear vibration.

They validated the solution methodology by comparing the obtained results with those of Reddy's TSDT. Zhao et al. [252] used the element-free kp-Ritz method to study the free vibration behavior of FGM cylindrical shells based on the Sander's first-order shear deformation theory. Again, Zhao et al. [253] used the same method to study the static response and the free vibration behavior of FGM plates based on the FSDT and subjected to thermo-mechanical load. Based on the numerical results they concluded that the different values of shear correction coefficients had no pronounced effect on the frequency when side-to-thickness ratio was greater than or equal to 10. A geometric mapping technique in conjunction with the differential quadrature method was employed by Malekzadeh and Beni [254] for the free vibration analysis of FGM rectangular and skew plates in thermal environment based on the FSDT. Nie and Zhong [255] used the state-space method in combination with the differential quadrature method for dynamic analysis of multi-directional FGM plates with material properties exponentially varied along thickness, radial or both directions. A differential quadrature method in conjunction with trigonometric functions was employed by Zahedinejad et al. [256] for the 3D free vibration analysis of FGM curved panels. A differential quadrature method was used by Malekzadeh et al. [257] to study the free vibration behavior of FGM thick annular plates in thermal environment based on the 3D elasticity theory. It was observed that for the same value of temperature rise, the uniform temperature rise has more effect than non-uniform temperature rise on the frequency parameters and increasing the temperature rise, the discrepancy between the results of two cases increase dramatically. Yas and Aragh [258] presented an elasticity solution using the generalized differential quadrature method for the free vibration analysis of a four-parameter functionally gradation cylindrical panel. Wu et al. [259] employed a meshless collocation and an element-free Galerkin method along with the differential reproducing kernel interpolation for the 3D free-vibration of FGM sandwich plates. A local Kriging meshless technique was employed by Zhu and Liew [260] to construct shape functions which possess Kronecker delta function property. The FSDT and local Petrov–Galerkin formulation were used by the authors to study the free vibration behavior of FGM plates. Alijani et al. [261] studied the effect of thermal loads on the nonlinear vibration of doubly curved FGM shells based on two different higher-order theories using the multi-model energy approach along with a pseudo arclength continuation method and collocation scheme. Neves et al. [262] used the Carrera's Unified Formulation (CUF) and radial basis functions collocation method for the free vibration analysis of FGM shells based on a higher-order shear deformation theory that accounts for through-the-thickness deformation. Zhu and Liew [263] developed a meshless method using the Kriging interpolation technique for the free vibration analysis of FGM plates based on the FSDT and von-Kármán nonlinearity.

4. Buckling analysis

When an edge compressive load is applied to a flat plate along its middle plane, the plate is deformed but remains flat as long as the edge forces are adequately small and there is no initial imperfection in geometry of the plate. As the load increases an unstable state reaches as a result of which the plate bends slightly. The minimum in-plane edge compressive load required just to initiate such instability is called critical buckling load. As in the case of stress analysis and vibration analysis, here too, the entire research work has been brought under two classes namely analytical methods and numerical methods. Similarly, methods based on three-dimensional elasticity theory and two-dimensional elasticity theory are included in analytical methods and finite element methods and meshless methods are included in numerical methods.

4.1. Analytical methods

There has been extensive investigations carried out for the buckling analysis of FGM plates and FGM sandwich plates to predict the critical buckling loads under various boundary and loading conditions. A review of various analytical and numerical methods used for buckling analysis are presented in the following sections.

4.1.1. Two-dimensional plate theories

In order to obtain the critical buckling load eigenvalue problem has to be solved. Unfortunately, buckling analysis of FGM plates using three-dimensional elasticity theory which is very accurate method of analysis hitherto not yet reported in the literature. Various two-dimensional plate theories employed by numerous investigators to obtain the critical buckling load for FGM plates and FGM sandwich plates under various boundary conditions are discussed separately depending on the applied load: (i) Mechanical load case; (ii) Thermo-mechanical load case.

4.1.1.1. Mechanical load case. Birman [264] was the first person who attempted to solve the buckling problem of FGM hybrid composite plates. The problem was formulated based on the multi-cell model proposed by Chamis [265] and Hopkiins and Chamis [266]. Results were presented for the critical buckling load of FGM hybrid composite plates made of 3 different types of fibers viz., Silicon Carbide, Boron and Nicalon fibers. It was illustrated from the results that the buckling loads can be significantly increased by using piecewise reinforcement of fibers in composite plates. The elastic bifurcation buckling of FGM plates with nonuniformly distributed fibers under uniaxial compressive loading was studied by Feldman and Aboudi [267] who combined micromechanical and structural approach. An improvement in buckling load up to 100% as compared to the corresponding uniformly reinforced structure was presented. Ma and Wang [268] established the TSDT plate solution of the axisymmetric bending and buckling of FGM circular plates in terms of the CPT solutions of isotropic circular plates. The TSDT solutions were compared with those obtained using FSDT and CPT to show that FSDT is enough to consider the effect of shear deformation on the axisymmetric bending and buckling of FGM plates. Zenkour [269] used the SSDT to study the buckling and free vibration of simply-supported FGM sandwich plates. The obtained results were validated by comparing with those obtained using CPT, FSDT and TSDT. The critical buckling load obtained using nonsymmetric FGM plates was found to be higher than those of symmetric counterparts. Aydogdu [270] investigated the conditions for the bifurcation buckling of FGM plates based on the CPT. It was demonstrated that, bending moment is required to keep the simply-supported plates in flat under in-plane loading whereas, clamped plates can provide flatness before buckling. Saidi et al. [271] studied the axisymmetric bending and buckling using the UTST. In addition, relationships between the UTST solutions for thick FGM circular plates and the CPT solutions of homogeneous thin plates were also presented for the axisymmetric bending and buckling. It was observed that UTST results were much closer to TSDT results than FSDT ones when bending analysis is considered. Mohammadi et al. [272] presented the Levy solution using the principle of minimum potential energy for the buckling analysis of thin FGM plates based on the CPT subjected to different mechanical loads under various boundary conditions. The pre-buckling configuration of FGM plates based on the Mindlin theory was discussed by Naderi and Saidi [273]. It was concluded that the simply-supported FGM plates exhibit the bifurcation buckling when the external in-plane loads are applied on physical neutral surface of the plate. A new refined hyperbolic shear deformation theory was presented by Meiche et al. [274] using the Navier's solution technique for the buckling and free vibration analyses of FGM sandwich plates. The obtained results were validated by comparing with those obtained using CPT, FSDT, parabolic shear deformation theory, SSDT and 3D elasticity theory. Thai and Choi [275] extended the refined theory proposed by Shimpi [276] for the buckling analysis of FGM plates subjected to in-plane loading. The theory was found to have strong similarity with CPT in many aspects which can account for quadratic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The accuracy of the results obtained was demonstrated by comparing them with those of CPT, FSDT and TSDT. Latifi et al. [277] used the CPT based on physical neutral surface along with a double Fourier series expansion of displacement functions to investigate the buckling behavior of FGM plates subjected to proportional biaxial compressive loadings with arbitrary edge supports. Fereidoon et al. [278] extended the Kantorovich method for the bending analysis of FGM thin annular sector plates based on the CPT. The validity and the accuracy of obtained results were examined by using commercial FEM software package ABAQUS. A new refined plate theory with four unknowns was used by Fekrar et al. [279] to study the mechanical buckling of FGM plates.

Najafizadeh and Eslami [280] presented the buckling analysis of FGM circular plates based on the Love–Kirchhoff hypothesis and the Sander's non-linear strain–displacement relation with either simply-supported or clamped edges subjected to uniform radial compression. It was concluded by the authors that the mechanical instability of FGM plates was lower than fully ceramic plates. A perturbation technique along with one-dimensional differential quadrature approximation and Galerkin procedure was employed by Yang and Shen [281] to investigate the postbuckling behavior of fully clamped FGM rectangular plates based on the CPT under transverse and in-plane loads. The authors have concluded that though the mechanical performance of FGM plates is quite similar to homogeneous isotropic ones, they do exhibit some unique and interesting characteristics due to the grading of material composition. Again Yang et al. [282] extended the same work to study the effect of initial geometric imperfection on the post-buckling behavior of FGM plates based on the Reddy's TSDT under various boundary conditions. It was concluded that the effect of local imperfection becomes much less as its center deviates from the center of the plate. A three nodded shear flexible plate element based on the field-consistency principles was used by Prakash and Ganapathi [283] for the asymmetric flexural vibration analysis of FGM plates based on the FSDT. It was concluded that the non-linear temperature variation through the thickness results in higher critical buckling loads compared to constant through the thickness variation of temperature. Shariat et al. [284] studied the buckling behavior of geometrically imperfect FGM plates based on the CPT. A closed form solution was presented by Najafizadeh and Heydari [285] for the buckling of FGM circular plates based on the Reddy's TSDT subjected to uniform radial compression. The obtained results were compared with those of CPT and FSDT and it was observed that the buckling values predicted by TSDT were the lowest.

4.1.1.2. Thermo-mechanical load case. Javaheri and Eslami [286] derived the equilibrium and stability equations for the rectangular FGM plates based on the Reddy's TSDT and then obtained the closed form solution for the critical buckling temperature. It was concluded that the employed TSDT underestimates the buckling load compared with CPT. The Navier's solution technique was employed by Lanhe [287] to study the thermal buckling of simply-supported moderately thick rectangular FGM plates based on the FSDT and subjected to two types of temperature fields; uniform temperature rise and gradient across the thickness of the plate. Najafizadeh and Heydari [288] presented the closed form solution for the critical buckling temperature of clamped FGM circular plates based on the Reddy's TSDT. A plane thermoelastic problem was solved by Morimoto et al. [82] using the Galerkin method to study the buckling behavior of FGM rectangular plates subjected to partial heating in a plane and uniform temperature rise through the thickness. It was concluded that there is no stretching–bending coupling in constitutive equations if the reference surface is selected properly. Shen [289] used a two-step perturbation technique along with the Reddy's TSDT to study the thermal postbuckling behavior of perfect and imperfect FGM plates and showed that for heat conduction, the postbuckling path for geometrically perfect plates is no longer of the bifurcation type. The Galerkin's method was employed by Navazi and Haddadpour [290] to determine the aero-thermoelastic stability margins of FGM plates based on the CPT and subjected to combined thermal and aerodynamic loads using a quasi-steady supersonic piston theory. It was found that in both, the absence and the presence of thermal loads, the aeroelastic response of FGM plates is an intermediate value of those of ceramic and metal plates. Matsunaga [291] used a two-dimensional global higher-order shear deformation theory to study the buckling behavior of FGM plates subjected to thermal loads. The Navier's solution technique along with several set of truncated approximate theories was used to obtain the solution for eigenvalue problem. Saidi and Baferani [292] presented an analytical solution for the thermal buckling analysis of FGM annular sector plates based on the FSDT. It was assumed that the plate is simply-supported in radial edges and has arbitrary boundary conditions along the circular edges. Zenkour and Sobhy [293] presented an analytical solution to find the critical buckling temperature difference of various symmetric FGM sandwich plates based on the SSDT and von-Kármán nonlinearity. Three types of thermal loadings were considered viz., uniform, linear and non-linear distribution through-the-thickness. The obtained results were compared with those of CPT, TSDT and FSDT. It was found that the results predicted by are higher than those predicted using rest of the shear deformation theories. Bodaghi and Saidi [294] used a Levy-type solution method to investigate the thermoelastic buckling behavior of thick FGM rectangular plates based on the Reddy's TSDT. The five highly coupled governing equations obtained using the principle of minimum potential energy were decoupled into two new uncoupled partial differential equations in terms of lateral deflection function and edge zone function. The parameters such as rotations, in-plane displacements and stress resultants were obtained in terms of these two functions. The average stress method was used by Chen et al. [295] to study the thermally induced buckling of initially stressed FGM hybrid composite plates based on the FSDT. Kettaf et al. [296] used the new hyperbolic displacement model with four unknowns to study the thermal buckling behavior of sandwich plates consisting of FGM face sheets and homogeneous core made of isotropic ceramic material.

Najafizadeh and Hedayati [297] studied the axisymmetric thermal and mechanical buckling of FGM circular plates based on the FSDT. Three types of thermal loadings were considered viz., uniform temperature rise, linear and nonlinear gradient through the thickness and uniform radial compression. Woo et al. [298] used a mixed Fourier series solution to derive analytical solutions to study the postbuckling behavior of moderately thick FGM plates and shallow shells under edge compressive load and specified temperature field. The problem was formulated using the von Karmen nonlinear theory for large transverse deflection and Reddy's TSDT. Shariat and Eslami [299,300] studied the effect of geometrical imperfections on the thermal buckling of rectangular FGM plates based on the FSDT. The plate was assumed to be under three types of thermal loadings namely uniform temperature rise, nonlinear temperature rise through the thickness and axial temperature rise. A finite double Chebyshev polynomial which is capable of solving plates with non-Levy type boundary conditions was employed by Wu et al. [301] to study the postbuckling response of FGM plates under different kinds of boundary conditions subjected to uniaxial compression or uniform temperature rise based on the FSDT. It was observed that no significant difference exists between the postbuckling response of FGM plates with aspect ratio equal to 3 and 4. Shariat and Eslami [302] proposed a closed-form solution for the buckling analysis of rectangular thick FGM plates based on the TSDT under mechanical and thermal loads. Three types of mechanical loadings were considered namely; uniaxial compression, biaxial compression, and biaxial compression and tension along with two types of thermal loadings viz., uniform temperature rise and non-uniform temperature rise through the thickness. The Navier's solution technique was presented by Bouazza et al. [303] for the stability analysis of simply-supported FGM plates subjected to uniform and linear temperature rise through the thickness based on the FSDT. Bodaghi and Saidi [304] presented a Levy-type solution using a boundary layer function for the buckling analysis of thick FGM plates under various boundary conditions based on the Reddy's TSDT.

4.2. Numerical methods

As it is already stated, the analytical solution is not easy to obtain for all plate geometry and boundary conditions. Solution becomes even more tedious when material properties are varied nonlinearly through the thickness. Therefore, numerical methods such as finite element methods and meshless methods are being extensively used for complex engineering problems.

4.2.1. Finite element method

Various finite element techniques used for linear and nonlinear buckling analyses of FGM plates due to mechanical load and thermo-mechanical load are discussed separately under two heading viz., (i) Mechanical load case; (ii) Thermo-mechanical load case

4.2.1.1. Mechanical load case. The existence of bifurcation buckling under various thermal and mechanical loads was investigated by Bateni et al. [305]. A four-variable refined plate theory was used to derive the governing equations of equilibrium. A multi-term Galerkin solution was presented to deduce the critical buckling loads/temperatures along with the buckled shape of the plate. The effect of temperature dependency of material properties on the critical buckling load was investigated for both uniform temperature rise and heat conduction cases. It was found that in both cases temperature dependency results in underestimation of the critical buckling temperature.

Naei et al. [306] used the energy method based on Love–Kirchhoff hypothesis along with the Sander's non-linear strain–displacement relation to study the buckling analysis of a radially-loaded circular FGM plates with variable thickness. Lee and Kim [307] investigated the postbuckling behavior of FGM plates in hygrothermal environments based on the FSDT and von-Kármán nonlinearity. Material properties were graded in the thickness direction using a power law function. It was concluded that the effect of moisture on the postbuckling behavior significantly increases with the increase in the value of power law parameter.

4.2.1.2. Thermo-mechanical load case. An 18-node solid element and assumed strain mixed formulation was employed by Na and Kim [308–310] for the 3D thermal buckling and postbuckling analysis of FGM plates with temperature-dependent material properties. However, a sinusoidal and linear through-the-thickness distribution of temperature was considered which is independent of time and does not reflect the actual temperature distribution in FGM plates. The EFG method and approximation were used by Jaberzadeh et al. [311] to study the thermal buckling of FGM skew and trapezoidal plates based on CPT.

A pseudo-spectral method that involves a finite element method and collocation method was used by Jalali et al. [312] to study the thermal stability of laminated FGM circular plates based on the FSDT and subjected to uniform temperature rise. Prakash et al. [313] used an eight-nodded C^0 shear flexible quadrilateral plate element to study the nonlinear bending/pseudo-post-buckling behavior of FGM plates based on the Mindlin formulation under thermo-mechanical load and concluded that temperature dependent material properties overestimates the thermal postbuckling resistance. Later, Prakash et al. [314] extended the same work to study the influence of the position of the neutral surface on the stability behavior of FGM plates. A nine nodded rectangular element was used by Sohn and Kim

[315] to study the static and dynamic stability of FGM panels based on the FSDT and also subjected to thermal and aerodynamic loads simultaneously.

4.2.2. Meshless method

Various meshless methods used for linear and nonlinear buckling analyses of FGM plates and FGM sandwich plates are again divided into two classes depending on the applied load: (i) Mechanical load case; (ii) Thermo-mechanical load case.

4.2.2.1. Mechanical load case. A two-dimensional elastic plane stress problem of FGM plates based on the Mindlin's plate assumption and subjected to nonlinearly distributed in-plane edge loads was investigated by Chen and Liew [316] for the buckling behavior using the radial basis function. Zhang [317] investigated stability and local bifurcation behaviors for a simply-supported FGM rectangular plate subjected to the transversal and in-plane excitations in the uniform thermal environment using both analytical and numerical methods. Three kinds of degenerated equilibrium points of the bifurcation response equations were considered.

Mahdavian [318] used the Airy stress field approach and Galerkin's approach for the stability analysis of FGM plates and subjected to non-uniform in-plane compressive loads based on the CPT and von-Kármán nonlinearty. Four types of loadings were considered viz., concentrated load, triangular load, uniform load, reverse triangular load and sinusoidal load. The critical buckling load coefficient was found to be maximum in case of sinusoidal load and minimum for concentrated load.

4.2.2.2. Thermo-mechanical load case. Liew et al. [319] employed the principle of minimum potential energy and differential quadrature method along with the FSDT to study the thermal buckling and postbuckling behavior of FGM hybrid plates with different kind of boundary conditions under uniform temperature rise. It was concluded that the unsymmetric FGM laminated plates do not have the bifurcation-type thermal buckling due to the presence of stretching-bending coupling and their post-buckling behavior is different from those of their symmetric counterparts. Ganapathi and Prakash [320] studied the thermal buckling of simply-supported FGM skew plates based on the FSDT considering linear and nonlinear forms of temperature-rise across the thickness. Two types of boundary conditions were considered namely simply-supported and clamped boundary conditions. A moving least square differential quadrature method was employed by Lanhe et al. [321] to investigate the dynamic stability of thick FGM plates based on the FSDT and subjected to aero-thermo-mechanical loads. A Bolotin's method was used to obtain the boundaries of instability region. Zhao et al. [322] employed an element-free kp-Ritz method to study the mechanical and thermal buckling response of FGM arbitrary straight-sided quadrilateral plates with a square/circular hole based on the FSDT. The bending stiffness was evaluated using the nodal integration technique whereas the shear and membrane terms were computed using a direct nodal integration technique. Later, Lee et al. [323] extended the same method for postbuckling analysis of FGM plates based on the FSDT under edge compression and temperature field conditions. The 3D thermal buckling analysis was presented by Malekzadeh [324] using the differential quadrature technique. The obtained results were validated by comparing them with those obtained using the higher-order shear deformation theory of Matsunaga [291]. Sepahi et al. [325] presented the thermal buckling and postbuckling analysis of annular FGM plates with temperature dependent material properties based on the FSDT using the differential quadrature method. It was observed that for annular plates with the same inner edge boundary condition, plates with clamped outer edge have greater critical temperature rise than those with simply-supported edge. Ghannadpour et al. [326] used the finite strip method for buckling analysis of FGM plates under thermal loadings based on the CPT. Three types of thermal loadings were considered namely; uniform temperature rise, linear temperature change across the thickness and nonlinear temperature change across the thickness. A semi-analytical method comprising of Galerkin and differential quadrature approach was used by Yang et al. [327] along with the Reddy's TSDT to study the dynamic stability of laminated FGM plates under combined action of uniform temperature change and a periodic in-plane load. It was concluded that the effect of temperature rise and material composition in FGM layers is more significant in thick plates. Zhang et al. [328] studied the mechanical and thermal buckling behaviors of FGM plates based on FSDT using the MLPG approach along with moving Kriging interpolation technique which can possess Kronecker delta function property.

Park and Kim [329] studied the thermal postbuckling and vibration behaviors of FGM plates with temperature-dependent material properties in the pre- and post-buckled regions based on the FSDT and incremental strain–displacement relationship. A von-Kármán type of nonlinearity was incorporated for the analysis. A shooting method along with the von-Kármán's plate theory was employed by Li et al. [330] to study the postbuckling of an imperfect FGM plates subjected to mechanical load and transverse non-uniform temperature rise as well. It was observed that the effect of mechanical load on the deflection decreases with the increase in the value of thermal load, and mechanical load becomes dominant in contributing on the central deflection when the effect of thermal load is very small. Liew et al. [331] used the element-free kp-Ritz method to investigate the postbuckling response of FGM cylindrical shells under axial compression and thermal loads based on the FSDT and von-Kármán nonlinearity. The system bending stiffness was evaluated using a stabilized conforming nodal integration method and the membrane and shear terms were estimated using direct nodal integration to eliminate shear locking and reduce the computational cost.

5. Future direction of research

Because of the flexibility in design and excellent performance under thermal and mechanical loading, FGMs are considered as a potential structural material for future engineering applications. Therefore, it becomes absolutely necessary to improve the solution techniques and methodologies for the analysis of FGM plates. As a step towards achieving this objective, there is a need to undertake the following studies and extensions.

1. A detailed study of the effect of geometric nonlinearity on the behavior of FGM plates using higher-order theories capable of considering the effect of transverse shear deformation is required.
2. A comparative study is required for analytical validation of various refined higher-order theories and to study their advantages and disadvantages with respect to accuracy and computational cost involved in evaluating the stresses and displacement of FGM plates with complex geometry and boundary conditions under various types of loading cases.
3. A focused attention is required for the development of numerical techniques for 3D analysis of FGM structures using advanced computational techniques that can effectively reduce both computational time and cost.
4. Analytical formulation and solution methodology using 3D elasticity theory have to be developed for FGM plates with power law type of variation of material properties.

5. Development of analytical solution using 3D elasticity theory for the stability analysis of FGM plates.

6. Conclusions

A review is presented on recent advances in analytical and numerical methods for the stress, vibration and buckling analyses of single and multi-layered FGM plates. Most of the approaches employed for the analysis of FGM plates are the extensions of the similar approaches used for composite/isotropic plates. Analytical solutions using 3D elasticity equations are restricted only for FGM plates with exponential and some other simple type of material gradients. 3D analytical solutions for FGM plates with the power law variation of material properties are still not available. Also, 3D analytical solution methods to study the nonlinear response and stability behavior of FGM plates are not yet available. Use of direct displacement method for 3D exact analysis is restricted only for FGM plates under uniform load. Adoption of the series solution limits the problem only to the rectangular plates. 3D analysis using a numerical technique requires more computational effort and larger computer core memory than 2D analysis. Thus, more attention was given to restrict 2D analysis with little compromise in the accuracy of results in comparison with 3D analysis.

Among the various 2D plate theories CPT, FSDT, TSDT, SSDT, Soldato plate theory, Levinson plate theory, UTST, FOST, CUF etc., employed for the analysis of FGM plates, FSDT and TSDT were extensively used. FSDT can provide reasonably accurate results at less computational cost. However, FSDT requires a shear correction factor to rectify unrealistic variation of shear stress/strain through the thickness. The classical shell theory based on Love–Kirchhoff's assumption fails to predict the realistic behavior of FGM and multilayered magneto-electro-elastic shells.

It has been proved by various researchers that meshless methods are reliable alternative to finite element methods, regardless of the variation of plate thickness and ceramic/metal contents. In spite of having some advantages over FEM, meshless methods suffer greatly because of their high computational cost in 3D analysis, difficulty in treating boundary conditions and instability in certain methods.

The gradients in material properties play an important role in determining the response of FGM plates. The combined effect of anisotropy and assumed function for gradation of material properties makes the prediction of nonlinear behavior of FGM structures complicated. The basic response of FGM plates that correspond to properties intermediate to that of the metal and ceramic necessarily lies in between that of the ceramic and the metal irrespective of the boundary conditions when only mechanical load case is considered whereas for thermal load case the deflection of the plate does not necessarily lie in between that of the ceramic and metal.

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