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Effective properties of electro-elastic composites with multi-coating inhomogeneities

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Abstract. This work presents a micromechanics-based model to investigate the effective thermo-electric properties of piezoelectric composite materials. The effective thermo-electric properties are derived by considering a multi-coated ellipsoidal inhomogeneity embedded in a host material in the framework of the generalized self-consistent method (GSCM). An incremental scheme, in which the reinforcements are incrementally put in the host material, is implemented. The validation of the micromechanical model is performed with experimental data. The model proposed has a wide range of applications and can be extended to other physical properties.

Introduction

Piezoelectric composite materials are widely used in many applications, like ultrasonic imaging devices, sensors and actuators. Such composites inherit the characteristics of functional materials, such as the piezoelectric effect, which can be tailored to meet specific applications [1]. To obtain piezoelectric materials with the competing properties, many conventional piezoelectric materials are engineered to incorporate material inhomogeneities, and in some cases even voids, to achieve the desired electro-elastic properties. However, the introduction of material inhomogeneities or voids into base media will generally lead to the material being anisotropic, complicated and, in some situations, even detrimental to the performance of the composite. Therefore, in for the composite to offer a favourable behaviour, it is necessary to clearly examine the electro-elastic responses from a micromechanics point of view, considering the influence of each material parameter. Different techniques have been adopted in the literature for estimating the effective properties of piezoelectric composites, see [2,3] among others. These techniques are generally obtained as extension of the ones developed for pure elastic materials.

Motivate by providing a realistic numerical model for piezoelectric composite materials with inhomogeneous (or functionally graded) interphase, the micromechanics multi-coating inhomogeneous approach in [4] is extended to the electro-elastic coupled problem.

Micromechanical model

Topology and constitutive equations. The topology of the present multi-coated inhomogeneity problem (see Fig. 1) consists of an ellipsoidal inhomogeneity phase whose behaviour is described by the electro-elastic moduli pseudo-tensor L^1. Surrounding this inhomogeneity phase are $n-1$ layers of coatings of another materials whose behaviors are described by their respective electro-elastic moduli tensor L^i, i in $\{2, 3, ..., n\}$. Note that the coating n is a shell of the matrix material. This multi-coated inhomogeneity is then embedded in the effective material described by the unknown effective electro-elastic moduli pseudo-tensor L^{eff}.

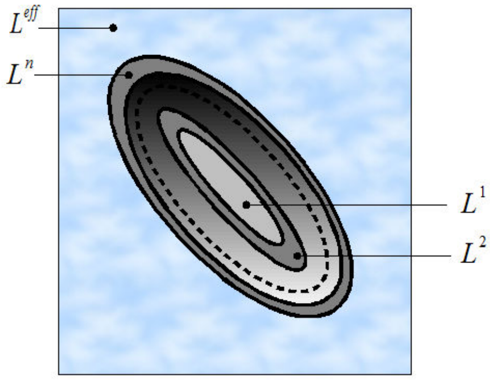

Fig. 1. Micromechanical model topology.

The interfaces matrix-coating, coating-coating and coating-inhomogeneity are assumed to be perfect, thus ensuring continuity of traction and displacements across these boundaries.

The constitutive equations for the stationary linear response of the electro-elastic material can be expressed as

$$\begin{cases} \sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k \\ D_i = e_{ikl}\varepsilon_{kl} + \kappa_{ik}E_k \end{cases} \quad (1)$$

Here σ_{ij} and ε_{kl} are the components of the stress and the strain tensors, E_k and D_i are the components of the electric field and the electric displacement vectors, C_{ijkl} are the components of the elastic modulus tensor measured at a constant electric field, κ_{ik} are the components of the dielectric tensor at fixed strain, e_{ikl} are the components of the piezoelectric constants tensor measured at fixed strain or electric field. The strain and the electric field are derivable from the displacement field u_i and electric potential ϕ, with comma denoting partial differentiation :

$$\begin{cases} \varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \\ E_i = -\phi_{,i} \end{cases} \quad (2)$$

We introduce the following generalized notations:

$$\Sigma_{i\alpha} = \begin{cases} \sigma_{i\alpha}, & \alpha = 1,2,3 \\ D_i, & \alpha = 4 \end{cases}, \quad \xi_{i\alpha} = \begin{cases} \varepsilon_{i\alpha}, & \alpha = 1,2,3 \\ -E_i, & \alpha = 4 \end{cases}, \quad U_\alpha = \begin{cases} u_\alpha, & \alpha = 1,2,3 \\ \phi, & \alpha = 4 \end{cases} \quad (3)$$

The constitutive Eq. 1, introducing Eq. 2 and Eq. 3 can be rewritten as

$$\Sigma_{i\alpha} = L_{i\alpha j\beta}\xi_{j\beta} = L_{i\alpha j\beta}U_{\beta,j} \quad (4)$$

where the electro-elastic properties pseudo-tensor L is defined by:

$$L_{i\alpha j\beta} = \begin{cases} C_{i\alpha j\beta}, & \alpha,\beta = 1,2,3 \\ e_{ji\alpha}, & \alpha = 1,2,3; \beta = 4 \\ e_{ij\beta}, & \beta = 1,2,3; \alpha = 4 \\ -\kappa_{ij}, & \alpha = 4; \beta = 4 \end{cases} \quad (5)$$

Note that in the above and in the rest of this paper, Roman indices range from 1 to 3 and Greek indices range from 1 to 4.

Integral equation, localizations and effective properties. Micromechanical schemes are based on two distinct steps: localization, which determines the relationship between the microscopic (local) fields and the macroscopic (global) loading, and homogenization, which employs averaging techniques to approximate the macroscopic behaviour. The equations of the elastic equilibrium and the Gauss's law of electrostatics in the absence of body forces and free charges are, based on the above shorthand notation:

$$\Sigma_{i\alpha,j} = 0 \tag{6}$$

Using the symmetries of C , κ, e, σ, and ε, Eq. 6 becomes

$$L^{eff}{}_{i\alpha j\beta} U_{\beta,ij} = -(\delta L_{\,i\alpha j\beta} \xi_{j\beta})_{,j} \tag{7}$$

where $\delta L_{\,i\alpha j\beta}$ are the spatially dependent electro-elastic moduli variations and $L^{eff}{}_{i\alpha j\beta}$ represent the unknown electro-elastic moduli of the effective material which are constant for the entire homogeneous medium. Using the Green's formalism, one gets the simplified equation for the strain and the electrical fields $\xi_{i\alpha}(r)$ at any point in the medium V as:

$$\xi_{j\alpha}(r) = \xi^{\infty}{}_{j\alpha} - \int_V \Gamma^{eff}{}_{i\alpha j\beta}(r - r')\, \delta L_{\,j\beta k\gamma}(r')\xi_{k\gamma}(r')dr' \tag{8}$$

where $\Gamma^{eff}{}_{i\alpha j\beta}(r - r')$ is the modified Green's electro-elastic pseudo-tensor, $\xi^{\infty}{}_{j\alpha}$ is the macroscopic strain and electric fields that have no spatial dependence. The different steps for the derivation of the localization pseudo-tensors A^k of the k^{th} phase defined by

$$\bar{\xi}^k = A^k : \xi^{\infty} \tag{9}$$

are similar to that of the pure elastic problem. The derivation details are given in [4]. $\bar{\xi}^k$ is the average of $\xi^k(r)$ over the volume of the k^{th} phase. Once the localization pseudo-tensors A^k of all the phases are computed, the effective electro-elastic properties pseudo-tensor L^{eff} can be obtained as follows

$$L^{eff} = \sum_{k=0}^{n} \varphi_k L^k : A^k \tag{10}$$

where φ_k is the volume fraction of the phase k. Two homogenization techniques are used herein, i.e. the non-incremental scheme and the incremental scheme. In the incremental scheme [5] the construction of the material is made by placing finite increments of the volume fractions of the inhomogeneities in a certain effective medium, in manner similar to the differential scheme, obtaining a progressive construction of the composite material.

Results

The results obtained with the micromechanical model are compared with experimental results in Fig. 2. The matrix is in Epoxy and the inclusions are of different piezoelectric materials. The inclusions are fibre shaped (aspect ratio $\Psi = 1000$). In this case there aren't significant differences between results obtained with the non-incremental scheme and the incremental scheme. The influences of parameters of the coated inclusion such as polarization orientation, aspect ratio and coatings properties on the effective properties can be easily investigated with the proposed model.

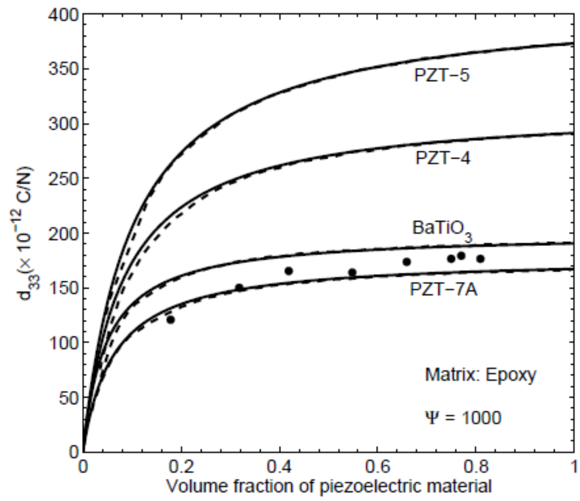

Fig. 2. Modulus d_{33} calculated with inhomogeneity of different materials. Incremental scheme in solid lines (-), non-incremental scheme in dashed lines (- -), experimental results for piezoelectric material PZT-7A in Epoxy matrix from [6] in dots (•).

Conclusions

A micromechanical model to predict the effective electro-elastic properties of piezo-electric composite materials is proposed. It is based on a generalized self consistent approach in which the composite inhomogeneities have an ellipsoidal shape and the number of coatings is a free parameter. This model proves to give results which are in good agreement with experimental data. The multi-coating formulation has the potential to study the effect of inclusions made of functionally graded materials.

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