Journal of Thermal Stresses

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/uths20

Solution in Elementary Functions to a BVP of Thermoelasticity: Green's Functions and Green's-Type Integral Formula for Thermal Stresses within a Half-Strip

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Published online: 02 Jun 2014.

Journal of Thermal Stresses, 37: 947–968, 2014

ISSN: 0149-5739 print/1521-074X online
DOI: 10.1080/01495739.2014.913389

SOLUTION IN ELEMENTARY FUNCTIONS TO A BVP OF THERMOELASTICITY: GREEN'S FUNCTIONS AND GREEN'S-TYPE INTEGRAL FORMULA FOR THERMAL STRESSES WITHIN A HALF-STRIP

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This article presents new elementary Green's functions for displacements and stresses created by a unit heat source applied in an arbitrary interior point of a half-strip. We also obtain the corresponding new integration formulas of Green's and Poisson's types which directly determine the thermal stresses in the form of integrals of the products of internal distributed heat source, temperature, or heat flux prescribed on boundary and derived thermoelastic influence functions (kernels). All these results are presented in terms of elementary functions in the form of a theorem. Based on this theorem and on derived early by author general Green's type integral formula, we obtain a new solution to one particular boundary value problem of thermoelasticity for half-strip. The graphical presentation of thermal stresses created by a unit point heat source and of thermal stresses for one particular boundary value problem of thermoelasticity for half-strip is also included. The proposed method of constructing thermoelastic Green's functions and integration formulas are applicable not only for a half-strip but also for many other two- and three-dimensional canonical domains of Cartesian system of coordinates.

Keywords: Green's functions; Green's type integral formula in thermoelasticity; Heat conduction; Thermoelastic influence functions; Volume dilatation

INTRODUCTION

About Some Works on Green's Functions and Green's Matrices

The most difficult problem of Green's function method (GFM) that plays a leading role in finding solutions in integrals for boundary value problems (BVPs) is the construction of Green's functions (GFs). This is why the constructing of a new Green's function, especially in closed form, is considered as substantial contribution in this field. In monographs [1–4], some methods for deriving GFs for ordinary and

Received 24 September 2013; accepted 10 October 2013.

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partial differential scalar equations are presented. The derivation and application of GFs and Green's matrices (GMs) for two dimensional (2D) BVPs for Lame's equations in theory of elasticity are presented in the monographs [5, 6].

Green's functions for advanced materials are derived in the monographs [7, 8]. A large list of GFs for 2D BVPs of Poisson's equation, constructed for canonical domains described in Cartesian or polar coordinates is given in the encyclopedia [9]. Respectively, in handbook [10], there is a large table of GFs and GMs for two-dimensional and three-dimensional (3D) BVPs in the theory of elasticity derived for canonical Cartesian domains. So, until present the most GFs were derived for BVPs of heat conduction theory and for BVPs of the theory of elasticity for Cartesian canonical domains.

About Some Integral Formulas of Thermoelasticity

Currently, a number of theories of thermoelasticity have been developed and described in literature [11–17]. But many new developments of thermoelasticity and many new references are included in the book [18]. The best developed theory which is widely used in practical calculations is the theory of thermal stresses, when the temperature field does not depend on the field of elastic displacements. In this theory, the following observations are worth to be mentioned. In the theory of uncoupled heat conduction to solve a BVP, a Green's integral formula provides the temperature field resulted from a given thermal exposure. The analogous Green's integral formula determines the elastic displacements, produced by the known mechanical actions. In the theory of thermoelasticity, that is a synthesis of the theory of heat conduction and of the theory of elasticity, the situation in not analogical as in their component parts mentioned previously.

As example in the Maysel's integral formula [13, 15, 16] the solution of a BVPs is not represented directly via the known data, but via the temperature field, which in the most cases is necessary to be found. This fact introduces certain inconvenience in application of Maysel's integral formula except the case when the temperature field is known already. To avoid the inconvenience of the Maysel's formula mentioned above, the author for the first time has proposed the following generalization of the Maysel's and Green's integral formulas in thermoelasticity [10, 19–23]:

$$u_i(\xi) = a^{-1}\int_V F(x)U_i(x,\xi)dV(x) - \int_{\Gamma_D} T(y)\frac{\partial U_i(y,\xi)}{\partial n_y}d\Gamma_D(y) + \int_{\Gamma_N}\frac{\partial T(y)}{\partial n_y}U_i(y,\xi)d\Gamma_N(y) + a^{-1}\int_{\Gamma_M}\left[\alpha T(y) + a\frac{\partial T(y)}{\partial n_y}\right]U_i(y,\xi)d\Gamma_M(y);\quad i = 1, 2, 3 \tag{1}$$

where Γ_D, Γ_N and Γ_M denote the surfaces on which the boundary conditions of Dirichlet's, Neumann's and mixed type are prescribed respectively: temperature $T(y)$, heat flux $a(\partial T(y)/\partial n_y)$ and a heat exchange between exterior medium and surface of the body represented by $\alpha T(y) + a[\partial T(y)/\partial n_y]$ law; $F(x)$ is the heat source; a is thermal conductivity; α is the coefficient of convective heat conductivity; $\gamma = \alpha_t(2\mu + 3\lambda)$ is the thermoelastic constant; λ, μ are Lamé's constants of elasticity;

α_t is the coefficient of the linear thermal expansion. The main advantage of formula (1) is that the unknown thermoelastic displacements $u_i(\xi)$ are determined in the integral form directly via the prescribed inner heat source and other thermal data, given on the boundary. However the elegant Maysel's integral formula remain still very important because it was generalized to piezoelectric vibrations [24], to piezoelastic bodies [25], to anisotropic thermoelastic bodies [26], to the dynamic eigenstrain problem [27], etc.

The MTGFs and Other Influence Functions

The considered MTGFs in Eq. (1) have the physical sense that as thermoelastic displacements at an arbitrary inner point generated by a unit heat source applied at another arbitrary inner point described by the Dirac-delta function. The functions $U_i = U_i(x, \xi)$ are determined by the following formula [10, 19–23]:

$$U_i(x, \xi) = \gamma \int_V G_T(x, z)\, \Theta^{(i)}(z, \xi) dV(z); \quad x, z, \xi \in V \tag{2}$$

where G_T is the GF for a heat conduction BVP corresponding to an unit internal point heat source, and $\Theta^{(i)}$ are functions of influence of unit concentrated body forces onto elastic volume dilatation.

The proposed integral formula (1) can be treated as a generalization of the Mayzel's formula [13, 15, 16] to those cases when the temperature field satisfies the BVP of heat conduction. The main advantage of formula (1) is that it allows us to unite the two-staged process of solving a BVP of thermoelasticity (the first stage comprises finding temperature fields, and the second ones comprises finding thermoelastic displacements) in one single stage. Also, the advantage of the formula (1) in comparison with the well-known Maysel's integral formula is that the thermoelastic displacements are determined directly via given heat actions.

Besides, for any concrete type of BVP of thermoelasticity we can obtain all possible solutions for different laws describing the above mentioned heat actions. Note, that, using formulas (1) and (2), the author derived, in elementary functions, some new very useful thermoelastic GFs and Green's type integral formulas for quadrant [28, 29], half-space [30, 31], quarter-space [32, 33], wedge [34, 35] and half-wedge [36]. For all these BVPs the difficulties associated with the constructing of the additional influence functions for elastic volume dilatation and with the computing of the volume integral (2) have been successfully overcome. Furthermore, the author has observed that for more complicated BVPs of thermoelasticity these difficulties are substantially more complex. It should be noted that in contrast to the 3D BVPs [11–16], for the calculation of integrals of the type (2) in the case of 2D problems of thermoelasticity for a half-strip.

For this reason, the author sought other methods to derive MTGFs. However, the preliminary investigations made by author have shown that the classical methods [11–18] such as: method of body-force analogy [11, 16], Goodier's method [11], method of thermoelastic potentials [11, 15, 16] and many other methods [12–18] leads to necessity to solve additional elastic BVPs or to calculate the complicate volume integral (2).

Objectives

The first main objective of this research is to prove a theorem on derivation of MTGFs and Green- type integral formula for a specific BVP for a half-strip with different types of mixed homogeneous mechanical and thermal boundary conditions. To reach this objective it is necessary first to use the general integral representations for MTGFs in Cartesian system of coordinates [37]. The other important objective is to solve a concrete particular BVP of thermoelasticity by using the derived MTGFs and Green's type integral formula.

Further, based on the derived analytical expressions for thermoelastic Green's functions, it is necessary to obtain solutions in the integrals of the type (1). Moreover, this article first proposed new MTGFs and Green's type integral formula for thermal stresses. This is due to the fact that in a number of BVPs of thermoelasticity it is necessary to determine the thermal stresses directly from respectively derived integral formulas, without preliminary determination the field of thermoelastic displacements. So, in this article the following Green's-type integral formula is proposed:

$$\sigma_{ij}(\xi) = a^{-1}\int_V F(x)\sigma^*_{ij}(x,\xi)dV(x) - \int_{\Gamma_D} T(y)\frac{\partial\sigma^*_{ij}(y,\xi)}{\partial n_y}d\Gamma_D(y) + \int_{\Gamma_N}\frac{\partial T(y)}{\partial n_y}\sigma^*_{ij}(y,\xi)d\Gamma_N(y) + a^{-1}\int_{\Gamma_M}\left[\alpha T(y) + a\frac{\partial T(y)}{\partial n_y}\right]\sigma^*_{ij}(y,\xi)d\Gamma_M(y); \quad i,j = 1,2,3 \qquad (3)$$

to determine thermal stresses $\sigma_{ij}(\xi)$. The thermal stresses $\sigma_{ij}(\xi)$ and $\sigma^*_{ij}(x,\xi)$ in Eq. (3) are determined on the base of Duhamel–Neumann law:

$$\sigma_{ij} = \mu(u_{i,j} + u_{j,i}) + \delta_{ij}(\lambda\theta - \gamma T);\ \theta = u_{k.k};\quad i,j,k = 1,2,3 \qquad (4)$$

$$\sigma^*_{ij} = \mu(U_{i,j} + U_{j,i}) + \delta_{ij}(\lambda\Theta - \gamma G);= \Theta = U_{k,k}(x,\xi) \qquad (5)$$

It should be noted that the main advantage of the integral formula (3) is that it allow us to determine directly (without predetermining the thermoelastic displacements and without prior determination of the temperature field) the thermal stresses in the form of integrals of the products of the internal heat source and surface known thermal actions and pre-computed thermoelastic influence functions (kernels), σ^*_{ij}. A specific example of making and using the formula (3) to the determination of thermal stresses of a BVP of thermoelasticity for a half-strip is given later.

Finally the last objective of this research is the computer evaluations and graphical presentations of the derived MTGFs and of the solution to particular BVP of thermoelasticity for thermal stresses σ^*_{ij} and σ_{ij}, respectively (see the Appendix).

DERIVATION OF MTGFs AND GREEN'S TYPE INTEGRAL FORMULA FOR A HALF-STRIP IN TERMS OF GFPE

Here, we consider the problem of static equilibrium of the thermoelastic half-strip, located in the plane strain conditions. The half-strip is exposed by a unit point internal heat source or a unit boundary point temperature or a unit boundary point heat flux (in this case, if the relevant functions of influence need to be constructed), and distributed thermal actions (in this case we derive appropriate Green's-type integral formula for the solution of BVPs of thermoelasticity). Later we derive the MTGFs for thermoelastic displacements U_i followed by the derivation of the MTGFs for thermal stresses, σ^*_{ij}. Finally, we derive the Green's-type integral formula for thermal stresses σ_{ij} within the half-strip. The preceding results are formulated and proved in the following theorem.

Theorem. *Let the field of displacements $u_i(\xi)$ at inner points $\xi \equiv (\xi_1, \xi_2)$ of the thermoelastic half-strip $V(0 \le x_1 < \infty; 0 \le x_2 \le a_2)$ be determined by non-homogeneous Lamé equations*

$$\mu\nabla^2 u_i(\xi) + (\lambda + \mu)\theta_{,i}(\xi) - \gamma T_{,i}(\xi) = 0 \tag{6}$$

and in the points $y \equiv (0, y_2)$, $y \equiv (y_1, 0)$, and $y \equiv (y_1, a_2)$ of boundary lines $\Gamma_{10}(y_1 = 0; 0 \le y_2 \le a_2)$, $\Gamma_{20}(0 \le y_1 < \infty; y_2 = 0)$ and $\Gamma_{21}(0 \le y_1 < \infty; y_2 = a_2)$ the following homogeneous locally mixed mechanical boundary conditions are given:

$$\begin{aligned}&\sigma_{11} = u_2 = 0, \quad \xi_1 = 0, \quad 0 \le \xi_2 \le a_2; \quad \sigma_{22} = u_1 = 0; \quad \xi_2 = 0, \quad 0 \le \xi_1 \le \infty;\\ &u_2 = \sigma_{21} = 0, \quad \xi_2 = a_2, \quad 0 \le \xi_1 \le \infty\end{aligned} \tag{7}$$

where σ_{11}, σ_{22} and σ_{21} are the normal and the tangential stresses which are determined by the well-known Duhamel–Neumann law

$$\sigma_{ij} = \mu(u_{i,j} + u_{j,i}) + \delta_{ij}(\lambda u_{k,k} - \gamma T); \quad i, j = 1, 2 \tag{8}$$

Let also the temperature field $T(\xi)$ in Eq. (6), generated by the inner heat source $F(\xi)$, boundary temperature $T_{20}(y)$ and heat flux $S_{21}(y)$ satisfy the following BVP of heat conduction

$$\nabla^2 T(\xi) = -a^{-1}F(\xi), \quad \xi \in V \tag{9a}$$

$$\begin{aligned}&T(y) = T_{10}(y), \quad y \equiv (0, y_2) \in \Gamma_{10}; \quad T(y) = T_{20}(y), \quad y \equiv (y_1, 0) \in \Gamma_{20};\\ &a(\partial T/\partial n_{y2}) = S_{21}(y), \quad y \equiv (y_1, a_2) \in \Gamma_{21}\end{aligned} \tag{9b}$$

If the inner heat source and boundary thermal data satisfy the conditions:

$$\begin{aligned}&\int_0^{+\infty}\int_0^{a_2} |F(x)|dx_1dx_2 < \infty; \quad x \equiv (x_1, x_2); \quad \int_0^{a_2} |T_{10}(0, y_2)|dy_2 < \infty,\\ &\int_0^{+\infty} |T_{20}(y_1, 0)|dy_1 < \infty, \quad \int_{-\infty}^{+\infty} |S_{21}(y_1, a_2)|dy_1 < \infty\end{aligned} \tag{10}$$

then the solution of this BVP in Eqs. (6)–(10) *of thermoelasticity for unknown thermal stresses* $\sigma_{ij}(\xi)$ *exists and it can be presented by the following Green's-type integral formula, written in the matrix form*:

$$\sigma_{ij}(\xi) = \frac{1}{a}\left[\int_0^{+\infty}\int_0^{a_2} F(x)\sigma^*_{ij}(x,\xi)dx_1dx_2 + \int_{-\infty}^{\infty} S_{20}(y_1,a_2)\Pi_{ij}(y_1,a_2;\xi)dy_1\right] - \int_0^{a_2} T_{10}(0,y_2)K_{ij}(0,y_2;\xi)dy_2 - \int_0^{+\infty} T_{20}(y_1,0)Q_{ij}(y_1,0;\xi)dy_1 \quad (11)$$

Kernels $\sigma^*_{ij}(x,\xi)$, $\Pi_{ij}(y_1,0;\xi)$, $K_{ij}(0,y_2;\xi)$ *and* $Q_{ij}(y_1,0;\xi)$ *have the following formulas*:

$$\sigma^*_{11}(x,\xi) = -\frac{\mu\gamma}{4\pi(\lambda+2\mu)}\left[\left(1-\xi_1\frac{\partial}{\partial\xi_1}\right)\ln\frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} + x_1\frac{\partial}{\partial\xi_1}\ln\frac{\overline{E}\overline{E}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}}\right] \quad (12a)$$

$$\sigma^*_{22}(x,\xi) = -\frac{\mu\gamma}{4\pi(\lambda+2\mu)}\left[\left(1+\xi_1\frac{\partial}{\partial\xi_1}\right)\ln\frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} - x_1\frac{\partial}{\partial\xi_1}\ln\frac{\overline{E}\overline{E}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}}\right] \quad (12b)$$

$$\sigma^*_{12}(x,\xi) = \frac{\gamma\mu}{4\pi(\lambda+2\mu)}\frac{\partial}{\partial\xi_2}\left[\xi_1\ln\frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} - x_1\ln\frac{\overline{E}\overline{E}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}}\right] \quad (12c)$$

where the functions $\overline{E},\overline{E}_2,\widetilde{E},\widetilde{E}_2,\overline{E}_1,\overline{E}_{12},\widetilde{E}_1,\widetilde{E}_{12}$ *are determined by expressions*:

$$\overline{E} = \overline{E}(x,\xi) = 1 + 2e^{(\pi/2a_2)(x_1-\xi_1)}\cos(\pi/2a_2)(x_2-\xi_2) + e^{(\pi/a_2)(x_1-\xi_1)},$$
$$\overline{E}_2 = \overline{E}_2(x,\xi) = \overline{E}(x;\xi_1,-\xi_2) \quad (12d)$$
$$\widetilde{E} = \widetilde{E}(x,\xi) = 1 - 2e^{(\pi/2a_2)(x_1-\xi_1)}\cos(\pi/2a_2)(x_2-\xi_2) + e^{(\pi/a_2)(x_1-\xi_1)},$$
$$\widetilde{E}_2 = \widetilde{E}_2(x,\xi) = \widetilde{E}(x;\xi_1,-\xi_2) \quad (12e)$$
$$\overline{E}_1 = \overline{E}_1(x,\xi) = 1 + 2e^{-(\pi/2a_2)(x_1+\xi_1)}\cos(\pi/2a_2)(x_2-\xi_2) + e^{-(\pi/2a_2)(x_1+\xi_1)};$$
$$\overline{E}_{12} = \overline{E}_{12}(x,\xi) = \overline{E}_1(x;-\xi_1,\xi_1) \quad (12f)$$
$$\widetilde{E}_1 = \widetilde{E}_1(x,\xi) = 1 - 2e^{-(\pi/2a_2)(x_1+\xi_1)}\cos(\pi/2a_2)(x_2-\xi_2) + e^{-(\pi/2a_2)(x_1+\xi_1)};$$
$$\widetilde{E}_{12} = \widetilde{E}_{12}(x,\xi) = \widetilde{E}_1(x;-\xi_1,\xi_1) \quad (12g)$$

– *for kernels* $\sigma^*_{ij}(x,\xi)$;

$$\Pi_{11}(y_1,a_2;\xi) = \sigma^*_{11}(y_1,a_2;\xi) = -\frac{\gamma\mu}{2\pi(\lambda+2\mu)}\left[1-\xi_1\frac{\partial}{\partial\xi_1}\ln\frac{\overline{E}_{a_2}\widetilde{E}_{1a_2}}{\widetilde{E}_{a_2}\overline{E}_{1a_2}} + y_1\frac{\partial}{\partial\xi_1}\ln\frac{\overline{E}_{a_2}\overline{E}_{1a_2}}{\widetilde{E}_{a_2}\widetilde{E}_{1a_2}}\right] \quad (13a)$$

$$\Pi_{22}(y_1,a_2;\xi) = \sigma^*_{22}(y_1,a_2;\xi) = -\frac{\gamma\mu}{2\pi(\lambda+2\mu)}\left[1+\xi_1\frac{\partial}{\partial\xi_1}\ln\frac{\overline{E}_{a_2}\widetilde{E}_{1a_2}}{\widetilde{E}_{a_2}\overline{E}_{1a_2}} - y_1\frac{\partial}{\partial\xi_1}\ln\frac{\overline{E}_{a_2}\overline{E}_{1a_2}}{\widetilde{E}_{a_2}\widetilde{E}_{1a_2}}\right] \quad (13b)$$

$$\Pi_{12}(y_1, a_2; \xi) = \sigma_{12}^*(y_1, a_2; \xi) = \frac{\gamma\mu}{2\pi(\lambda+2\mu)} \frac{\partial}{\partial \xi_2}\left[\xi_1 \ln \frac{\overline{E}_{a_2}\widetilde{E}_{1a_2}}{\widetilde{E}_{a_2}\overline{E}_{1a_2}} - y_1 \ln \frac{\overline{E}_{a_2}\overline{E}_{1a_2}}{\widetilde{E}_{a_2}\widetilde{E}_{1a_2}}\right] \quad (13c)$$

where functions $\overline{E}_{a_2}$, $\overline{E}_{1a_2}$, $\widetilde{E}_{a_2}$ *and* $\widetilde{E}_{1a_2}$ *are determined from functions* (12d)–(12g) *by changing point* $x \equiv (x_1, x_2) \bigcup V$ *with point* $y \equiv (y_1, y_2 = a_2) \bigcup \Gamma_{21}$- *for kernels* $\Pi_{ij}(y_1, 0; \xi)$

$$K_{11}(0, y_2; \xi) = -(\partial/\partial y_1)\sigma_{11}^*(y, \xi)|_{y_1=0} = -\frac{\mu\gamma}{2\pi(\lambda+2\mu)}\xi_1 \frac{\partial^2}{\partial \xi_1^2} \ln \frac{\widetilde{E}_0\overline{E}_{20}}{\overline{E}_0\widetilde{E}_{20}} \quad (14a)$$

$$K_{22}(0, y_2; \xi) = -(\partial/\partial y_1)\sigma_{22}^*(y, \xi)|_{y_1=0} = \frac{\mu\gamma}{2\pi(\lambda+2\mu)}\left[\left(2+\xi_1\frac{\partial}{\partial \xi_1}\right)\frac{\partial}{\partial \xi_1} \ln \frac{\widetilde{E}_0\overline{E}_{20}}{\overline{E}_0\widetilde{E}_{20}}\right] \quad (14b)$$

$$K_{12}(0, y_2; \xi) = -(\partial/\partial y_1)\sigma_{12}^*(y, \xi)|_{y_1=0} = -\frac{\gamma\mu}{4\pi(\lambda+2\mu)}\frac{\partial}{\partial \xi_2}\left[1+\xi_1\frac{\partial}{\partial \xi_1}\right] \ln \frac{\widetilde{E}_0\overline{E}_{20}}{\overline{E}_0\widetilde{E}_{20}} \quad (14c)$$

where functions $\overline{E}_0$, $\overline{E}_{20}$, $\widetilde{E}_0$ *and* $\widetilde{E}_{20}$ *are determined from functions* (12d)–(12g) *by changing point* $x \equiv (x_1, x_2) \bigcup V$ *with point* $y \equiv (y_1 = 0, y_2) \bigcup \Gamma_{10}$- *for kernels* $K_{ij}(0, y_2; \xi)$, *and*

$$Q_{11}(y_1, 0; \xi) = -\frac{\partial}{\partial y_2}\sigma_{11}^*(y, \xi)\,|_{y_2=0} = \frac{\gamma\mu}{2\pi(\lambda+2\mu)}\frac{\partial}{\partial \xi_2}\left[\left(1-\xi_1\frac{\partial}{\partial \xi_1}\right)\ln \frac{\widetilde{E}_0\overline{E}_{10}}{\overline{E}_0\widetilde{E}_{10}} + y_1\frac{\partial}{\partial \xi_1}\ln \frac{\widetilde{E}_0\widetilde{E}_{10}}{\overline{E}_0\overline{E}_{10}}\right] \quad (15a)$$

$$Q_{22}(y_1, 0; \xi) = -\frac{\partial}{\partial y_2}\sigma_{22}^*(y, \xi)\,|_{y_2=0} = \frac{\gamma\mu}{2\pi(\lambda+2\mu)}\frac{\partial}{\partial \xi_2}\left[\left(1+\xi_1\frac{\partial}{\partial \xi_1}\right)\ln \frac{\widetilde{E}_0\overline{E}_{10}}{\overline{E}_0\widetilde{E}_{10}} - y_1\frac{\partial}{\partial \xi_1}\ln \frac{\widetilde{E}_0\widetilde{E}_{10}}{\overline{E}_0\overline{E}_{10}}\right] \quad (15b)$$

$$Q_{12}(y_1, 0; \xi) = -\frac{\partial}{\partial y_2}\sigma_{12}^*(y, \xi)\,|_{y_2=0} = -\frac{\gamma\mu}{2\pi(\lambda+2\mu)}\frac{\partial^2}{\partial \xi_2^2}\left[\xi_1 \ln \frac{\widetilde{E}_0\overline{E}_{10}}{\overline{E}_0\widetilde{E}_{10}} - y_1 \ln \frac{\widetilde{E}_0\widetilde{E}_{10}}{\overline{E}_0\overline{E}_{10}}\right] \quad (15c)$$

where functions $\overline{E}_0$, $\overline{E}_{20}$, $\widetilde{E}_0$ *and* $\widetilde{E}_{20}$ *are determined from functions* (12d)–(12g) *by changing point* $x \equiv (x_1, x_2) \bigcup V$ *with point* $y \equiv (y_1, y_2 = 0) \bigcup \Gamma_{20}$- *for kernels* $Q_{ij}(0, y_2; \xi)$.

Proof. Derivation of the MTGFs for thermoelastic displacements U_i.

To derive MTGFs for the half-strip we need to solve the Lamé equations

$$\mu\nabla_\xi^2 U_i(x,\xi) + (\lambda+\mu)\Theta_{,\xi_i}(x,\xi) - \gamma G_{T,\xi_i}(x,\xi); \quad i = 1,2 \tag{16}$$

and Poisson-type equation:

$$\nabla^2 G_T(x,\xi) = -\delta(x-\xi), \quad x,\xi \in V \tag{17}$$

at the following homogeneous mechanical and thermal conditions with respect to MTGFs U_i, respective thermal stresses σ_{ij}^* and Green functions G_T:

$$\sigma_{11}^*(x,y) = U_2(x,y) = 0, \quad G_T(y,\xi) = 0; \quad x \in V; \quad y \equiv (0,y_2) \in \Gamma_{10} \tag{18a}$$

$$U_1(x,y) = \sigma_{22}^*(x,y) = 0, \quad G_T(y,\xi) = 0; \quad x \in V; \quad y \equiv (y_1,a_2) \in \Gamma_{20} \tag{18b}$$

$$\sigma_{21}^*(x,y) = U_2(x,y) = 0, \quad x \in V; \quad \partial G_T(y,\xi)/\partial n_{y2} = 0; \quad y \equiv (y_1,0) \in \Gamma_{21} \tag{18c}$$

shown in the Figure 1.

Let us prove that the boundary conditions (18a)–(18c) lead to the following equivalent conditions:

$$U_2 = \sigma_{11}^* = 0; \quad G_T = 0; \Rightarrow U_2 = U_{2,2} = U_{1,1} = 0 \Rightarrow \Theta = G_2 = G_{1,1} = G_\Theta = G_T = 0 \tag{19}$$

– for locally mixed boundary conditions on the marginal segment of straight line Γ_{10},

$$U_1 = \sigma_{22}^* = 0; \quad G_T = 0; \Rightarrow U_1 = U_{1,1} = U_{2,2} = 0 \Rightarrow \Theta = G_1 = G_2 = G_\Theta = G_T = 0 \tag{20}$$

– for locally mixed boundary conditions on the marginal straight line Γ_{20}, and

$$\sigma_{21}^* = U_2 = 0; \quad G_{T,2} = 0 \Rightarrow U_{1,2} = 0; \quad U_2 = 0; U_{2,1} = 0 \Rightarrow \Theta_{,2} = 0; \quad G_{1,2} = 0;$$
$$G_2 = 0; \quad G_{\Theta,2} = 0; \quad G_{T,2} = 0 \tag{21}$$

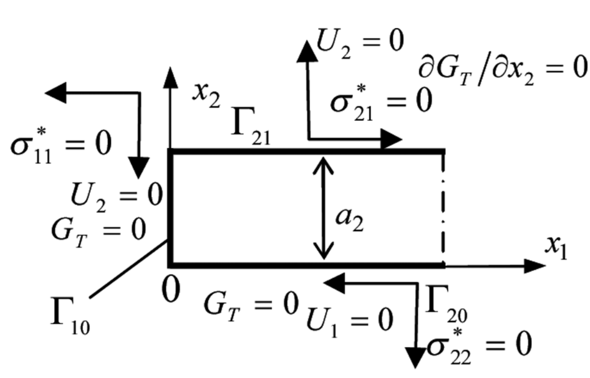

Figure 1 The scheme of the half-strip with boundary straight lines Γ_{10}, Γ_{20}, Γ_{21} and with the mechanical and thermal boundary conditions for U_1, U_2, σ_{11}^*, σ_{22}^*, σ_{21}^* and G_T.

– for locally mixed boundary conditions on the marginal straight line Γ_{21}, where the functions G_i, G_Θ, and G_T are Green's functions for Poison equation, those homogeneous boundary conditions are the similar to the boundary conditions for U_i, Θ andG_T, respectively. So, it means that, if on a marginal line are known U_i and Θ, T, then $G_i = 0$ and $G_\Theta = G_T = 0$; and if on a marginal line are known $U_{i,n}$ and $\Theta_{,n}$, $T_{,n}$, then$G_{i,n} = 0$ and $G_{\Theta,n} = G_{T,n} = 0$.

First, let us prove that if the boundaries of some domains of Cartesian system of coordinates represent straight lines, then are valid the following two sentences:

1. If on the marginal straight lines are given zero normal displacements, zero tangential stresses and zero normal derivative of Green's function for temperature (18c), then the normal derivative of volume dilatation is equal to zero:

$$\Theta_{,2} = 0 \to [\partial\Theta(y, \xi)/\partial n_{y2}] = 0 \tag{22}$$

2. If on the marginal straight lines are given zero normal stresses, zero tangential displacements and zero Green's function for temperature (18a) or (18b), then the volume dilatation is zero, $\Theta = 0$.

To prove the first sentence (22) we can observe that from the boundary conditions, with respect to tangential stresses in Eq. (18c) and the equilibrium equation,

$$\sigma^*_{2j,j} = 0; \quad j = 1, 2 \tag{23}$$

follows the relation

$$\sigma^*_{21,1} = 0 \Rightarrow \sigma^*_{22,2} = 0 \tag{24}$$

Next, from the Duhamel–Neumann law (5) rewritten for thermal stresses σ^*_{22}

$$\sigma^*_{22} = 2\mu U_{2,2} + (\lambda\Theta - \gamma G_T); \quad \Theta = U_{i,i} \tag{25}$$

from Eq. (25) for Θ and from equation (24) for $\sigma^*_{22,2}$ it follows:

$$\sigma^*_{22,2} = 2\mu U_{2,22} + \lambda\Theta_{,2} = (\lambda + 2\mu)\Theta_{,2} - 2\mu U_{1,12} = 0 \tag{26}$$

From the boundary conditions in Eq. (18c) and Duhamel–Neumann law (5) for the tangential stresses σ^*_{21} in Eq. (24) it follows

$$\left.\begin{array}{l} U_2 = 0 \to U_{2,11} = 0; \\ \sigma^*_{21,1} = 0 \to \mu(U_{2,11} + U_{1,21}) = 0 \end{array}\right\} \Rightarrow U_{1,12} = 0 \tag{27}$$

Finally, from Eq. (26) and the last equality in Eq. (27) it follows that the boundary conditions in Eq. (18c) lead to zero normal derivative from volume dilatation on the boundary Γ_{21} in Eq. (22).

To prove the second sentence, we can observe that from the boundary conditions for normal stresses in Eq. (18b) and from Duhamel–Neumann law (5)

$$\sigma_{22}^{*} = 2\mu U_{2,2} + (\lambda\Theta - \gamma G_T) = (\lambda + 2\mu)U_{2,2} + \lambda U_{1,1} - \gamma G_T; \quad \Theta = U_{k,k} \tag{28}$$

and boundary conditions for tangential displacements follows:

$$\left.\begin{array}{l} U_1 = 0 \Rightarrow U_{1,1} = 0; \\ \sigma_{22}^{*} = (\lambda + 2\mu)\, U_{2,2} + \lambda U_{1,1} - \gamma G_T = 0,\, G_T = 0; \Rightarrow U_{2,2} = 0 \end{array}\right\}$$
$$\Rightarrow U_{1,1} = 0, \quad U_{2,2} = 0; \quad \Theta = U_{k,k} \Rightarrow \Theta = 0 \tag{29}$$

So, the second sentence about volume dilatation is proved. Next we can prove the Eqs. (20) and (21).

Indeed, from each boundary condition (18a) and (18b) and proved two sentences it follows identical conditions:

$$G_T = 0; \quad \left\{U_2 = 0 \Rightarrow U_{2.2} = G_2 = 0; \sigma_{11} = 0 \Rightarrow U_{1,1} = 0; G_{1,1} = 0\right\} \Rightarrow \Theta = 0;$$
$$G_\Theta = 0 \tag{30}$$

– from locally mixed boundary conditions (18a) on the marginal segment of straight line Γ_{10},

$$G_T = 0; \quad \left\{U_1 = 0 \Rightarrow U_{1,1} = G_1 = 0; \sigma_{22} = 0; U_{2,2} = 0; G_{2,2} = 0\right\} \Rightarrow \Theta = 0;$$
$$G_\Theta = 0 \tag{31}$$

– from locally mixed boundary conditions (18b) on the marginal straight line Γ_{20}, and

$$G_{T,2}=0; \quad \left\{U_2 = 0 \Rightarrow U_{2,1}=0; G_2=0; \sigma_{21}=0 \Rightarrow U_{1,2}=0; G_{1,2} = 0; \right\} \Rightarrow \Theta_{,2} = 0;$$
$$G_{\Theta,2} = 0 \tag{32}$$

– from locally mixed boundary conditions (18c) on the marginal straight line Γ_{21}, that coincide with Eqs. (19)–(21). Next, let us rewrite the general integral representations [37] in the case of the half-strip V in the following form:

$$\Theta(x,\xi) = \frac{\gamma}{\lambda + 2\mu} G_\Theta(x,\xi) + \int_{\Gamma_{10}} \left[\frac{\partial\Theta(x,y)}{\partial n_{\Gamma_{10}}} - \Theta(x,y)\frac{\partial}{\partial n_{\Gamma_{10}}}\right] G_\Theta(y,\xi)\, d\Gamma_{10}(y)$$
$$+ \sum_{i=0}^{1} \int_{\Gamma_{2i}} \left[\frac{\partial\Theta(x,y)}{\partial n_{\Gamma_{2i}}} - \Theta(x,y)\frac{\partial}{\partial n_{\Gamma_{2i}}}\right] G_\Theta(y,\xi)\, d\Gamma_{2i}(y) \tag{33}$$

– for thermoelastic volume dilatation $\Theta(x,\xi)$, where

$$y \equiv (0, y_2) \in \Gamma_{10}, \quad d\Gamma_{10}(y) \equiv dy_2; \quad \partial/\partial n_{\Gamma_{10}} = -\partial/\partial y_1;$$

$$
\begin{aligned}
&U_j(x,\xi)\\
&= -\frac{\lambda+\mu}{2\mu}\xi_j\Theta(x,\xi) - \frac{\gamma}{2(\lambda+2\mu)}x_jG_j(x,\xi) + \frac{\gamma\xi_j}{2\mu}G_T(x,\xi)\\
&\quad - \int_{\Gamma_{10}}\left\{[U_j(x,y)+(2\mu)^{-1}y_j((\lambda+\mu)\Theta(x,y)-\gamma G_T(x,y))]\frac{\partial G_j(y,\xi)}{\partial n_{y_1}}\right.\\
&\quad \left. - \frac{\partial}{\partial n_{y_1}}[U_j(x,y)+(2\mu)^{-1}y_j((\lambda+\mu)\Theta(x,y)-\gamma G_T(x,y))]G_j(y,\xi)\right\} d\Gamma_{10}(y)\\
&\quad - \sum_{i=0}^{1}\int_{\Gamma_{2i}}\left\{[U_j(x,y)+(2\mu)^{-1}y_j((\lambda+\mu)\Theta(x,y)-\gamma G_T(x,y))]\frac{\partial G_j(y,\xi)}{\partial n_{y_2}}\right.\\
&\quad \left. - \frac{\partial}{\partial n_{y_2}}[U_j(x,y)+(2\mu)^{-1}y_j((\lambda+\mu)\Theta(x,y)-\gamma G_T(x,y))]G_j(y,\xi)\right\} d\Gamma_{2i}(y);\\
&\qquad i,j=1,2
\end{aligned}
\tag{34}
$$

– for MTGFs $U_j(x,\xi)$.

Substituting the values of the volume dilatation Θ and respective GFPE G_Θ from Eqs. (19)–(21) into representation (33) we can see that integrals on segment Γ_{10} and lines Γ_{20}, Γ_{21} are zero, so that we obtain

$$
\Theta(x,\xi) = \frac{\gamma}{\lambda+2\mu}G_\Theta(x,\xi) \tag{35}
$$

As from Eqs. (19)–(21) follows that boundary conditions for $G_\Theta(x,\xi)$ and $G_T(x,\xi)$ on the marginal segment Γ_{10} and lines Γ_{20}, Γ_{21} are the same, then from Eq. (35) we obtain the following formula for volume dilatation:

$$
\Theta(x,\xi) = \gamma(\lambda+2\mu)^{-1}G_T(x,\xi) \tag{36}
$$

Next, if we use boundary conditions (30)–(32) and expression (35) in representations (34) we can see:

a. In integral representation (35) rewritten for MTGF $U_1(x,\xi)$ all line integrals are zero, and

$$
U_1(x,\xi) = \gamma[2(\lambda+2\mu)]^{-1}[\xi_1G_T(x,\xi) - x_1G_1(x,\xi)] \tag{37}
$$

b. In the integral representation (35) rewritten for MTGFs $U_2(x,\xi)$ all integrals on marginal line Γ_{20} only are zero, so that we obtain:

$$
\begin{aligned}
U_2(x,\xi) &= -\frac{\lambda+\mu}{2\mu}\xi_2\frac{\gamma}{\lambda+2\mu}G_T(x,\xi) - \frac{\gamma}{2(\lambda+2\mu)}x_2G_T(x,\xi) + \frac{\gamma\xi_2}{2\mu}G_T(x,\xi)\\
&\quad - \int_{\Gamma_{21}}\left[U_2(x,y)+(2\mu)^{-1}y_2\left((\lambda+\mu)\frac{\gamma}{\lambda+2\mu}G_T(x,y)-\gamma G_T(x,y)\right)\right]\\
&\quad \times \frac{\partial}{\partial n_{y_2}}G_2(y,\xi)d\Gamma_{21}(y)
\end{aligned}
\tag{38}
$$

or due to boundary conditions (32):

$$U_2(x,\xi)=\frac{\gamma}{2(\lambda+2\mu)}\left[-(x_2-\xi_2)G_T+\int_{\Gamma_{21}}a_2G_T(x,y)\frac{\partial}{\partial n_{y_2}}G_2(y,\xi)d\Gamma_{21}(y)\right] \quad (39)$$

Note that in Eqs. (37)–(39) the expressions for GFPE for half-strip with boundary conditions (30)–(32) can be rewritten from the book [9]:

$$G_1=\frac{1}{4\pi}\ln\frac{\overline{EE}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}};\quad G_2(x,\xi)=\frac{1}{4\pi}\ln\frac{\overline{EE}_2\widetilde{E}_1\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_2\overline{E}_1\overline{E}_{12}},$$
$$G_\Theta(x,\xi)=G_T(x,\xi)=\frac{1}{4\pi}\ln\frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} \quad (40)$$

Next, integral in Eq. (39) can be taken using boundary conditions for MTGFs:

$$U_{1,1}(x,\xi)=U_2(x,\xi)=0;\quad x\equiv(x_1,x_2)\in V;\quad \xi\equiv(0,\xi_2)\in\Gamma_{10}, \quad (41a)$$
$$U_1(x,\xi)=U_{2,2}(x,\xi)=0;\quad x\equiv(x_1,x_2)\in V;\quad \xi\equiv(\xi_1,a_2)\in\Gamma_{20}, \quad (41b)$$
$$U_{1,2}(x,\xi)=U_2(x,\xi)=0;\quad x\equiv(x_1,x_2)\in V;\quad \xi\equiv(\xi_1,0)\in\Gamma_{21} \quad (41c)$$

with respect to point $\xi\in\Gamma=\Gamma_{10}\bigcup\Gamma_{20}\bigcup\Gamma_{21}$ that follows from boundary conditions (29)–(32) for MTGFs, and

$$U_i(y,\xi)=0,\quad y\equiv(0,y_2)\in\Gamma_{10};\quad U_i(y,\xi)=0,\quad y\equiv(y_1,0)\in\Gamma_{20};$$
$$\partial U_i(y,\xi)/\partial n_{21}=0,\quad y\equiv(y_1,a_2)\in\Gamma_{21} \quad (42)$$

with respect to point $x=y\in\Gamma=\Gamma_{10}\bigcup\Gamma_{20}\bigcup\Gamma_{21}$, that follows from boundary conditions (29)–(32) for GFPE and results from [31–33] (with respect to point $x\equiv(x_1,x_2)$ MTGFs have to satisfy boundary conditions for GFPE. Finally, we obtain

$$I_2=\int_{\Gamma_{21}}a_2G_T(x,y)G_{2,2}(y,\xi)d\Gamma_{21}(y)=x_2G_2(x,\xi)-\xi_2G_T(x,\xi)$$
$$-x_1\int\frac{\partial}{\partial x_2}G_2(x,\xi)dx_1+\int\xi_1\frac{\partial}{\partial\xi_2}G_T(x,\xi)d\xi_1 \quad (43)$$

where

$$\int\xi_1\frac{\partial}{\partial\xi_2}G_T(x,\xi)d\xi_1=\xi_1\int\frac{\partial G_T(x,\xi)}{\partial\xi_2}d\xi_1-\iint\frac{\partial G_T(x,\xi)}{\partial\xi_2}d^2\xi_1 \quad (44)$$

because integral $I_2(x,\xi)$ is a harmonic function with respect to coordinates of both points $x\equiv(x_1,x_2)\in V$ and $\xi\equiv(\xi_1,\xi_2)\in V$ and it have the following values on marginal lines:

$$I_2(x,\xi)=0;\quad \xi\equiv(\xi_1,\xi_2)\in V,\quad x\equiv(x_1=y_1=0,x_2)\in\Gamma_{10};$$
$$x\equiv(x_1,x_2=y_2=0)\in\Gamma_{20} \quad (45a)$$

$$\partial I_2(x,\xi)/\partial n_{x_2} = \partial I_2(x,\xi)/\partial x_2 = a_2 G_{2,2}(y,\xi); \quad \xi \equiv (\xi_1, \xi_2) \in V,$$
$$x \equiv (x_1 = y_1,\ x_2 = y_2 = a_2) \in \Gamma_{21} \tag{45b}$$

with respect to point $x = y \in \Gamma = \Gamma_{10} \bigcup \Gamma_{20} \bigcup \Gamma_{21}$, and

$$I_2(x,\xi) = 0; \quad x \equiv (x_1, x_2) \in V; \quad \xi \equiv (\xi_1 = 0, \xi_2 = y_2) \in \Gamma_{10};$$
$$\partial I_2(x,\xi)/\partial n_{\xi_2} = \partial I_2(x,\xi)/\partial \xi_2 = 0; \quad \xi \equiv (\xi_1, \xi_2 = y_2) \in \Gamma_{20} \tag{46a}$$
$$I_2(x,\xi) = a_2 G(x,\xi); \quad x \equiv (x_1, x_2) \in V; \quad \xi \equiv (\xi_1, \xi_2 = a_2) \in \Gamma_{21} \tag{46b}$$

with respect to point $\xi \in \Gamma = \Gamma_{10} \bigcup \Gamma_{20} \bigcup \Gamma_{21}$, that result from the boundary conditions (41a)–(41c). So, substituting (43) and (44) in (39) we obtain the intermediary constructive formula for MTGFs $U_2(x,\xi)$:

$$U_2(x,\xi) = -\frac{\gamma}{2(\lambda+2\mu)}\Bigg[(x_2-\xi_2)G_T - (x_2-\xi_2)G_T + x_1 \int \frac{\partial G_2(x,\xi)}{\partial x_2} dx_1 - \xi_1 \int \frac{\partial G_T(x,\xi)}{\partial \xi_2} d\xi_1 + \iint \frac{\partial G_T(x,\xi)}{\partial \xi_2} d^2\xi_1\Bigg] \tag{47}$$

So, from (37) and (47), we obtain the final constructive formulas for MTGFs $U_i(x,\xi)$:

$$U_1(x,\xi) = \frac{\gamma}{2(\lambda+2\mu)}[\xi_1 G_T(x,\xi) - x_1 G_1(x,\xi)] \tag{48a}$$

$$U_2(x,\xi) = -\frac{\gamma}{2(\lambda+2\mu)}\left[x_1 \int \frac{\partial G_2(x,\xi)}{\partial x_2} dx_1 - \xi_1 \int \frac{\partial G_T(x,\xi)}{\partial \xi_2} d\xi_1 + \iint \frac{\partial G_T(x,\xi)}{\partial \xi_2} d^2\xi_1\right] \tag{48b}$$

Note, that by using expressions for Green's functions $G_T(x,\xi)$,$G_1(x,\xi)$ and $G_2(x,\xi)$ from (40) we can see that all terms of MTGFs (48a) and (48b), excepting term contains double integral are presented in elementary functions. Thus, taking into account Eq. (40) and calculating the respective integrals we obtain the following expressions for MTGFs:

$$U_1(x,\xi) = \frac{\gamma}{8\pi(\lambda+2\mu)}\left[\xi_1 \ln \frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} - x_1 \ln \frac{\overline{E}\overline{E}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}}\right] \tag{49a}$$

$$\begin{aligned} U_2(x,\xi) = -\gamma[8\pi(\lambda+2\mu)]^{-1}\Big[&(x_1+\xi_1)\Big(\widetilde{I}(x,\xi) + \widetilde{I}_{12}(x,\xi) + \widetilde{I}(x,\xi) \\ &+ \overline{I}_2(x,\xi) + \overline{I}_1(x,\xi)\Big) - (x_1-\xi_1)\Big(\widetilde{I}_1(x,\xi) + \widetilde{I}_2(x,\xi) + \overline{I}(x,\xi) + \overline{I}_{12}(x,\xi)\Big) \\ &+ \int \Big(\widetilde{I}_1(x,\xi) - \widetilde{I}_{12}(x,\xi) + \widetilde{I}_2(x,\xi) - \widetilde{I}(x,\xi) + \overline{I}(x,\xi) - \overline{I}_2(x,\xi) \\ &- \overline{I}_1(x,\xi) + \overline{I}_{12}(x,\xi)\Big) d\xi_1\Big], \end{aligned} \tag{49b}$$

where

$$\overline{I}(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \overline{E}(x,\xi) d\xi_1 = -\int \frac{\partial}{\partial x_2} \ln \overline{E}(x,\xi) d\xi_1$$
$$= -\mathrm{arctg} \frac{e^{(\pi/2a_2)(x_1-\xi_1)} + \cos(\pi/2a_2)(x_2-\xi_2)}{\sin(\pi/2a_2)(x_2-\xi_2)} \quad (50a)$$

$$\overline{I}_1(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \overline{E}_1(x,\xi) d\xi_1 = -\int \frac{\partial}{\partial x_2} \ln \overline{E}_1(x,\xi) d\xi_1 = \overline{I}(x;-\xi_1,\xi_2) \quad (50b)$$

$$\overline{I}_2(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \overline{E}_2(x,\xi) d\xi_1 = \int \frac{\partial}{\partial x_2} \ln \overline{E}_2(x,\xi) d\xi_1 = \overline{I}(x;\xi_1,-\xi_2) \quad (50c)$$

$$\overline{I}_{12}(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \overline{E}_{12}(x,\xi) d\xi_1 = \int \frac{\partial}{\partial x_2} \ln \overline{E}_{12}(x,\xi) d\xi_1 = \overline{I}(x;-\xi_1,-\xi_2) \quad (50d)$$

$$\widetilde{I}(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \widetilde{E}(x,\xi) d\xi_1 = -\int \frac{\partial}{\partial x_2} \ln \widetilde{E}(x,\xi) d\xi_1$$
$$= \mathrm{arctg} \frac{e^{(\pi/2a_2)(x_1-\xi_1)} - \cos(\pi/2a_2)(x_2-\xi_2)}{\sin(\pi/2a_2)(x_2-\xi_2)} \quad (50e)$$

$$\widetilde{I}_1(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \widetilde{E}_1(x,\xi) d\xi_1 = -\int \frac{\partial}{\partial x_2} \ln \widetilde{E}_1(x,\xi) d\xi_1 = \widetilde{I}(x;-\xi_1,\xi_2) \quad (50f)$$

$$\widetilde{I}_2(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \widetilde{E}_2(x,\xi) d\xi_1 = \int \frac{\partial}{\partial x_2} \ln \widetilde{E}_2(x,\xi) d\xi_1 = \widetilde{I}(x;\xi_1,-\xi_2) \quad (50g)$$

$$\widetilde{I}_{12}(x,\xi) = \int \frac{\partial}{\partial \xi_2} \ln \widetilde{E}_{12}(x,\xi) d\xi_1 = \int \frac{\partial}{\partial x_2} \ln \widetilde{E}_{12}(x,\xi) d\xi_1 = \widetilde{I}(x;-\xi_1,-\xi_2) \quad (50h)$$

Derivation of the MTGFs for Thermal Stresses σ^*_{ij}

By using the Duhamel–Neumann law (5), constructive formulas (48a), (48b) and the expressions (40) for $G_T(x,\xi)$ and $G_1(x,\xi)$ we obtain the following expressions for MTGFs for thermal stresses:

$$\sigma^*_{ij} : \sigma^*_{11}(x,\xi)$$
$$= -\frac{\mu\gamma}{(\lambda+2\mu)} \left[\left(1 - \xi_1 \frac{\partial}{\partial \xi_1} \right) G_T + x_1 \frac{\partial}{\partial \xi_1} G_1 \right]$$
$$= -\frac{\mu\gamma}{4\pi(\lambda+2\mu)} \left[\left(1 - \xi_1 \frac{\partial}{\partial \xi_1} \right) \ln \frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} + x_1 \frac{\partial}{\partial \xi_1} \ln \frac{\overline{E}\overline{E}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}} \right] \quad (51a)$$

$$\sigma^*_{22}(x,\xi)$$
$$= -\frac{\gamma\mu}{(\lambda+2\mu)} \left[\left(1 + \xi_1 \frac{\partial}{\partial \xi_1} \right) G_T - x_1 \frac{\partial}{\partial \xi_1} G_1 \right]$$
$$= -\frac{\mu\gamma}{4\pi(\lambda+2\mu)} \left[\left(1 + \xi_1 \frac{\partial}{\partial \xi_1} \right) \ln \frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} - x_1 \frac{\partial}{\partial \xi_1} \ln \frac{\overline{E}\overline{E}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}} \right] \quad (51b)$$

$$\sigma^*_{12}(x,\xi) = \frac{\gamma\mu}{(\lambda+2\mu)}\frac{\partial}{\partial\xi_2}[\xi_1 G_T(x,\xi) - x_1 G_1(x,\xi)] = \frac{\gamma\mu}{4\pi(\lambda+2\mu)}\frac{\partial}{\partial\xi_2}\left[\xi_1 \ln\frac{\overline{E}\widetilde{E}_1\widetilde{E}_2\overline{E}_{12}}{\widetilde{E}\overline{E}_1\overline{E}_2\widetilde{E}_{12}} - x_1\ln\frac{\overline{E}\overline{E}_1\widetilde{E}_2\widetilde{E}_{12}}{\widetilde{E}\widetilde{E}_1\overline{E}_2\overline{E}_{12}}\right] \quad (51c)$$

that coincide with kernels (12a)–(12g).

Note, that by omitting functions $\widetilde{E}_1$ $\widetilde{E}_{12}$ $\overline{E}_1$ $\overline{E}_{12}$, which contain the inferior index "1" (this means that the boundary Γ_{10} will be placed at infinity), we obtain the expressions for thermal stresses for respective BVP of thermoelasticity for the strip.

Derivation of the Green's Type Integral Formula for Thermal Stresses Within Half-Strip

The Green's type integral formula (11) can be obtained by using the rewritten for half-strip the general integral formula (3) taking into account boundary conditions (9b) and expressions (51a)–(51c) for σ^*_{ij}. Finally, calculating by using expressions (51a)–(51c) the other influence functions: $\Pi_{ij}(y_1, a_2; \xi) = \sigma^*_{ij}(y_1, a_2; \xi)$ (on marginal line Γ_{21}), $K_{ij}(0, y_2; \xi) = \partial\sigma^*_{ij}(0, y_2; \xi)/\partial n_{y1}$ (on marginal line Γ_{10}) and $Q_{ij}(y_1, 0; \xi) = \partial\sigma^*_{ij}(y_1, 0; \xi)/\partial n_{y2}$ (on marginal line Γ_{20}) and substituting them in the rewritten for half-strip formula (3) we obtain the integral solution (11)–(15b) of respective non-homogeneous BVP (6)–(10) for the thermoelastic half-strip.

EXPLICIT THERMAL STRESS OF A PARTICULAR BVP WITHIN HALF-STRIP

Here we present an example of application of the integral formula for the thermal stresses to solution of a particular BVP of thermoelasticity for the half-strip V. Suppose we want to determine the thermal stresses $\sigma_{ij}(\xi)$; $i, j = 1, 2$ in the half-strip $V \equiv (0 \le x_1 < \infty,\ 0 \le x_2 \le a_2)$, caused by the following thermal boundary conditions given on marginal lines Γ_{20} and Γ_{21}:

$$T(y) = \begin{cases} T_{10}(y) = T_0 = const, & y \equiv (0, y_2) \in \Gamma_{10}; \quad T_0 > 0 \\ T_{20}(y) = 0, & y \equiv (y_1, 0) \in \Gamma_{20}; \end{cases}$$
$$\partial T(y_1, 0)/\partial n_{y_2} = \partial T(y_1, a_2)/\partial y_2 = S_{21}(y_1, a_2) = 0; \quad y \equiv (y_1, a_2) \in \Gamma_{21} \quad (52)$$

shown in Figure 2.

Thus, on the boundary lines are defined homogeneous mechanical boundary conditions (7). Traditionally, to solve this BVP of thermoelasticity, consisting of a BVP of heat conduction (9a) in absence of heat source $F(\xi)$, boundary conditions (52) and of the elastic BVP (6) and (7) it is necessary first to determine from (9a) and (52) the temperature, then from (6) and (7) the thermal displacements and finally to determine thermal stresses by using Duhamel–Neumann law (8). However, due to the obtained in the proved Theorem results, the solution of the mentioned above BVP of thermoelasticity can be obtained using the Green's type integral formula

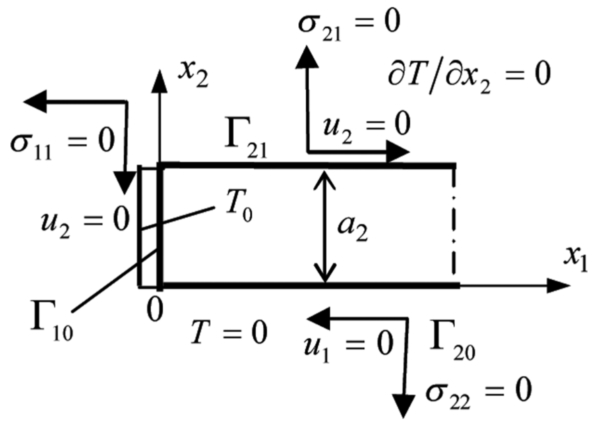

Figure 2 The scheme of the half-strip with boundary straight lines Γ_{10} Γ_{20}, Γ_{21} and with the mechanical and thermal boundary conditions for u_1, u_2, σ_{21}, σ_{22}, and T.

(11) in the absence of inner heat source, heat flux on Γ_{20} and at a temperature $T_{21}(y_1)$ on Γ_{21}:

$$\sigma_{ij}(\xi) = -\int_0^{a_2} T_{10}(0, y_2) K_{ij}(0, y_2; \xi) dy_2 \tag{53}$$

where the kernels $K_{ij}(0, y_2; \xi)$ are defined by Eqs. (14a)–(14c). Thus, substituting (14a)–(14c) and (52) in (53) we obtain the following integral formula for the determining the thermal stresses:

$$\begin{aligned}\sigma_{11}(\xi) &= -\int_0^{a_2} T_{10}(0, y_2) K_{11}(0, y_2; \xi) dy_2 = \frac{\mu\gamma T_0}{2\pi(\lambda+2\mu)} \xi_1 \int_0^{a_2} \frac{\partial^2}{\partial \xi_1^2} \ln \frac{\widetilde{E}_0 \overline{E}_{20}}{\overline{E}_0 \widetilde{E}_{20}} dy_2 \\ &= -\frac{\mu\gamma T_0}{2\pi(\lambda+2\mu)} \xi_1 \frac{\partial}{\partial \xi_2} \ln \frac{\overline{E}_{0a_2} \overline{E}_{20a_2} \widetilde{E}_{00} \widetilde{E}_{200}}{\widetilde{E}_{0a_2} \widetilde{E}_{20a_2} \overline{E}_{00} \overline{E}_{200}} \end{aligned} \tag{54a}$$

$$\begin{aligned}\sigma_{22}(\xi) &= -\int_0^{a_2} T_{10}(0, y_2) K_{22}(0, y_2; \xi) dy_2 \\ &= -\frac{\mu\gamma T_0}{2\pi(\lambda+2\mu)} \int_0^{a_2} \left(2 + \xi_1 \frac{\partial}{\partial \xi_1}\right) \frac{\partial}{\partial \xi_1} \ln \frac{\widetilde{E}_0 \overline{E}_{20}}{\overline{E}_0 \widetilde{E}_{20}} dy_2 \\ &= -\frac{\mu\gamma T_0}{2\pi(\lambda+2\mu)} \left(2\int_0^{a_2} \frac{\partial}{\partial \xi_1} \ln \frac{\widetilde{E}_0 \overline{E}_{20}}{\overline{E}_0 \widetilde{E}_{20}} dy_2 - \xi_1 \frac{\partial}{\partial \xi_2}\right) \ln \frac{\overline{E}_{0a_2} \overline{E}_{20a_2} \widetilde{E}_{00} \widetilde{E}_{200}}{\widetilde{E}_{0a_2} \widetilde{E}_{20a_2} \overline{E}_{00} \overline{E}_{200}}, \end{aligned} \tag{54b}$$

$$\begin{aligned}\sigma_{12}(\xi) &= -\int_0^{a_2} T_{10}(0, y_2) K_{12}(0, y_2; \xi) dy_2 \\ &= \frac{\gamma\mu}{2\pi(\lambda+2\mu)} T_0 \int_0^{a_2} \frac{\partial}{\partial \xi_2} \left(1 + \xi_1 \frac{\partial}{\partial \xi_1}\right) \ln \frac{\widetilde{E}_0 \overline{E}_{20}}{\overline{E}_0 \widetilde{E}_{20}} dy_2 \\ &= \frac{\gamma\mu}{2\pi(\lambda+2\mu)} T_0 \left(1 + \xi_1 \frac{\partial}{\partial \xi_1}\right) \ln \frac{\overline{E}_{0a_2} \overline{E}_{20a_2} \widetilde{E}_{00} \widetilde{E}_{200}}{\widetilde{E}_{0a_2} \widetilde{E}_{20a_2} \overline{E}_{00} \overline{E}_{200}}, \end{aligned} \tag{54c}$$

where

$$\overline{E}_{00} = \overline{E}_{200} = \overline{E}_{0a_2} = \widetilde{E}_{20a_2} = 1 + 2e^{-\frac{\pi}{2a_2}\xi_1}\cos\frac{\pi\xi_2}{2a_2} + e^{-\frac{\pi}{a_2}\xi_1};$$

$$\widetilde{E}_{00} = \widetilde{E}_{200} = \overline{E}_{20a_2} = \widetilde{E}_{0a_2} = 1 - 2e^{-\frac{\pi}{2a_2}\xi_1}\cos\frac{\pi\xi_2}{2a_2} + e^{-\frac{\pi}{a_2}\xi_1} \tag{55}$$

After computing the integrals (54a)–(54c) we obtain the following final analytical expressions for thermoelastic stresses, presented in terms of elementary functions:

$$\sigma_{11}(\xi) = -\frac{\mu\gamma}{\pi(\lambda+2\mu)}T_0\xi_1\frac{\partial}{\partial\xi_2}\left[\ln\left(1 - 2e^{-\frac{\pi}{2a_2}\xi_1}\cos\frac{\pi\xi_2}{2a_2} + e^{-\frac{\pi}{a_2}\xi_1}\right)\right.$$
$$\left. - \ln\left(1 + 2e^{-\frac{\pi}{2a_2}\xi_1}\cos\frac{\pi\xi_2}{2a_2} + e^{-\frac{\pi}{a_2}\xi_1}\right)\right] \tag{56a}$$

$$\sigma_{22}(\xi) = -\frac{\gamma\mu T_0}{\pi(\lambda+2\mu)}\left\{2\left[\operatorname{arctg}\frac{e^{(\pi/2a_2)(-\xi_1)} + \cos(\pi/2a_2)(\xi_2)}{\sin(\pi/2a_2)(\xi_2)}\right.\right.$$
$$\left. + \operatorname{arctg}\frac{e^{(\pi/2a_2)(-\xi_1)} - \cos(\pi/2a_2)(\xi_2)}{\sin(\pi/2a_2)(\xi_2)}\right]$$
$$- \xi_1\frac{\partial}{\partial\xi_2}\left[\ln\left(1 - 2e^{-(\pi/2a_2)\xi_1}\cos(\pi\xi_2/2a_2) + e^{-(\pi/a_2)\xi_1}\right)\right.$$
$$\left.\left. - \ln\left(1 + 2e^{-(\pi/2a_2)\xi_1}\cos(\pi\xi_2/2a_2) + e^{-(\pi/a_2)\xi_1}\right)\right]\right\}, \tag{56b}$$

$$\sigma_{12}(\xi) = \frac{\gamma\mu T_0}{\pi(\lambda+2\mu)}\left(\xi_1\frac{\partial}{\partial\xi_1} + 1\right)\left[\ln\left(1 - 2e^{-(\pi/2a_2)\xi_1}\cos\frac{\pi\xi_2}{2a_2} + e^{-(\pi/a_2)\xi_1}\right)\right.$$
$$\left. - \ln\left(1 + 2e^{-(\pi/2a_2)\xi_1}\cos\frac{\pi\xi_2}{2a_2} + e^{-(\pi/a_2)\xi_1}\right)\right] \tag{56c}$$

Note that graphics of thermal stresses (56a)–(56c) constructed by using computer program Maple 15 are presented in the Figures A1(b), A2(b), and A3(b) of the Appendix.

CONCLUSIONS

A theorem about the constructing new MTGFs $U_i(x, \xi)$ and new Green's type integral formula (11) is proved for a specific BVP for half-strip in terms of GFPE. All results are obtained in terms of elementary functions. The solution in elementary functions for a particular BVP of thermoelasticity for half-strip is included. Both, the derived MTGFs for thermal stresses $\sigma^*_{11}(x, \xi)$,$\sigma^*_{22}(x, \xi)$, $\sigma^*_{12}(x, \xi)$ and for thermal stresses $\sigma_{11}(\xi)$, $\sigma_{22}(\xi)$, $\sigma_{12}(\xi)$ for a particular BVP for thermoelastic half-strip were evaluated numerically and graphically by using computer program Maple 15. The main advantages of the proposed approach in comparison with the $G\Theta$ convolution method for MTGFs constructing are: a. It is not necessary to derive the functions of influence of a unit concentrated force onto elastic volume dilatation – $\Theta^{(i)}$ and, b. It is not necessary to calculate a complicate volume integral of the product of the function $\Theta^{(i)}$ and Green's function G_T in heat conduction. Also the proposed approach may be extended onto many canonical domains of Cartesian system of coordinates.

APPENDIX

Graphics of Normal and Tangential Thermal Stresses within Half-Strip, Caused by the Unit Point Heat Source and by a Constant Boundary Temperature

Graphics of thermal stresses σ^*_{11}, σ^*_{22}, σ^*_{12}, caused by the unit point heat source and σ_{11}, σ_{22}, σ_{12}, caused by a constant boundary temperature were constructed at the following values of elastic and thermal constants: the Poisson ratio $\nu = 0.3$, modulus of elasticity $E = 2.1 \times 10^5$ MPa and coefficient of linear thermal

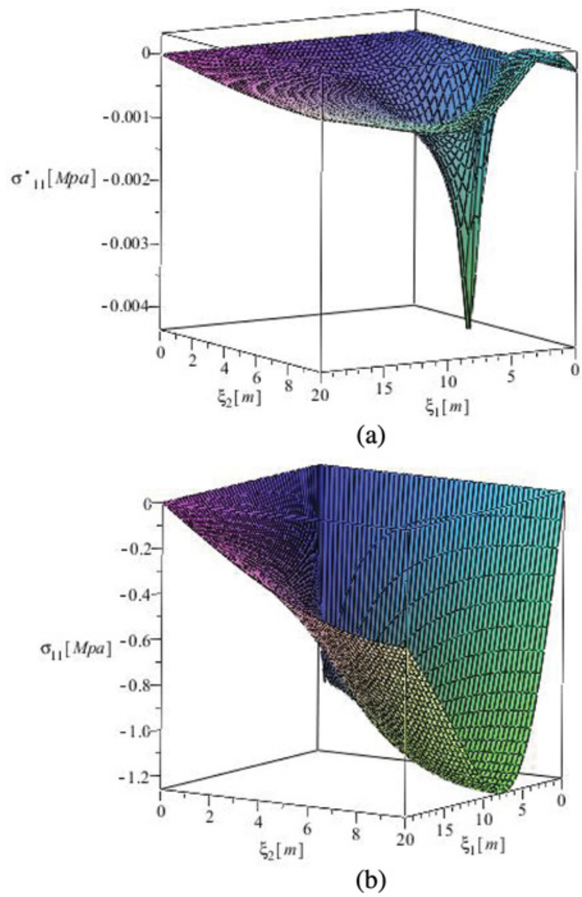

Figure A1 Graphics of normal thermal stresses σ^*_{11} and σ_{11} in the half-strip V in dependence of $0 \leq \xi_1 \leq 20m$, $0 \leq \xi_2 \leq 10m$, created by a unit heat source applied in the point $x_1 = 2m$, $x_2 = 5m$ (a); and by the constant temperature $T_0 = 303°$ K, acting on a segment $0 \leq y_1 \leq 10m$ of the boundary line Γ_{10} (b).

expansion $\alpha = 6.57 \times 10^{-8}(\mathrm{K})^{-1}$. The behavior of the normal thermal stresses σ^*_{11}, caused by the unit point heat source and of the thermal stresses σ_{11}, caused by the boundary temperature, calculated by the formulas (51a) and (56a) are shown in the Figure A1(a) and Figure A1(b), respectively.

The behavior of the normal thermal stresses σ^*_{22}, caused by the unit point heat source and of thermal stresses σ_{22}, caused by the boundary temperature, calculated by the formulas (51a) and (56b) are shown in the Figure A2(a) and in the Figure A2(b), respectively.

The behavior of the tangential thermal stresses σ^*_{12}, caused by the unit point heat source and of the tangential thermal stresses σ_{12}, caused by the boundary temperature, calculated by the formulas (51c) and (56c) are showed in the Figure A3(a) and in the Figure A3(b), respectively.

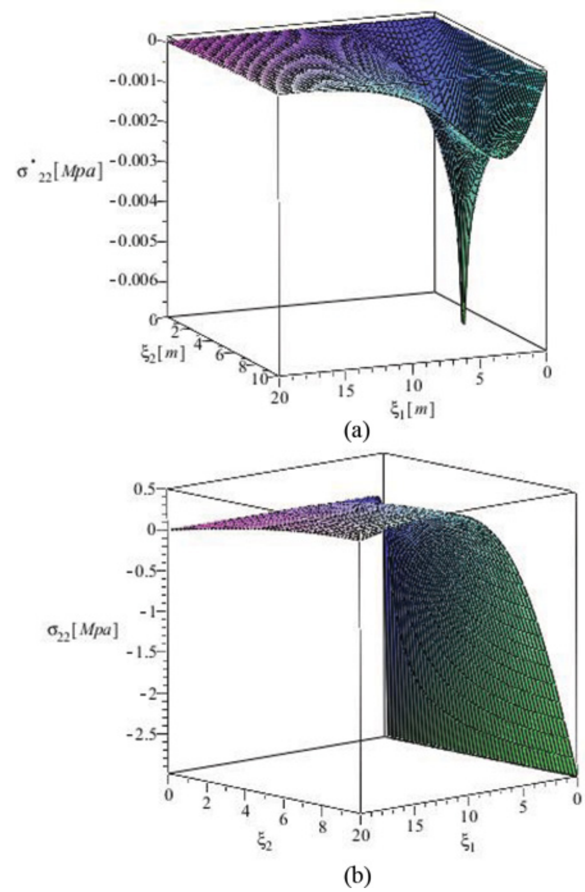

Figure A2 Graphics of normal thermal stresses σ^*_{22} and σ_{22} in the half-strip V in dependence of $0 \le \xi_1 \le 20m$, $0 \le \xi_2 \le 10m$, created by a unit heat source applied in the point $x_1 = 2m$, $x_2 = 5m$ (a); and by the constant temperature $T_0 = 303°\,\mathrm{K}$, acting on a segment $0 \le y_1 \le 10m$ of the boundary line Γ_{10} (b).

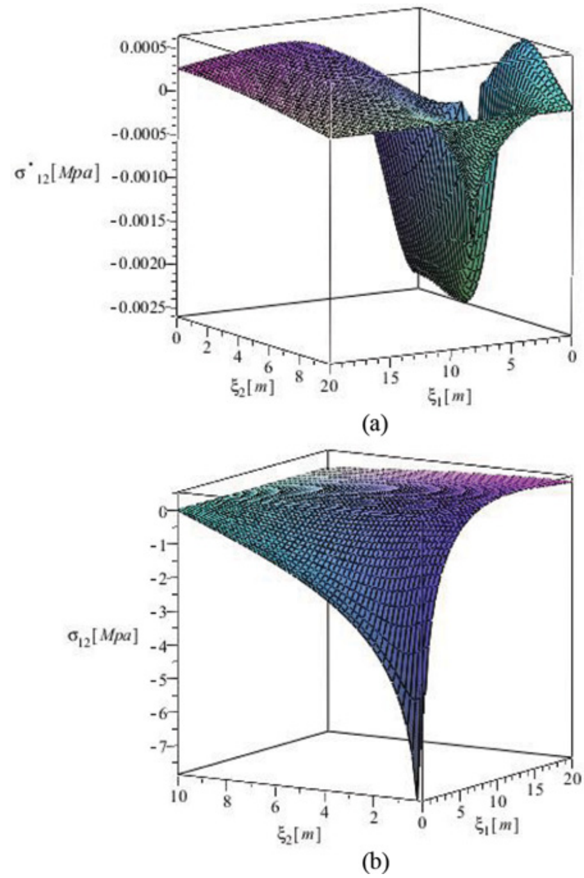

Figure A3 Graphics of tangential thermal stresses σ_{12}^{*} and σ_{12} in the half-strip V in dependence of $0 \leq \xi_1 \leq 20m$, $0 \leq \xi_2 \leq 10m$, created by a unit heat source applied in the point $x_1 = 2m$, $x_2 = 5m$ (a); and by the constant temperature $T_0 = 303°$ K, acting on a segment $0 \leq y_1 \leq 10m$ of the boundary line Γ_{10} (b).

The main observation that can be seen in Figures A1–A3 consist in the confirmation the satisfaction of the boundary conditions in Eqs. (51b), (51c) for thermal stresses σ_{22}, σ_{12}, created the constant temperature $T_0 = 303°$ K and of the boundary conditions in Eqs. (56b), (56c) for thermal stresses σ_{22}^{*}, σ_{12}^{*}, created by a unit source applied in the point $x_1 = 2m$, $x_2 = 5m$. Note that for the construction of graphics in Figures A1–A3 was used the computer program Maple 15.

ACKNOWLEDGMENTS

The Grantee Victor Seremet is grateful to Professor Erasmo Carrera for his excellent research supervision of the project: "Investigation of Thermal Displacements and Stresses by Generalized Green's Function Method."

FUNDING

This project has been funded with support from the European Commission. This article was written in the mobility period from 02 September to 02 October 2013 of the Grantee of the Erasmus Mundus Action 2 EUROEAST: Professor Victor Seremet at Host Institution Politecnico di Torino; Contact Person, Professor Erasmo Carrera. This paper reflects the views of the authors only, and the Commission cannot be held reasonable for any use which may be made of the information contained therein.

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