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Improved Response of Unsymmetrically Laminated Sandwich Plates by Using Zig-zag Functions

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ABSTRACT: This article shows the advantages of using the zig-zag function $(-1)^k\zeta_k$ (ZZF) in the bending analysis of unsymmetrically laminated sandwich flat panels with a soft core. Higher order theories are developed by adding ZZF to displacement fields of known theories. From linear to seventh order cases in displacement are considered. The main advantage of ZZF lies in the fact that it introduces a discontinuity in the first derivative (so called zig-zag effect) of the displacement distribution corresponding to the core-face interfaces. Results including and discarding ZZF are compared in the bending response of sandwich plates loaded by an harmonic distribution of transverse pressure at the top surface. Different values of face-to-core stiffness ratio (FCSR) as well as length-to-thickness ratio (LTR) have been analyzed. It is concluded that: (1) ZZF is highly recommended in the bending analysis of unsymmetrically laminated sandwich plates; (2) the use of ZZF makes the error almost independent of the FCSR parameter; (3) ZZF is easy to implement and its use should be considered with respect to other theories.

KEY WORDS: sandwich plates, zig-zag effect, Murakami's zig-zag function, higher order theories, soft core, length-to-thickness ratio, face-to-core stiffness ratio.

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Figures 3–6 appear in color online: http://jsm.sagepub.com

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INTRODUCTION

SANDWICH STRUCTURES ARE used to build large portions of advanced structural elements used in aerospace, automotive and ship structures. These structures are characterized by a soft core sandwiched between two stiffer faces. The discontinuity of mechanical properties between faces and core introduces a discontinuity in the deformed core-faces planes at the interfaces, (Figure 1). This is also known as the zig-zag effect in laminated structures [1]. Such discontinuities make difficult the use of classical theories such as Kirchhoff [2] or Mindlin [3] type theories, to accurately describe the response of sandwich structures, see books by Zenkert [4], Bitzer [5], and Vinson [6]. So called layer-wise models, in which the three layers are treated as three independent layers, can be used to capture elements of zig-zag effect, see the overview by Burton and Noor [7] also, Noor et al. [8], Altenbach [9], Librescu and Hause [10], Vinson [11], Carrera and Brischetto [12], Demasi [13]. However, such models result in computational expensive.

In the framework of mixed multilayered plate theories, Murakami [14] proposed a zig-zag function (ZZF) able to model the described slope discontinuity. Equivalent single layer models (that is the three layers are treated as one-layer) with only displacement unknowns can be developed on the basis of ZZF. The advantages, however of using ZZF in multilayered anisotropic plates and shells, as well as in Finite Element implementation, have been discussed by Carrera [15] and Demasi [16], respectively. In the present article, attention is restricted to the application of ZZF to the bending analyses of unsymmetrically laminated sandwich plates.

Various values of thickness ratios (thin and thick plates) are considered along with several face-to-core stiffness ratios (FCSR) to highlight the capability of ZZF. Comparisons with elasticity solutions [13] are proposed. This article consists of a further contribution of a work that aims to establish

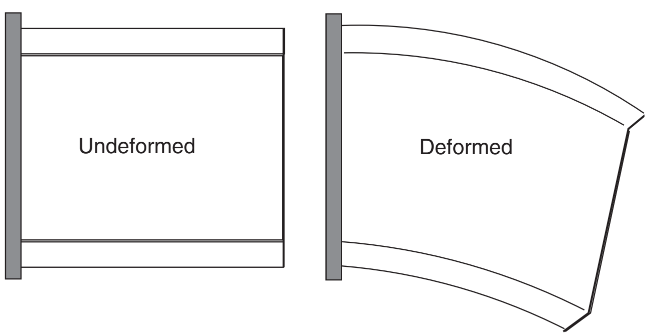

Figure 1. *Undeformed and deformed plane of a sandwich structure.*

the limitations and advantages of ZZF in the analysis of laminated sandwich structures. Previous works have been restricted to bending and vibration analysis of symmetrically laminated sandwich plates and shells [17,18].

THE ZIG-ZAG FUNCTION

Let us consider a sandwich, unsymmetrically laminated plate, composed of three layers, perfectly bonded together (z is the thickness coordinate of the whole plate, while z_k is the layer thickness coordinate. a and b are the lengths of the sandwich plate, h is the thickness). The nondimensioned layer coordinate $\zeta_k = (2z_k)/h_k$ is further introduced (h_k is the thickness of the k^{th} layer). Murakami's ZZF $Z(z)$ has been defined according to the following formula in [14] as:

$$Z(z) = (-1)^k \zeta_k. \tag{1}$$

$Z(z)$ has the following properties:

(1) It is a piece-wise linear function of the layer coordinates z_k;
(2) $Z(z)$ has a unit amplitude for the layers;
(3) The slope $Z'(z) = \mathrm{d}Z/\mathrm{d}z$ assumes an opposite sign between two-adjacent layers.

A plot of $Z(z)$ is also given in Figure 2. ZZF can then be used to introduce discontinuous slopes with correspondence to layer interfaces, for the displacement $u(z)$. If a *linear function* is considered for the displacement along the thickness, one has:

$$u(z) = c^0 + c^1 z, \tag{2}$$

where c^0, c^1 are the amplitudes of the uniform and linear terms of $u(z)$, respectively. By adding ZZF one has:

$$u(z) = c^0 + c^1 z + c^Z Z(z), \tag{3}$$

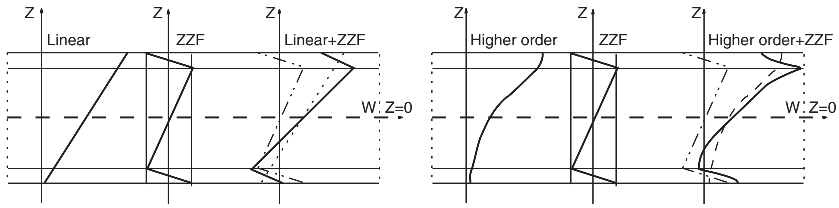

Figure 2. *Effect of adding the ZZF: linear case (left) and higher order case (right).*

where c^Z is the amplitude of ZZF. Displacements with and without ZZF are compared in Figure 2, which shows how $Z(z)$ emulates ZZ effects, and is very relevant in sandwich structures. $Z(z)$ can be also used in conjunction with higher N-order expansions (Figure 2),

$$u(z) = c^0 + c^1 z + c^2 z^2 + \cdots + c^{N-1} z^{N-1} + c^N z^N + c^Z Z(z). \tag{4}$$

CONSIDERED THEORIES

Classical theories for sandwich plates, such Kirchhoff (Classical Lamination Theory, CLT), Reissner-Mindlin (First order Shear Deformation Theory, FSDT) and Higher order Shear Deformation Theory, HSDT, (see the review paper by Librescu and Hause [10] and Vinson [11]) do not account for the ZZ effect. A possible use of ZZF would consist of enhancing classical models by 'simply' adding $Z(z)$ to their displacement fields. Let us first consider a linear distribution of a displacement component in the thickness direction z:

$$u = u^0 + zu^1, \tag{5}$$

in which: u is the displacement component along an assigned direction of the generic point P in a given Cartesian reference system; u^0 is the value of u corresponding to the reference surface Ω for which one has $z = 0$ and u^1 is an additional variable (u^1 has the symbolic meaning of rotation of the normal to Ω in P).

If FSDT applications are considered, Equation (5) is retained only for the in-plane components (u^1 coincides to the derivative of the transverse displacement with respect to the in-plane coordinates in the CLT case). ZZF offers the possibility to introduce the ZZ effect in Equation (5), thus,

$$u = u^0 + zu^1 + (-1)^k \zeta_k u^Z. \tag{6}$$

The following remarks can be made:

(1) The additional degree of freedom u^Z relates to displacement;
(2) The amplitude u^Z is layer independent: u^Z has, in fact, an intrinsic equivalent single layer description. At a first glance this fact could appear as a strong limitation of ZZF. In reality ZZF does not differ from other zig-zag theories, such as Ambartsumian's Multilayered Theories as well as Lekhnitskii's Multilayered Theories, detailed in reference [1];
(3) ZZF can be used for both in-plane and out-of-plane displacement components.

Refinements of FSDT

The application of ZZF to FSDT leads to the following displacement field model:

$$
\begin{aligned}
u_1 &= u_1^0 + z u_1^1 + (-1)^k \zeta_k u_1^Z \\
u_2 &= u_2^0 + z u_2^1 + (-1)^k \zeta_k u_2^Z \\
u_3 &= u_3^0
\end{aligned}
\tag{7}
$$

Subscripts 1, 2, and 3 denote displacement components in the three orthogonal directions of a given plate/shell reference system. The third one refers to the thickness-transverse z direction. The enhanced FSDT model has seven degrees of freedom, two more than the classical FSDT model that will be considered in the present analysis.

Refinement of FSDT by inclusion of ZZ effects and transverse normal strain ε_{zz}

The displacement model which includes transverse normal strain and ZZ effect in FSDT is:

$$
\begin{aligned}
u_1 &= u_1^0 + z u_1^1 + (-1)^k \zeta_k u_1^Z \\
u_2 &= u_2^0 + z u_2^1 + (-1)^k \zeta_k u_2^Z \\
u_3 &= u_3^0 + z u_3^1 + (-1)^k \zeta_k u_3^Z
\end{aligned}
\tag{8}
$$

For this model the acronym EDZ1 could be used, where *E* means Equivalent Single Layer approach (the three layers are treated as one layer), *D* states for displacements formulation, *Z* indicates the inclusion of ZZF, 1 means linear expansion in z direction for the three displacement components.

Higher order theories with ZZ function

ZZF can be used to introduce ZZ effect in any HSDT type expansions. The expansions considered in this work make use of power of z polynomials:

$$
u_i = u_i^0 + z u_i^1 + z^2 u_i^2 + \cdots + z^N u_i^N + (-1)^k \zeta_k u_i^Z, \quad i = 1, 2, 3. \tag{9}
$$

where N is the order of the expansion. The cases $N = 1, 2, 3, 4, 5, 6, 7$ will be considered in the numerical discussion. Previous works were restricted to $N = 4$. The proposed theories are denoted with the acronym EDN, where *E* means Equivalent Single layer approach, *D* indicates that a displacements formulation is employed, *N* is the order of expansion in the thickness direction.

Table 1. Number of degrees of freedom for the considered sandwich plate theories.

Theory	d.o.f.
CLT	3
FSDT	5
ED1	6
EDZ1	9
ED2	9
EDZ2	12
ED3	12
EDZ3	15
ED4	15
EDZ4	18
ED5	18
EDZ5	21
ED6	21
EDZ6	24
ED7	24
EDZ7	27

EDN theories including the ZZF are denoted as EDZN. A summary of the degrees of freedom of the considered theories is given in Table 1. It appears clear that the number of degrees of freedom of a EDZN theory is the same of the correspondent ED(N+1). EDZN theories improve the bending response of sandwich plates with respect to EDN theories.

NUMERICAL RESULTS AND DISCUSSION

The various models described above have been developed in the framework of Carrera's Unified Formulation CUF [19,20]. Closed form solutions are discussed herein for the case of simply supported, unsymmetrically laminated, flat sandwich plates made of isotropic layers and loaded by a transverse distribution of in-plane bisinusoidal pressure, as shown in Figure 3. Details of related governing equations and solution procedures are herein omitted. Poisson's thickness locking problem has been approached according to results discussed in [21]. Attention here has been focused in evaluating the effectiveness of the ZZ function in improving the bending response of sandwich plates with respect to classical and refined theories that do not make use of ZZ function. FSDT and higher order theories (ED1-ED7) are therefore compared to theories that make use of ZZ functions (EDZ1-EDZ7). A 3D elasticity solution [13] has been also provided.

Two values of length-to-thickness ratio (LTR) ($a\,/\,h$) have been considered: thick ($a/h = 4$) and thin ($a/h = 100$) sandwich plates. The physical reason of the zig-zag form for displacements field in the thickness direction is due to the

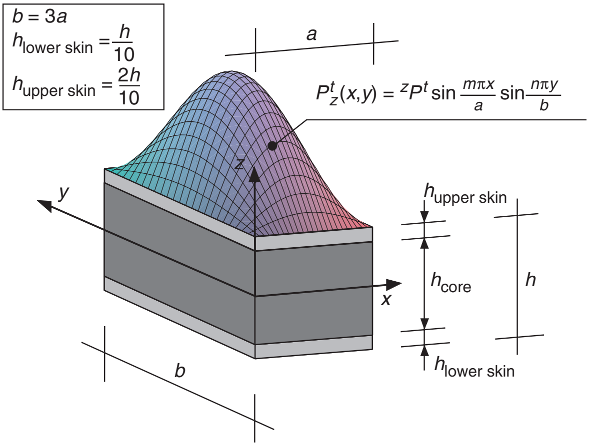

Figure 3. *Geometry of the plate sandwich structure.*

variation of mechanical properties between core and faces. A mechanical paramete FCSR has been therefore introduced to make evident such a discontinuity.

The multilayered structure is a sandwich plate (Figure 3) made of two skins and a core ($h_{\text{lower skin}} = h/10$; $h_{\text{upper skin}} = 2h/10$; $h_{\text{core}} = (7/10)h$), there $\frac{E_{\text{lower skin}}}{E_{\text{upper skin}}} = 5/4$. The plate is simply supported and the load is a sinusoidal pressure applied at the top surface of the plate ($m = n = 1$). The elastic modulus of the skins is increased and the effect on the displacements and stresses evaluated. Different cases are discussed here:

- $\text{FCSR} = \frac{E_{\text{lower skin}}}{E_{\text{core}}} = 10^1$; $a/h = 4, 100$;
- $\text{FCSR} = \frac{E_{\text{lower skin}}}{E_{\text{core}}} = 10^5$; $a/h = 4, 100$.

As far as Poisson's ratio is concerned, the following values are used: $\upsilon_{\text{lower skin}} = \upsilon_{\text{upper skin}} = \upsilon_{\text{core}} = \upsilon = 0.34$. In all cases $b = 3a$. In this test case there is no symmetry with respect the plane $z = 0$. The following nondimensional quantities have been introduced:

$$\widehat{u}_x = u_x \frac{E_{\text{core}}}{{}^zP^t h\left(\frac{a}{h}\right)^3}; \quad \widehat{u}_z = u_z \frac{100E_{\text{core}}}{{}^zP^t h\left(\frac{a}{h}\right)^4}; \quad \widehat{\sigma}_{zx} = \frac{\sigma_{zx}}{{}^zP^t\left(\frac{a}{h}\right)}; \quad \widehat{\sigma}_{xx} = \frac{\sigma_{xx}}{{}^zP^t\left(\frac{a}{h}\right)^2}. \quad (10)$$

Figure 4 shows the distribution of the in-plane displacement in the sandwich thickness direction. The advantages of using ZZF are clearly evident. The ability of ZZF to describe the discontinuity in slope, as shown in [17], is validated for the case of unsymmetrically laminated sandwich plates. Table 2 compares the transverse displacement of the considered theories for two values of LTR and FCSR. The error with respect to the 3D elasticity solution has been given.

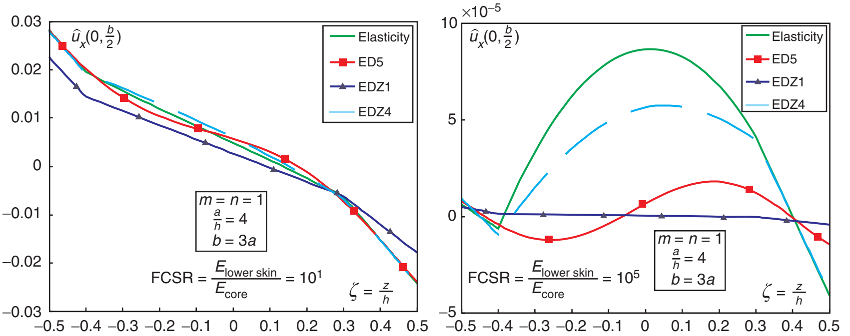

Figure 4. *Dimensionless in-plane displacement $\hat{u}_x$. Comparison between higher order models and the 3D elasticity solution.*

Table 2. Comparison of various theories to evaluate the transverse displacements amplitude (center plate deflection) $\hat{u}_z = u_z(100E_{core}/^zP^th\,(a/h)^4$ in $z = z^{upper\ skin}_{bottom} = \frac{3}{10}h$. The 3D elasticity solution is given in [13].

LTR	4	(Err. %)	100	(Err. %)
		FCSR=10^1		
3D	3.01123	–	1.51021	–
FSDT	1.58218	(−47.5)	1.10845	(−26.6)
ED1	1.58218	(−47.5)	1.10845	(−26.6)
ED4	2.79960	(−7.03)	1.50989	(−0.02)
ED5	2.84978	(−5.36)	1.50996	(−0.02)
ED6	2.85423	(−5.21)	1.50997	(−0.02)
ED7	2.86875	(−4.73)	1.50999	(−0.01)
EDZ1	2.34412	(−22.2)	1.15866	(−23.3)
EDZ4	2.97886	(−1.07)	1.51017	(−0.00)
EDZ5	2.98737	(−0.79)	1.51018	(−0.00)
EDZ6	2.99508	(−0.54)	1.51019	(−0.00)
EDZ7	2.99670	(−0.48)	1.51019	(−0.00)
		FCSR=10^5		
3D	$1.31593 \cdot 10^{-02}$	–	$2.08948 \cdot 10^{-03}$	–
FSDT	$1.79831 \cdot 10^{-04}$	(−98.6)	$1.19941 \cdot 10^{-04}$	(−94.3)
ED1	$1.79831 \cdot 10^{-04}$	(−98.6)	$1.19941 \cdot 10^{-04}$	(−94.3)
ED4	$1.16851 \cdot 10^{-03}$	(−91.1)	$1.64835 \cdot 10^{-04}$	(−92.1)
ED5	$4.29224 \cdot 10^{-03}$	(−67.4)	$1.73120 \cdot 10^{-04}$	(−91.7)
ED6	$9.17599 \cdot 10^{-03}$	(−30.3)	$2.19552 \cdot 10^{-04}$	(−89.5)
ED7	$1.08119 \cdot 10^{-02}$	(−17.8)	$2.96304 \cdot 10^{-04}$	(−85.8)
EDZ1	$8.36735 \cdot 10^{-04}$	(−93.6)	$1.63329 \cdot 10^{-04}$	(−92.2)
EDZ4	$1.26288 \cdot 10^{-02}$	(−4.03)	$1.16305 \cdot 10^{-03}$	(−44.4)
EDZ5	$1.30409 \cdot 10^{-02}$	(−0.90)	$1.78411 \cdot 10^{-03}$	(−14.6)
EDZ6	$1.31159 \cdot 10^{-02}$	(−0.33)	$1.97505 \cdot 10^{-03}$	(−5.48)
EDZ7	$1.31363 \cdot 10^{-02}$	(−0.17)	$2.02060 \cdot 10^{-03}$	(−3.30)

The following conclusions can be made: – FSDT and ED1-ED7 theories are quite accurate for the analyses of the sandwich plate if low values of FCSR are considered, the error becomes significant in the case of a soft core plate; – the use of ZZF leads to significant benefits in both thin and thick geometries as well as in the cases of soft and hard core; - for thin plates the percentage error is reduced for both values of FCSR, for example in case of FCSR = 10^5 (soft core), comparing the ED7 with the EDZ6 (same number of d.o.f as indicated in Table 1) the error is reduced from 85.8% to 5.48%. By adding a further order of expansion in EDZ6 in order to obtain a EDZ7 theory, the error is reduced from 5.48% to 3.30%. Significant improvements are also obtained in the case of thick plate geometries. In other words it appears inconvenient to introduce higher order terms (z^2, z^3, and z^4) in the displacement field since a simple ZZ function leads to a much better description.

Additional analysis are quoted in Figures 5 and 6 and Table 3 for transverse shear and in-plane normal stress evaluations. The same conclusions obtained for the displacement are there by confirmed.

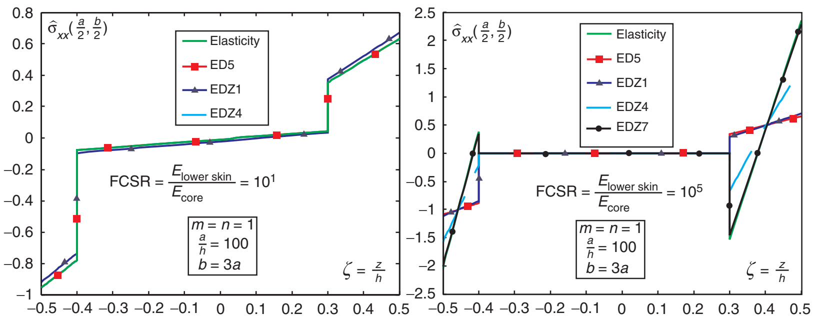

Figure 5. *Dimensionless in-plane stress $\hat{\sigma}_{xx}$. Comparison between higher order models and the 3D elasticity solution.*

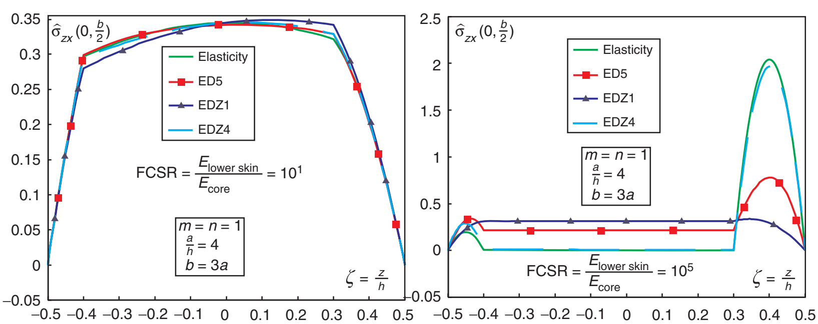

Figure 6. *Dimensionless out-of-plane shear stress $\hat{\sigma}_{zx}$. Comparison between higher order models and the 3D elasticity solution.*

Table 3. Comparison of various theories to evaluate the transverse shear stress $\hat{\sigma}_{zx} = (\sigma_{zx}/{}^{z}P^{t}\ (a/h))$ in $z = z_{bottom}^{upper\ skin} = \frac{3}{10}h$. The indefinite equilibrium equations have been integrated along the thickness. The 3D elasticity solution is given in [13].

LTR	4	(Err.%)	100	(Err.%)
		FCSR=10^1		
3D	0.32168	–	0.33176	–
FSDT	0.33178	(+3.14)	0.33178	(+0.01)
ED1	0.33178	(+3.14)	0.33178	(+0.01)
ED4	0.33240	(+3.33)	0.33178	(+0.01)
ED5	0.32884	(+2.23)	0.33178	(+0.00)
ED6	0.32706	(+1.67)	0.33177	(+0.00)
ED7	0.32707	(+1.68)	0.33177	(+0.00)
EDZ1	0.34184	(+6.27)	0.34497	(+3.98)
EDZ4	0.32913	(+2.31)	0.33178	(+0.00)
EDZ5	0.32755	(+1.82)	0.33177	(+0.00)
EDZ6	0.32539	(+1.15)	0.33177	(+0.00)
EDZ7	0.32530	(+1.12)	0.33177	(+0.00)
		FCSR=10^5		
3D	$5.40842 \cdot 10^{-04}$	–	0.27797	–
FSDT	0.33242	(>100)	0.33242	(+19.6)
ED1	0.33242	(>100)	0.33242	(+19.6)
ED4	0.30529	(>100)	0.33238	(+16.6)
ED5	0.21639	(>100)	0.33214	(+19.5)
ED6	$8.15220 \cdot 10^{-02}$	(>100)	0.33082	(+19.0)
ED7	$3.96907 \cdot 10^{-02}$	(>100)	0.32865	(+18.2)
EDZ1	0.30971	(>100)	0.33077	(+19.0)
EDZ4	$6.84336 \cdot 10^{-03}$	(>100)	0.30392	(+9.33)
EDZ5	$1.87520 \cdot 10^{-03}$	(>100)	0.28655	(+3.09)
EDZ6	$1.00683 \cdot 10^{-03}$	(+86.2)	0.28120	(+1.16)
EDZ7	$8.02443 \cdot 10^{-04}$	(+48.4)	0.27994	(+0.71)

CONCLUSIONS

The present article has demonstrated the convenience of using the ZZF to build higher order plate theories for the analysis of unsymmetrically laminated sandwich structures. The numerical investigation has shown that very significant improvements are obtained by introducing ZZF for both the displacement and stress evaluations. The use of ZZF is highly recommended for sandwich structures with soft cores. Such improvements are almost independent of the FCSR, exception made for those configurations in which Layer-wise models are mandatory, as illustrated in [12]. Scientists or engineers working in the field of sandwich structures modeling are encouraged to try ZZF. It is the authors' opinion that they will be surprised by the effectiveness and benefits as well as by the simplicity of its implementation.

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