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MIXED MULTILAYERED PLATE ELEMENTS FOR COUPLED MAGNETO-ELECTRO-ELASTIC ANALYSIS

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Abstract

Finite Elements FE based on the Reissner's Mixed Variational Theorem RMVT, for the analysis of multilayered plates subjected to magneto-electro-elastic MEE fields, are developed in this work. Accurate description of the various field variables has been provided by employing a variable kinematic model which is based on the Carrera's Unified Formulation CUF. Displacements, transverse shear/normal stresses, magnetic and electric potentials have been chosen as independent unknowns. Interlaminar continuity of mechanical variables is “a priori" guaranteed by the RMVT application. Layer-wise plate elements with linear up to fourth order distribution in the thickness direction have been compared. FE governing equations, according to CUF, are presented in terms of fundamental nuclei whose form is not affected by kinematic assumptions. Results show the effectiveness of the proposed elements, the superiority of mixed FEs with respect to the classical ones, as well as their capability, by choosing appropriate kinematics, to accurately trace the static response of laminated plates subject to magneto-electro-elastic fields.

Keywords
Finite Elements, multilayered plates, magneto-electro-elastic problems, mixed models, smart structures

1. Introduction

The introduction of non conventional materials has always played a key-role in the design and construction of lighter and more performant vehicles [1], [2]. Smart structures are characterized by the use of piezoelectric and/or piezomagnetic materials which can be used as both sensors and actuators to make a closed loop control on smart structures, see [1], [2]. Mechanical strains are measured on piezoelectric materials (PEMs) if an electrical field is applied; vice versa an electrical charge is obtained if mechanical loadings are applied to a piezoelectric made structure. The same happens if magnetical/electrical fields (actuator cases) or mechanical loadings (sensor cases) are applied to structures made by magneto-electro-elastic materials (MEEMs). In these applications both PEMs and MEEMs are embedded as patches or, more conveniently, as embedded layers in multilayered structures, which are often made by composite materials. According to the discussion already proposed in [3], as well as in the companion paper [4], the study of smart systems involves the analysis of multilayered structures MLS (often with anisotropic behaviors) under a combined action of multifield loadings (multifield problems MFP). A number of requirements must be taken into account for an accurate analysis of MFP and MLS, readers are addressed to the companion paper [4] for further discussions and literature references. The present work proposes a mixed variable kinematic finite element model which is based on the Carrera's Unified Formulation CUF [5] and on the Reissner's Mixed Variational Theorem RMVT [6]. In particular, the analysis in the companion paper [4] was restricted to classical variational statements based on the Principle of Virtual Displacements PVD. The extended RMVT is used to derive fundamental nuclei of coupled magneto-electric-elastic problems. Attention has been restricted to Layer-Wise LW description: displacements, transverse stresses, electric and magnetic potentials are assumed at layer level. The order of the expansion in the thickness layer/plate directions varies from linear to fourth order.

The paper has been organized as it follows. Sec.2 gives constitutive and geometrical relations. Extension of RMVT to MEE continua is given in Sec.3. CUF for the considered plate elements is presented and fundamental nuclei of finite element matrices are derived in Sec.4. Assessments and results for static bending problems are discussed in Sec.5.

2. Preliminary

The geometry and the coordinate system of the laminated plate made of N_l layers, including the MEE-layers has been represented in [4]. The reference system is denoted by *x*, *y* and *z*; the correspondent plate dimensions are denoted by letters *a, b* and *h*, respectively. The latter coordinate denotes the thickness direction. The number 1, 2 and 3 will be used in the case of the material system. The following set of constitutive equations, related to the k^{th} layer can be obtained accordingly to [4]:

$$
\begin{aligned}
\sigma_p^k &= C_{pp}^k \varepsilon_p^k + C_{pn}^k \varepsilon_n^k - e_p^{k^T} E^k - q_p^{k^T} H^k \\
\sigma_n^k &= C_{pn}^{k^T} \varepsilon_p^k + C_{nn}^k \varepsilon_n^k - e_n^{k^T} E^k - q_n^{k^T} H^k \\
D^k &= e_p^k \varepsilon_p^k + e_n^k \varepsilon_n^k + \varepsilon^k E^k + d^k H^k \\
B^k &= q_p^k \varepsilon_p^k + q_n^k \varepsilon_n^k + d^k E^k + \mu^k H^k .
\end{aligned} \tag{1}
$$

Superscripts k and T indicates the k^{th} layer of the plate and the array transposition, respectively. The explicit form of the introduced matrices can be found in [3] and [4].

3. Reissner's Mixed Variational Theorem for Magneto-Electro-Elastic Problems

In order to extend the Reissner's Mixed Variational Theorem RMVT [6] to MEE continua, Hamilton's principle can be considered, as done in [7] and [8] for the piezoelectric case. By omitting the algebra already illustrated in [4], the extension of the *Reissner's Mixed Variation Theorem to MEE problems* is obtained as it follows:

$$\sum_{k=1}^{N_l} \int_{\Omega_k} \int_{h_k} \left\{ \delta\varepsilon_p^{k^T} \sigma_p^k + \delta\varepsilon_n^{k^T} \sigma_n^k - \delta E^{k^T} D^k - \delta H^{k^T} B^k + \delta\sigma_{nM}^{k\ T} (\varepsilon_{nG}^k - \varepsilon_{nC}^k) \right\} d\Omega_k dz = \delta L_e - \delta L_{in} \tag{2}$$

Constitutive equations for the variational statement in Eq.(2) are obtained from Eqs.(1), by rewriting them in a consistent way for the RMVT application.

4. Carrera's Unified Formulation for Plate Theories and FE matrices

The Carrera's Unified Formulation (CUF) [5], [9] consists of a technique which permits to handle in a unified manner a large variety of plate models. This is made by expressing governing equations and/or finite element matrices in terms of a few fundamental nuclei which do not formally depend on:

- expansion N used in the z-direction;
- number of the nodes N_n of the element;
- variables description (layer wise or equivalent single layer theories).

Unknown variables are the displacements u, transverse stresses σ_n, electric potential Φ and the magnetic potential Ψ.

By considering the shape functions for the Finite Element approximation and the thickness functions for the CUF, we obtain:

$$\begin{aligned} &u^k(x,y,z) = F_\tau N_i q_{\tau i}^k; \quad \Phi^k(x,y,z) = F_\tau N_i g_{\tau i}^k; \\ &\Psi^k(x,y,z) = F_\tau N_i b_{\tau i}^k; \quad \sigma(x,y,z) = F_\tau N_i f_{\tau i}^k. \end{aligned} \tag{3}$$

All the necessary omitted steps can be found in [3] and [4]. However, the final form of governing equations is:

$$\begin{aligned} \delta q_{sj}^{k^T} &: \quad K_{uu}^{k\tau sij} q_{\tau i}^k + K_{ue}^{k\tau sij} g_{\tau i}^k + K_{uh}^{k\tau sij} b_{\tau i}^k + K_{u\sigma}^{k\tau sij} f_{\tau i}^k = P_{u\tau}^k - M_{u\ddot{u}}^{k\tau sij} \ddot{q}_{\tau i}^k \\ \delta g_{sj}^{k^T} &: \quad K_{eu}^{k\tau sij} q_{\tau i}^k + K_{ee}^{k\tau sij} g_{\tau i}^k + K_{eh}^{k\tau sij} b_{\tau i}^k + K_{e\sigma}^{k\tau sij} f_{\tau i}^k = P_{e\tau}^k \\ \delta b_{sj}^{k^T} &: \quad K_{hu}^{k\tau sij} q_{\tau i}^k + K_{he}^{k\tau sij} g_{\tau i}^k + K_{hh}^{k\tau sij} b_{\tau i}^k + K_{h\sigma}^{k\tau sij} f_{\tau i}^k = P_{h\tau}^k \\ \delta f_{sj}^{k^T} &: \quad K_{\sigma u}^{k\tau sij} q_{\tau i}^k + K_{\sigma e}^{k\tau sij} g_{\tau i}^k + K_{\sigma h}^{k\tau sij} b_{\tau i}^k + K_{\sigma\sigma}^{k\tau sij} f_{\tau i}^k = 0 \end{aligned} \tag{4}$$

5. Numerical Results

This section shows several numerical results employed to assess the developed FEs for static magneto-electrical-elastic plate problems based on the RMVT statement. Acronyms will be used in tables and diagrams to denote the considered plate elements. These will be denoted by LM1, LM2, LM3, LM4: L states that a layer description has been employed and M denotes that mixed approach based on RMVT has been used; 1-4 denote the order of expansion that has been introduced for the field variables in each layer (linear to fourth order). The corresponding elements, which were based on PVD applications, were denoted by acronyms LD1, LD2, LD3, LD4 as indicated in [4]. The reduced integration scheme has been preserved as done in several past studies, see [9].

In order to compare the analysis with an exact solution, the problem proposed by Pan [10] has been considered. The work considers a simply supported three layered square plate made by the combination of piezoelectric and magnetostrictive materials, loaded at the top. The piezoelectric is the $BaTiO_3$ (called B for brevity) and the magnetostrictive one is the $CoFe_2O_4$ (called F for brevity). The characteristics of these two materials can be seen in [4] and [10]. The considered stacking sequences are B/F/B and F/B/F; the load is of bi-sinusoidal type, each layer has thickness $h = 0.1$, square plate side-length is $a = 1$ and each parameter is calculated at $x = 0.75$ and $y = 0.25$, where $(x\ ,\ y)$ is the reference middle plane system.

Mechanical and electrical variables related to present FEs are compared in Tables 1 and 2 with other corresponding results in [10] and [4], B/F/B case is first considered. Transverse shear stresses are better evaluated by present RMVT applications. F/B/F case is analyzed in the Tables 3 and 4. It appears clear that FEs accuracy is very much dependent on the stacking sequence. RMVT applications permits the continuity of transverse shear/normal stresses in the thickness direction.

Table.1 Displacements and stresses for the B/F/B stacking sequence, 4X4 mesh

	$\sigma_{zz}[Pa]$ TOP	$\sigma_{xz}[Pa]$ MIDDLE	$u[m]$ BOT	$w[m]$ BOT
LM3	0.53	-4.13×10^{-1}	-2.05×10^{-12}	5.71×10^{-12}
LM4	0.59	-4.09×10^{-1}	-2.05×10^{-12}	5.72×10^{-12}
[10]	0.50	-3.96×10^{-1}	-2.01×10^{-12}	5.4×10^{-12}
LD1[4]	0.74	-4.14×10^{-1}	-2.02×10^{-12}	5.5×10^{-12}
LD4[4]	0.51	-4.23×10^{-1}	-2.05×10^{-12}	5.6×10^{-12}

Table.2 Stresses and potential for the B/F/B stacking sequence, 4X4 mesh

	$\phi_{MAX}[V]$	$\psi_{MAX}[C/S]$	$\sigma_{xx}[Pa]$ TOP	$\sigma_{xy}[Pa]$ BOT
LM3	8.43×10^{-4}	-7.18×10^{-7}	1.41	-5.47×10^{-1}
LM4	8.43×10^{-4}	-7.09×10^{-7}	1.43	-5.47×10^{-1}
[10]	1.54×10^{-3}	-2.61×10^{-6}	1.27	-5.77×10^{-1}
LD1[4]	8.33×10^{-4}	-7.07×10^{-7}	1.43	-5.38×10^{-1}
LD4[4]	8.39×10^{-4}	-7.12×10^{-7}	1.38	-5.48×10^{-1}

Table.3 Displacements and stresses for the F/B/F stacking sequence, 4X4 mesh

	$\sigma_{zz}[Pa]$ TOP	$\sigma_{xz}[Pa]$ MIDDLE	$u[m]$ BOT	$w[m]$ BOT
LM3	0.51	-3.98×10^{-1}	-1.58×10^{-12}	4.487×10^{-12}
LM4	0.51	-4.00×10^{-1}	-1.58×10^{-12}	4.488×10^{-12}
[10]	0.50	-3.86×10^{-1}	-1.57×10^{-12}	4.38×10^{-12}
LD1[4]	0.83	-3.97×10^{-1}	-1.55×10^{-12}	4.33×10^{-12}
LD4[4]	0.51	-4.06×10^{-1}	-1.585×10^{-12}	4.40×10^{-12}

Table.4 Stresses and potentials for the F/B/F stacking sequence, 4X4 mesh

	$\phi_{MAX}[V]$	$\psi_{MAX}[C/S]$	$\sigma_{xx}[Pa]$ TOP	$\sigma_{xy}[Pa]$ BOT
LM3	5.55×10^{-4}	-1.54×10^{-7}	1.433	-5.74×10^{-1}
LM4	5.54×10^{-4}	-1.52×10^{-7}	1.433	-5.79×10^{-1}
[10]	2.30×10^{-3}	-1.88×10^{-6}	1.33	-5.69×10^{-1}
LD1[4]	5.54×10^{-4}	-1.63×10^{-7}	1.60	-5.49×10^{-1}
LD4[4]	5.58×10^{-4}	-1.55×10^{-7}	1.42	-5.74×10^{-1}

6. Concluding Remarks

This paper has extended the Carrera's Unified Formulation (CUF) and the Reissner's Mixed Variational Theorem (RMVT) to develop finite elements for the static analysis of MEE plates. The following main conclusions have been outlined.

1. It has been confirmed that CUF consists of a valuable tool to provide hierarchical analysis of piezoelectric plates by finite element method. The implemented FEs, in fact, provide very accurate descriptions of mechanical, electrical and magnetical fields.
2. RMVT appears a very valuable tool to introduce “a priori” the requested continuity conditions at the interface between adjacent layers.

In addition to the analyses addressed in this paper, in which the interlaminar transverse stresses are continuous along the thickness direction, future works could direct to include “a priori” interlaminar continuous transverse electric displacement and transverse magnetic inductance, as well as to assess the proposed FEs for further applications.

References:

[1] I. Chopra, *SPIE Smart Structures and Materials Conference*, 1996, 20.
[2] I. Chopra, *AIAA Journal,* 40(2002)2, 2145.
[3] E. Carrera, S. Brischetto, P. Nali, Mech. *Adv. Mater. Struct.,* 15(2008), 182.
[4] E. Carrera, M. Di Gifico, P. Nali et al, *MMMS*, 5(2009), 119.
[5] E. Carrera, *Archives of Computational Methods in Engineering*, 10(2003), 215.
[6] E. Reissner, *Int. J. Numer. Methods Eng.*, 20(1984), 1366.
[7] M. D'Ottavio, B. Kröplin, *Mech. Adv. Mater. Struct.*, 13(2006), 139.

[8] E. Carrera, M. Boscolo, *Int. J. Numer. Methods Eng.*, 70(2007), 1135.
[9] E. Carrera, L. Demasi, *Int. J. Numer. Methods Eng.* , 55(2002), 253.
[10] E. Pan, *J. Appl. Mech.*, 68(2001), 608.