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Analysis of composite plates through cell-based smoothed finite element and 4-noded mixed interpolation of tensorial components techniques

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ABSTRACT

The static bending and the free vibration analysis of composite plates are performed with Carrera's Unified Formulation (CUF). We combine the cell-based smoothed finite element method (CSFEM) and the 4-noded mixed interpolation of tensorial components approach (MITC4). The smoothing method is used for the approximation of the bending strains, whilst the mixed interpolation allows the calculation of the shear transverse stress in a different manner. With a few numerical examples, the accuracy and the efficiency of the approach is demonstrated. The insensitiveness to shear locking is also demonstrated.

1. Introduction

Increasingly complex composite structures implies complex and effective means of analysis. Different approaches can be used in the study of laminated composite structures [1–4]. In recent years, two-dimensional (2D) theories using higher-order displacement functions had proven to be a true alternative to the computationally very expensive 3D models. The theories can be equivalent-single-layer (ESL) or layerwise [5]. For the general description of 2D formulations for multilayered plates and shells, a Unified Formulation was derived by Carrera, called 'Carrera Unified Formulation' (CUF) [6–8]. This formulation is a powerful tool to implement in a single software a large number of 2D model theories, ranging from ESL models to higher layerwise descriptions. The CUF can be used in the Finite Element Method (FEM) environment [8,9] or with meshless methods [10].

Nevertheless, even with the very useful CUF, there is an important shortcoming of the FEM. For thin structures, the inclusion of both the bending and the shear stiffness in a unique rotational degree of freedom cause the locking of the finite element solution, leading to inaccurate numerical results. This shear locking phenomena can be alleviated by the use of some techniques: taking optimal rules of integration [11]; using the assumed strain method [12,13]; using field redistributed shape functions [9]; using the mixed interpolation of tensorial components (MITC) technique and by incorporating the strain smoothing technique (SFEM) [14–19]. Alternative mixed methods with similar good results can be found in Moleiro et al. [20–24].

Another approach for the elimination of the shear locking phenomena is a combination of the previous remedies. Therefore, in this work a combination of the cell-based finite element method (CSFEM) with the 4-noded quadrilateral mixed interpolation of tensorial components technique (MITC4) and selective integration rule, is considered to study the global response of laminated composites within the CUF framework. By combining different technique, we take the advantage of each technique and aim to formulate an efficient and accurate methodology, that is free from shear locking syndrome. The displacements are approximated through a sinusoidal deformation theory, and a complete study of the influence of various model parameters is performed.

The paper is organized as follows. Section 2 introduces the cell-based smoothed finite element method. In Section 3, the shear strain field according to the 4-noded mixed interpolation tensorial components technique is presented. The separation between bending and shear contributions for the stiffness matrix is discussed and a brief overview of the Carrera's Unified Formulation is presented. The shear locking phenomena is discussed in Section 4. A few numerical examples are presented in Section 5 to show the

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effectiveness of the proposed approach, followed by concluding remarks in the last section.

2. Cell-based finite element method

In the strain smoothing technique, originally proposed for meshfree methods [25], later extended to FEM by Liu et al. [26], the strain field is written as a spatial average of the compatible strain field. Based on this technique, a series of smoothed finite element method (SFEM) can be derived [17]. One such SFEM is called the cell-based smoothed finite element method (CSFEM). In the CSFEM each element is subdivided into smoothing domains, called the *subcells*. Over each subcell, the smoothing technique is performed. By judicious choice of the smoothing function and by applying the Gauss divergence theory, the surface integrals can be transformed into line integrals around the boundary of the elements in 2D. This circumvents the need to compute the derivatives of shape functions, normally required in the partition of unity framework. The strain field, $\tilde{\varepsilon}_{ij}^h$ used to compute the stiffness matrix, is computed by a weighted average of the standard strain field ε_{ij}^h. At a point $\mathbf{x}_C$ in an element Ω^h, the smoothed strain field is given by:

$$\tilde{\varepsilon}_{ij}^h = \int_{\Omega^h} \varepsilon_{ij}^h(\mathbf{x})\Phi(\mathbf{x}-\mathbf{x}_C)d\mathbf{x} \tag{1}$$

where $\Phi(\mathbf{x}-\mathbf{x}_C)$ is a smoothing function and is chosen to be:

$$\Phi(\mathbf{x}-\mathbf{x}_C) = \begin{cases} \frac{1}{A_C} & \mathbf{x}_C \in \Omega_C \\ 0 & \mathbf{x}_C \notin \Omega_C \end{cases} \tag{2}$$

being A_C is the area of the subcell. For more detailed discussion see Refs. [26,17,16,27,28].

3. MITC4 under Carrera's Unified Formulation

The transverse shear strains, interpolated according to the 4-noded mixed interpolation of tensorial components (MITC4) technique, assume the following shear strain field:

$$\{\varepsilon_s\} = \left\{ \begin{array}{c} \{\varepsilon_{xz}\} \\ \{\varepsilon_{yz}\} \end{array} \right\} = \left\{ \begin{array}{c} \frac{1}{2}(1+\xi)\varepsilon_{xz}^N + \frac{1}{2}(1-\xi)\varepsilon_{xz}^Q \\ \frac{1}{2}(1+\eta)\varepsilon_{yz}^P + \frac{1}{2}(1-\eta)\varepsilon_{yz}^M \end{array} \right\} \tag{3}$$

where $M,\ N,\ P$ and Q are sample points in the element as shown in Fig. 1. The stiffness matrix K is cleaved in two contributions, bending and shear:

$$\begin{aligned} &[K] = [K_b] + [K_s] \\ &[K_b] = \langle [B_b]^T [Q_b][B_b] \rangle; \quad [K_s] = \langle [B_s]^T [Q_s][B_s] \rangle \end{aligned} \tag{4}$$

with the following notation:

$$\langle \cdots \rangle = \sum_{k=1}^{ns} \int_{V_k} (\ldots) dV_k \tag{5}$$

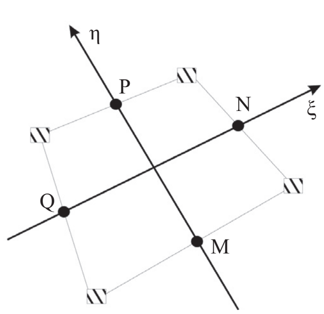

Fig. 1. Sample points (M,N,P,Q) to approximate the shear contribution in the MITC4 element.

The numerical code reflects this separation between the bending and the shear strains, within the framework of Carrera's Unified Formulation (CUF). According to CUF the displacements are expressed as a set of thickness functions depending only on the thickness coordinate z. So that the displacement field $\boldsymbol{u}(x,y,z)$ and its variation $\delta\boldsymbol{u}(x,y,z)$ are written according to the following general expansion:

$$\begin{aligned} &\boldsymbol{u}(x,y,z) = F_\tau(z)\boldsymbol{u}_\tau(x,y), \\ &\delta\boldsymbol{u}(x,y,z) = F_s(z)\delta\boldsymbol{u}_s(x,y), \quad \text{with} \quad \tau, s = 1,\ldots,N \end{aligned} \tag{6}$$

where the summing convention with repeated indexes τ and s is assumed. F_τ and F_s are the so-called the thickness functions and they can be generic functions of the coordinate z.

In this work, the derivation of the governing equations is based on the *Principle of Virtual Displacements* (PVD) in case of a multilayered plate subjected to mechanical loads. The CUF permits us to obtain the governing equations in terms of the so-called *fundamental nuclei*, which are simple matrices representing the basic element from which the stiffness matrix of the whole structure can be computed. The PVD for a plate with N_l layers, under mechanical loads, reads:

$$\sum_{k=1}^{N_l} \delta L_{int}^k = \sum_{k=1}^{N_l} \int_{\Omega_k} \int_{A_k} \left\{ \delta\boldsymbol{\varepsilon}_{pG}^{k\ T} \boldsymbol{\sigma}_{pC}^k + \delta\boldsymbol{\varepsilon}_{nG}^{k\ T} \boldsymbol{\sigma}_{nC}^k \right\} d\Omega_k dz = \sum_{k=1}^{N_l} \delta L_e^k \tag{7}$$

where Ω_k and A_k are the integration domains in plane (x,y) and z direction, respectively, k indicates the layer and T the transpose of a vector. δL_e^k is the external work for the k^{th} layer, G implies geometrical relations and C the constitutive relations.The first step to derive the governing equations is the substitution of *constitutive equations* (C) in the variational statement PVD:

$$\begin{aligned} &\boldsymbol{\sigma}_{pC}^k = \boldsymbol{C}_{pp}^k \boldsymbol{\varepsilon}_{pG}^k + \boldsymbol{C}_{pn}^k \boldsymbol{\varepsilon}_{nG}^k \\ &\boldsymbol{\sigma}_{nC}^k = \boldsymbol{C}_{np}^k \boldsymbol{\varepsilon}_{pG}^k + \boldsymbol{C}_{nn}^k \boldsymbol{\varepsilon}_{nG}^k \end{aligned} \tag{8}$$

with

$$\begin{aligned} &\boldsymbol{C}_{pp}^k = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix} \quad \boldsymbol{C}_{pn}^k = \begin{bmatrix} 0 & 0 & C_{13} \\ 0 & 0 & C_{23} \\ 0 & 0 & C_{36} \end{bmatrix} \\ &\boldsymbol{C}_{np}^k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{13} & C_{23} & C_{36} \end{bmatrix} \quad \boldsymbol{C}_{nn}^k = \begin{bmatrix} C_{55} & C_{45} & 0 \\ C_{45} & C_{44} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \end{aligned} \tag{9}$$

The second step is the substitution of *geometrical relations* which relate the strains to the displacement components $\boldsymbol{u} = (u_x, u_y, u_z)$:

$$\begin{aligned} &\boldsymbol{\varepsilon}_{pG}^k = \boldsymbol{D}_p^k \boldsymbol{u}^k, \\ &\boldsymbol{\varepsilon}_{nG}^k = (\boldsymbol{D}_{n\Omega}^k + \boldsymbol{D}_{nz}^k)\boldsymbol{u}^k \end{aligned} \tag{10}$$

wherein the differential operator arrays are defined as follows:

$$\boldsymbol{D}_p^k = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ \partial_y & \partial_x & 0 \end{bmatrix}, \quad \boldsymbol{D}_{n\Omega}^k = \begin{bmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_y \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{D}_{nz}^k = \begin{bmatrix} \partial_z & 0 & 0 \\ 0 & \partial_z & 0 \\ 0 & 0 & \partial_z \end{bmatrix}. \tag{11}$$

By introducing the Unified Formulation for the displacements, one has:

$$\begin{aligned} &\boldsymbol{\varepsilon}_{pG}^k = \boldsymbol{D}_p^k (F_\tau \boldsymbol{u}_\tau^k), \\ &\boldsymbol{\varepsilon}_{nG}^k = (\boldsymbol{D}_{n\Omega}^k + \boldsymbol{D}_{nz}^k)(F_\tau \boldsymbol{u}_\tau^k) = \boldsymbol{D}_{n\Omega}^k (F_\tau \boldsymbol{u}_\tau^k) + F_{\tau,z} \boldsymbol{u}_\tau^k \end{aligned} \tag{12}$$

Finally, it is possible to express the displacement $\boldsymbol{u}_\tau^k$ as a function of *nodal displacements* $\boldsymbol{q}_{\tau i}^k$, by means of the *shape functions*:

$$\boldsymbol{u}_\tau^k = N_i \boldsymbol{q}_{\tau i}^k \quad (i = 1,2,\ldots,N_n) \tag{13}$$

with $\boldsymbol{q}^k_{\tau i} = (q^k_{u_x\tau i}, q^k_{u_y\tau i}, q^k_{u_z\tau i})$. N_n is the number of nodes (in this case 4) and N_i is the shape function relative to node i. The MITC technique is introduced within this procedure and the shear strains are reinterpolated according to Eq. (3). Therefore, the governing equations are written in the following form:

$$\delta L^k_{int} = \delta \boldsymbol{q}^{kT}_{\tau i} \boldsymbol{K}^{k\tau sij} \boldsymbol{q}^k_{sj} \tag{14}$$

For the sake of brevity, the explicit expression of the fundamental nucleus $\boldsymbol{K}^{k\tau sij}$ is omitted here, interested readers are referred to the literature [29], where detailed derivation is given.

4. Shear locking phenomena

For thin structures, the inclusion of both bending and shear stiffness in a unique rotational degree of freedom, may cause the locking of the finite element, with oscillations in the shear and the membrane strains. There are some remedies for the locking phenomena: use an optimal rule of integration [11]; use the assumed strain method [12,13]; use field redistributed shape functions [9]. In this study, two procedures are combined to eliminate the locking: the cell-based smoothing technique (CSFEM), and the 4-noded mixed interpolation tensorial component (MITC4) technique that calculates the transverse shear stresses σ_{xz} and σ_{yz} in a different manner from other tensorial components. The bending strains are approximated with a cell-based smoothing technique. In the case of the shear strains the MITC4 approach is employed. If the thickness-to-side ratio of the structure is bigger than 0.1 a normal integration scheme (2×2 Gauss points) is used. It should be noted that the MITC4 technique by itself does not require any kind of selective integration in order to overcome the shear locking phenomena. In this paper, due to the combination with CSFEM technique, it was chosen a selective rule of integration providing some stiffness overestimation to compensate the inclusion of CSFEM technique, and that led to accurate solutions in less computational time, even though some spurious mode appeared.

5. Numerical examples

Static bending and free vibration analysis of composite laminate plate is performed as follows. The in-plane displacements u, v and the transverse displacement w are expressed by sinusoidal shear deformation theory denoted by SINUS:

$$u = u_o + zu_1 + \sin\left(\frac{\pi z}{h}\right) u_2$$
$$v = v_o + zv_1 + \sin\left(\frac{\pi z}{h}\right) v_2$$
$$w = w_o + zw_1 + \sin\left(\frac{\pi z}{h}\right) w_2 \tag{15}$$

where u_o, v_o and w_o are translations of a point at the middle-surface of the plate [30], w_2 is higher order translation, and u_1, v_1, u_3 and v_3 denote rotations. In this study a 20×20 structured quadrilateral mesh is considered for the pretended comparison with benchmark results. The present results are denoted by CSFEM-MITC4. Concerning the shear strains, the numerical integration rule depends on the thickness-to-side ratio, as mentioned above.

5.1. Static bending

In this section the static bending analysis of cross-ply laminated plates with three and four layers under following sinusoidal load:

$$p_z = P_o \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{a}\right) \tag{16}$$

is studied, where P_o is the amplitude of the mechanical load. In all cases, we present the normalized displacement as, unless otherwise states:

$$\overline{w} = w(a/2, a/2, 0) \frac{100 E_2 h^3}{P a^4} \tag{17}$$

5.1.1. Four layer $(0°/90°)_s$ square cross-ply laminated plate under sinusoidal load

A square simply supported laminate plate of thickness-to-side ratio h/a, composed of four equally thick layers oriented at $(0°/90°)_s$ is considered. The plate is subjected to a vertical pressure given by Eq. (16). The material properties are as follows: $E_1 = 25E_2$; $G_{12} = G_{13} = 0.5E_2$; $G_{23} = 0.2E_2$; $\nu_{12} = 0.25$. In Table 1, we present results for the SINUS theory with the combined CSFEM-MITC4 approach. We compare the results with higher order plate theories [31,32], first order theory [33], an exact solution [34], and the standard (FEM Q4) and smoothed (CS-FEM Q4) 4-noded element with field consistent approach [9]. It can be seen that the results from the CSFEM-MITC4 formulation show very good agreement with those in the literature and is insensitive to shear locking with the selective rule of integration.

5.1.2. Three layer (0°/90°/0°) square cross ply laminated plate under sinusoidal load

A square laminate plate of thickness-to-side ratio h/a, composed of three equally thick layers oriented at (0°/90°/0°) is considered. It is simply supported on all edges and subjected to a vertical pressure of the form (16). The material properties are: $E_1 = 132.38$ GPa, $E_2 = E_3 = 10.756$ GPa, $G_{12} = 3.606$ GPa, $G_{13} = G_{23} = 5.6537$ GPa, $\nu_{12} = \nu_{13} = 0.24$, $\nu_{23} = 0.49$. In Table 2, we present results for the SINUS-W2 theory with the present CSFEM-MITC4 approach. The results from the present approach are compared with an analytical solution [35,36], results from MITC4 formulation [29], and results from the standard (FEM Q4) and smoothed (CS-FEM Q4) 4-noded element with field consistent approach [9]. The numerical results from the present formulation are precise and agree with the existing solutions, being insensitive to shear locking, as the plate gets thinner.

Table 1
Normalized central deflection $\overline{w} = w(a/2, a/2, 0) \frac{100E_2h^3}{Pa^4}$ of a simply supported cross-ply laminated square plate [0°/90°/90°/0°], with $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu_{12} = 0.25$. Quadrilateral mesh with 20×20 elements for the present formulation.

Method	$a/h = 4$	$a/h = 10$	$a/h = 100$
HSDT [31]	1.8937	0.7147	0.4343
FSDT [33]	1.7100	0.6628	0.4337
Elasticity [34]	1.9540	0.7430	0.4347
RBF [32]	1.9783	0.7325	0.4307
FEM Q4 [9]	1.8949	0.7135	0.4302
CS-FEM Q4 (4 subcells) [9]	1.9089	0.7195	0.4304
Present (CSFEM-MITC4)	1.9086	0.7201	0.4304

Table 2
Transverse displacement $\overline{w} = w(a/2, a/2, h/2)$ at the center of a multilayered plate [0°/90°/0°] with $E_1 = 132.38$ GPa, $E_2 = E_3 = 10.756$ GPa, $G_{12} = 3.606$ GPa, $G_{13} = G_{23} = 5.6537$ GPa, $\nu_{12} = \nu_{13} = 0.24$, $\nu_{23} = 0.49$. Quadrilateral mesh with 20×20 elements for the present formulation.

$\overline{w}$	a/h				
	10	50	100	500	1000
Analytical (ESL-2) [35,36]	0.9249	0.7767	0.7720	0.7705	0.7704
MITC4 [29]	0.9195	0.7713	0.7666	0.7650	0.7650
FEM Q4 [9]	0.9152	0.7700	0.7651	0.7636	0.7635
CS-FEM Q4 (4 subcells) [9]	0.9235	0.7703	0.7655	0.7639	0.7639
Present (CSFEM-MITC4)	0.9238	0.7704	0.7655	0.7639	0.7639

Table 3
Normalized fundamental frequency $\Omega = \omega a^2/h\sqrt{\rho/E_2}$ of a simply supported cross-ply laminated square plate $(0°/90°)_s$ with $h/a = 0.2, \frac{E_1}{E_2} =$ 10, 20, 30 or 40, $G_{12} = G_{13} = 0.6E_2, G_{23} = 0.5E_2, \nu_{12} = 0.25$.

Method	Mesh	Subcell(s)	E_1/E_2			
			10	20	30	40
Liew [37]			8.2924	9.5613	10.3200	10.8490
Reddy, Khdeir [38]			8.2982	9.5671	10.3260	10.8540
FSDT [10]	21 × 21		8.2982	9.5671	10.3258	10.8540
HSDT [10] ($\nu_{23} = 0.18$)	21 × 21		8.2999	9.5411	10.2687	10.7652
FEM Q4 [9]	20 × 20		8.3651	9.5801	10.2980	10.7894
CS-FEM Q4 [9]	20 × 20	4	8.3639	9.5790	10.2970	10.7883
Present (CSFEM-MITC4)	20 × 20	4	8.3775	9.5857	10.3001	10.7892

Table 4
Variation of fundamental frequencies, $\Omega = \omega a^2/h\sqrt{\rho/E_2}$ with a/h for a simply supported square laminated plate $[0°/90°/90°/0°]$, with $E_1/E_2 = 40$, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$. Quadrilateral mesh with 20 × 20 elements for the present formulation.

Method	a/h					
	2	4	10	20	50	100
FSDT [39]	5.4998	9.3949	15.1426	17.6596	18.6742	18.8362
Model-1 (12dofs) [40]	5.4033	9.2870	15.1048	17.6470	18.6720	18.8357
Model-2 (9dofs) [40]	5.3929	9.2710	15.0949	17.6434	18.6713	18.8355
HSDT [31]	5.5065	9.3235	15.1073	17.6457	18.6718	18.8356
HSDT [41]	6.0017	10.2032	15.9405	17.9938	18.7381	18.8526
FEM Q4 [9]	5.4029	9.3005	15.1790	17.7578	18.7993	18.9657
CS-FEM Q4 (4 subcells) [9]	5.4026	9.2998	15.1766	17.7540	18.7947	18.9611
Present (CSFEM-MITC4)	5.3986	9.2975	15.1674	17.7471	18.7895	18.9561

5.2. Free vibration – cross-ply laminated plates

Consider a simply supported square plate with cross-ply lamination $(0°/90°)_s$ where all layers are assumed to be of the same thickness, density and made up of the same linear elastic material. The following material properties are considered for each layer

$$\frac{E_1}{E_2} = 10, 20, 30 \quad \text{or} \quad 40; \quad G_{12} = G_{13} = 0.6E_2; \quad G_{23} = 0.5E_2; \quad \nu_{12} = 0.25.$$

The subscripts 1 and 2 denote the directions normal and the transverse to the fiber direction in a lamina, which may be oriented at an angle to the plate axes. The ply angle of each layer is measured from the global x-axis to the fiber direction. In this example, the effect of Young's modulus ratio E_1/E_2 and the plate slenderness ratio a/h on the fundamental frequency is studied. In this study, we present the non-dimensionalized natural frequency as, unless specified otherwise:

$$\Omega = \omega a^2/h\sqrt{\rho/E_2} \tag{18}$$

Table 3 shows the fundamental frequency for a simply supported square laminated plate for different ratio of Young's modulus, E_1/E_2. The thickness-to-side ratio is $h/a = 0.2$. The results from the present CSFEM-MITC4 formulation are compared with the meshfree results [37], the results based on higher order theory [38], FSDT, higher order theories with radial basis functions [10] and the results using the standard (FEM Q4) and smoothed (CS-FEM Q4) 4-noded element with field consistent approach [9]. It can be observed that the present numerical procedure provide accurate results and similar to those in the literature.

Table 4 shows the effect of the thickness-to-side ratio of a simply supported cross-ply laminated square plate on the fundamental frequency, for Young's modulus $E_1/E_2 = 40$. The results from the present CSFEM-MITC4 formulation are compared with the results based on first order theory [39], analytical solutions [40], results from higher order theories [31,41]), and results using the standard (FEM Q4) and smoothed (CS-FEM Q4) 4-noded element with field consistent approach [9]. It can be seen that the present results are in very good agreement with the results available in the literature and they are accurate even for thin plates, which proves that the present methodology serves its propose of eliminating the shear locking.

6. Conclusion

In this work, the static bending and free vibration analysis of laminated composite plates is studied within the framework of Carrera's Unified Formulation (CUF). The plate kinematics is defined by sinsusoidal shear deformation theory. A 4-noded mixed interpolation of tensorial components approach (MITC4) was employed to treat the shear terms, whilst, a cell-based smoothed finite element method (CSFEM) was used for the bending terms. The efficiency and accuracy of the present formulation is demonstrated with a few numerical examples. From the numerical examples, it can be observed that the present approach yields results comparabled with the results available in the literature. The present approach, also shows insensitive to shear locking.

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