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Bending of FGM plates by a sinusoidal plate formulation and collocation with radial basis functions

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ABSTRACT

This paper addresses the static deformations analysis of functionally graded plates by collocation with radial basis functions, according to a sinusoidal shear deformation formulation for plates. The present plate theory approach accounts for through-the-thickness deformations. The equations of motion and the boundary conditions are obtained by the Carrera's Unified Formulation, and further interpolated by collocation with radial basis functions.

1. Introduction

Functionally graded plates (FGP) are obtained from gradual and continuous variation of material properties across the thickness direction. One advantage of FGP compared to laminated plates is that the material properties continuously vary in the thickness direction, as opposed to being discontinuous across adjoining layers as they are in laminated plates. This gradual variation avoids the delamination issues in laminated plates.

Typically FGP have been analysed neglecting the thickness stretching ϵ_{zz}, being the transverse displacement considered independent by thickness coordinates. Some recent work on the analysis of functionally graded plates was presented (Zenkour, 2006; Cheng and Batra, 2000; Loy et al., 1999; Reddy, 2000; Ferreira et al., 2005, 2006, 2007; Viola and Tornabene, 2009).

The effect of thickness stretching in FG plates has been investigated by Carrera et al. (2011), using finite elements.

The present paper addresses for the first time, the thickness stretching issue on FG plates, by a meshless technique based on collocation with radial basis functions. The technique is combined with the Carrera's Unified Formulation (CUF) (Carrera, 1996, 2001), in order to obtain the relevant equations of motion and natural boundary condition in strong form.

In recent years, radial basis functions (RBFs) proved to be an accurate technique for interpolating data and functions. A radial basis function, $\phi(||x-x_j||)$ depends on the Euclidian distance between distinct data centers $x_j, j=1,2,\ldots,N\in\mathbb{R}^n$, also called collocation points. Kansa (1990) introduced the concept of solving PDEs by an unsymmetric RBF collocation method based upon the MQ interpolation functions. The use of alternative methods to the Finite Element Methods for the analysis of plates, such as the meshless methods based on radial basis functions is attractive due to the absence of a mesh and the ease of collocation methods. The authors have recently applied the RBF collocation to the static deformations of composite beams and plates (Ferreira, 2003a,b; Ferreira et al., 2003).

The use of sinusoidal shear deformation plate theory was first presented by Touratier (1992, 1991, 1992), and later by Vidal and Polit (2008). The use of sinusoidal plate theories for functionally graded plates was presented by Zenkour (2006), where a $\epsilon_{zz}=0$ approach was used. The use of trigonometric shear deformation theory accounting for $\epsilon_{zz}\neq 0$ for the analysis of plates has not been used before. In this paper we consider an hybrid quasi-3D sinusoidal shear deformation theory. The expansion of both inplane displacements is defined as:

$$u=u_0+zu_1+\sin\left(\frac{\pi z}{h}\right)u_Z;\quad v=v_0+zv_1+\sin\left(\frac{\pi z}{h}\right)v_Z \qquad (1)$$

while the transverse displacement is defined as:

$$w=w_0+zw_1+z^2w_2 \qquad (2)$$

It is relevant to notice that the application of applied loads is now possible at the top (or bottom) surfaces.

2. Numerical examples

In this example, an isotropic FGM square plate with a polynomial material law, as given by Zenkour (2006) is considered. The plate is

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Table 1
FGM isotropic plate with polynomial material law (Zenkour, 2006). Effect of transverse normal strain ϵ_{zz} for a bending problem.

k	a/h	ϵ_{zz}	$\bar{\sigma}_{xx}(h/3)$			$\bar{u}_z(0,0)$		
			4	10	100	4	10	100
1	Carrera et al. (2008)	$\neq 0$	0.6221	1.5064	14.969	0.7171	0.5875	0.5625
	CLT	0	0.8060	2.0150	20.150	0.5623	0.5623	0.5623
	FSDT (k = 5/6)	0	0.8060	2.0150	20.150	0.7291	0.5889	0.5625
	GSDT (Zenkour, 2006)	0		1.4894			0.5889	
	Carrera (N = 4) (Carrera et al., 2011)	0	0.7856	2.0068	20.149	0.7289	0.5890	0.5625
	Carrera (N = 4) (Carrera et al., 2011)	$\neq 0$	0.6221	1.5064	14.969	0.7171	0.5875	0.5625
	Present 13 × 13 grid	$\neq 0$	0.5925	1.4939	14.901	0.6997	0.5844	0.5596
	Present 17 × 17 grid	$\neq 0$	0.5925	1.4945	14.957	0.6998	0.5845	0.5622
	Present 21 × 21 grid	$\neq 0$	0.5925	1.4945	14.969	0.6997	0.5845	0.5624
4	Carrera et al. (2008)	$\neq 0$	0.4877	1.1971	11.923	1.1585	0.8821	0.8286
	CLT	0	0.6420	1.6049	16.049	0.8281	0.8281	0.8281
	FSDT (k = 5/6)	0	0.6420	1.6049	16.049	1.1125	0.8736	0.828
	GSDT (Zenkour, 2006)	0		1.1783			0.8651	
	Carrera (N = 4) (Carrera et al., 2011)	0	0.5986	1.5874	16.047	1.1673	0.8828	0.8286
	Carrera (N = 4) (Carrera et al., 2011)	$\neq 0$	0.4877	1.1971	11.923	1.1585	0.8821	0.8286
	Present 13 × 13 grid	$\neq 0$	0.4404	1.1780	11.894	1.1178	0.8749	0.8251
	Present 17 × 17 grid	$\neq 0$	0.4404	1.1783	11.923	1.1178	0.8750	0.8284
	Present 21 × 21 grid	$\neq 0$	0.4404	1.1783	11.932	1.1178	0.8750	0.8286
10	Carrera et al. (2008)	$\neq 0$	0.3695	0.8965	8.9077	1.3745	1.0072	0.9361
	CLT	0	0.4796	1.1990	11.990	0.9354	0.9354	0.9354
	FSDT (k = 5/6)	0	0.4796	1.1990	11.990	1.3178	0.9966	0.9360
	GSDT (Zenkour, 2006)	0		0.8775			1.0089	
	Carrera (N = 4) (Carrera et al., 2011)	0	0.4345	1.1807	11.989	1.3925	1.0090	0.9361
	Carrera (N = 4) (Carrera et al., 2011)	$\neq 0$	0.1478	0.8965	8.9077	1.3745	1.0072	0.9361
	Present 13 × 13 grid	$\neq 0$	0.3227	1.1780	11.894	1.3490	0.8749	0.8251
	Present 17 × 17 grid	$\neq 0$	0.3227	1.1783	11.923	1.3490	0.8750	0.8284
	Present 21 × 21 grid	$\neq 0$	0.3227	1.1783	11.932	1.3490	0.8750	0.8286

simply supported with a bi-sinusoidal transverse mechanical load, of amplitude load $p_z = \bar{p}_z \sin(\pi x/a)\sin(\pi y/a)$ applied at the top of the plate, $z = h/2$, $\bar{p}_z = 1$.

The considered thickness ratios a/h are 4, 10 and 100, which means thickness h equals 0.25, 0.1 and 0.01, respectively. The plate is graded from aluminum (bottom) to alumina (top). The following functional relationship is considered for Young's modulus $E(z)$ in the thickness direction z (Zenkour, 2006):

$$E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k \tag{3}$$

where E_m = 70 GPa and E_c = 380 GPa are the corresponding properties of the metal and ceramic, respectively; k is the (positive number) volume fraction exponent. The Poisson ratio is considered constant (ν = 0.3).

The in-plane displacements, the transverse displacements, the normal stresses and the in-plane and transverse shear stresses are

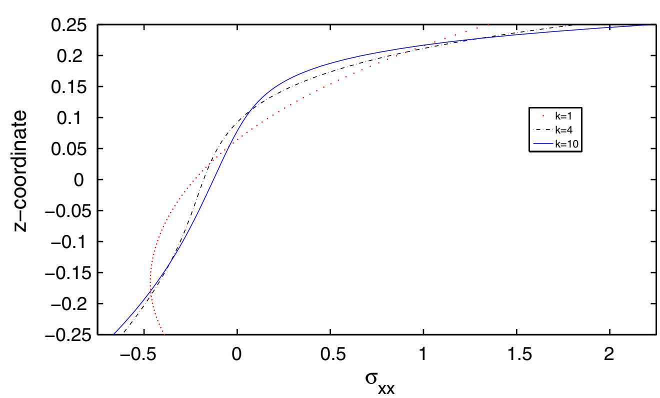

Fig. 2. FGM square plate subjected to sinusoidal load at the top, with a/h = 4. σ_{xx} through the thickness direction for different values of k.

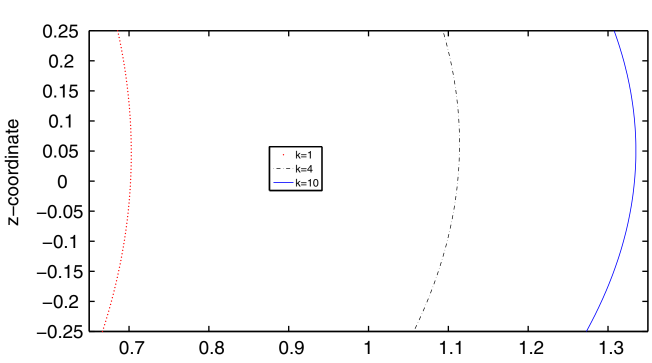

Fig. 1. FGM square plate subjected to sinusoidal load at the top, with a/h = 4. Displacement through the thickness direction for different values of k.

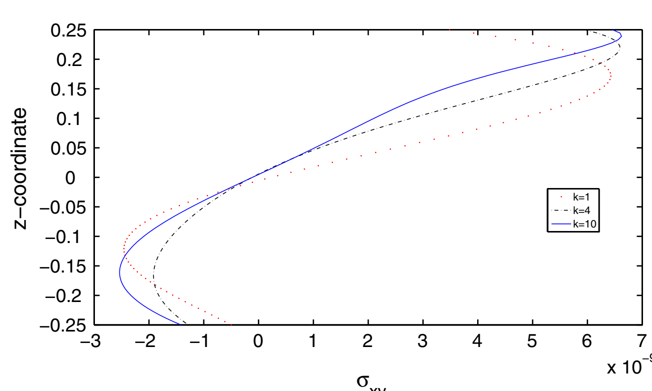

Fig. 3. FGM square plate subjected to sinusoidal load at the top, with a/h = 4. σ_{xy} through the thickness direction for different values of k.

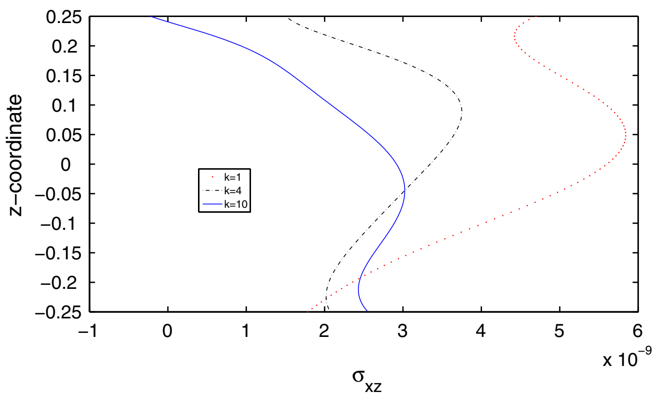

Fig. 4. FGM square plate subjected to sinusoidal load at the top, with $a/h=4$. σ_{xz} through the thickness direction for different values of k.

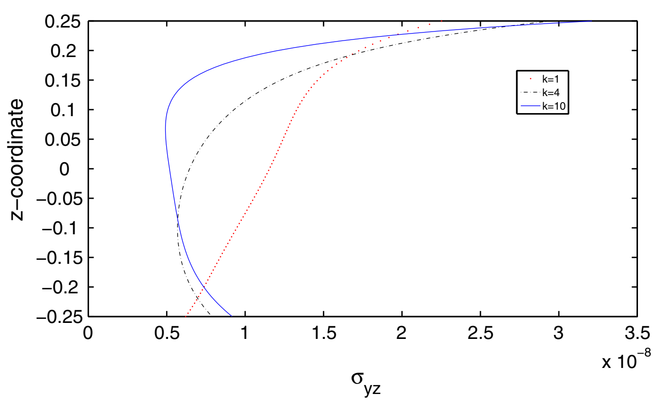

Fig. 5. FGM square plate subjected to sinusoidal load at the top, with $a/h=4$. σ_{yz} through the thickness direction for different values of k.

presented in normalized form as:

$$\bar{u}_z=\frac{10h^3E_c}{a^4\bar{p}_z}u_z,\quad \bar{\sigma}_{xx}=\frac{h}{a\bar{p}_z}\sigma_{xx},\quad \bar{\sigma}_{xz}=\frac{h}{a\bar{p}_z}\sigma_{xz},\quad \bar{\sigma}_{zz}=\sigma_{zz} \tag{4}$$

In Table 1 we analyse a FGM plate. We consider 90 mathematical layers, in order to model the continuous variation of properties across the thickness direction. We consider a Wendland C6 radial function, and a Chebyshev grid (see Ferreira and Fasshauer, 2006, for details). It is important to note that the load is applied at the top surface ($z=h/2$), which is not only physically correct as it makes all the difference in terms of the displacement and stresses evolution.

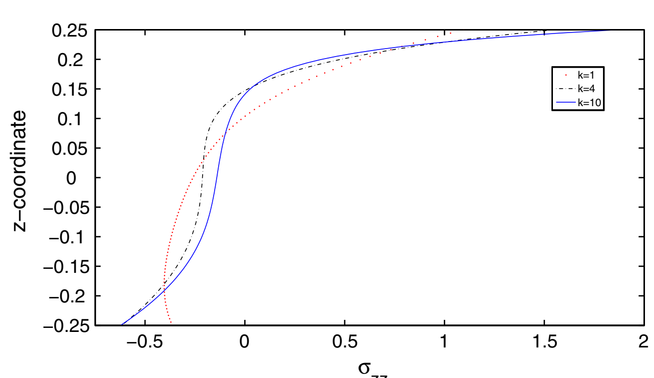

Fig. 6. FGM square plate subjected to sinusoidal load at the top, with $a/h=4$. σ_{zz} through the thickness direction for different values of k.

In Figs. 1–6 we present the evolution of the displacement and stresses across the thickness direction for various values of the exponent k, using a 21×21 grid.

It should be noted that the present numerical method presents very close results to those of Carrera et al. (2011) for a $N=4$ expansion. The consideration of a non-zero ϵ_{zz} strain produces a significant change in the transverse displacement as well as in the normal stress. This becomes evident when we compare the present approach with that of Zenkour (2006) who neglected the ϵ_{zz} strain in the formulation.

3. Conclusions

In this paper we presented a study using the radial basis function collocation method to analyse static deformations of functionally graded plates using a sinusoidal shear deformation plate formulation, allowing for through-the-thickness deformations. This has not been done before and serves to fill the gap of knowledge in this area.

The Unified Formulation by Carrera was used to generate the algebraic equations of equilibrium, later collocated with radial basis.

We analysed a square functionally graded plate in bending. The present results were compared with existing analytical solutions or competitive finite element solutions and excellent agreement was observed in all cases. It is relevant to notice the strong effect of considering the non-zero transverse normal deformations ϵ_{zz}.

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