AIAA JOURNAL
Vol. 43, No. 10, October 2005

Transverse Normal Strain Effects on Thermal Stress Analysis of Homogeneous and Layered Plates

E. Carrera*
Politecnico di Torino, 10129 Turin, Italy

A study of the transverse normal strain effect on the static thermoelastic response of homogeneous and multilayered plates is presented. Numerical evaluations have been given for classical, refined, and advanced zig–zag plate theories. Constant, linear, and higher-order forms of temperature profile in the plate thickness direction have been accounted for. Basic assumptions of the considered theories are quoted. The related governing equations are not given because these were derived from a unified formulation that was described elsewhere. Closed-form solutions are discussed by addressing three plate problems: a homogeneous plate made of an isotropic layer; a two-layered plate consisting of two layers made of different isotropic materials; a multilayered composite plate made of three cross-ply layers. It has been confirmed that any refinements of classical models are generally meaningless, unless the effects of transverse shear and normal strains are both taken into account in a plate theory. Furthermore, it has been found that transverse normal strains cannot be discarded even though thin plates are considered and the accurate description of the temperature profile in the plate thickness direction could result to be meaningless, unless transverse normal strains are taken into account, and vice versa.

I. Introduction

KOITER, in his lecture on two-dimensional modeling of traditional isotropic shells,[1] on the bases of energy considerations, stated the following Koiter's recommendation (KR): "a refinement of Love's first approximation theory is indeed meaningless, in general, unless the effects of transverse shear and normal strains (stresses) are taken into account at the same time." More general and systematic substantiation of Koiter's conclusion can be read in the books by Goldenvaizer[2] and Cicala[3] in which the method of asymptotic expansion of the three-dimensional governing equations is employed. The evaluation of transverse normal strain effects on the bending and vibration of multilayered plates and shells has recently been provided in Refs. 4 and 5. Amendments of Koiter's recommendation (KR1) were proposed in these works according to the statement: "any refinements of classical models are meaningless, in general, unless the effects of interlaminar continuous transverse shear and normal stresses are both taken into account in a multilayered plate/shell theory."

Transverse normal strain ϵ_{zz} (see Fig. 1 for geometrical notations) can, in general, exist even though transverse normal stress σ_{zz} is discarded. However, transverse normal strain is expected to be the most important contribution to σ_{zz}. In the present work, attention is therefore restricted to transverse normal strain. In spite of the preceding recommendations, most of the available refinements of classical theories [such as the classical lamination theory (CLT), which is based on Kirchhoff's assumptions, and the first-order shear deformation theory (FSDT), which is based on the so-called Reissner–Mindlin assumptions] that have been proposed for homogeneous (one-layered) and multilayered (anisotropic) plates and shells do not account for ϵ_{zz}. A discussion on this point, for the case of pure mechanical problems, can be found in Refs. 4 and 5. The main causes of violating KR lie in the intrinsic coupling experienced by isotropic and orthotropic materials between in-plane stresses σ_{xx}, σ_{yy}, and transverse strain ϵ_{zz} and vice versa. Two consequences of such a coupling are, in fact, 1) the differential equations that govern the static and dynamic behaviors of plate/shell structures are more difficult to obtain and solve, compared to those related to theories that discard ϵ_{zz}; and 2) it is difficult to develop multilayered plate/shell theories that are able to a priori fulfill interlaminar equilibria for both transverse shear (σ_{xz}, σ_{yz}) and normal stresses (σ_{zz}). See the recent discussion provided in Ref. 6.

The attention of this work has been focused on the evaluation of transverse normal strain ϵ_{zz} effects for static, thermoelastic problems of homogeneous and multilayered composite plates. The interest in thermal stress analysis of the structures of launch/reentry vehicles, fighter aircraft, biomedical retina, reactor vessels, turbines, advanced optical mirrors, semiconductor technologies, and space antennas is well established in the open literature.[7]

To introduce the discussion that will be detailed in the following sections, let us consider a temperature distribution $T(x, y, z)$ in the plate domain V (plate volume). It is assumed that $T(x, y, z)$ can be written in the form

$$T(x, y, z) = T_P(z) T_\Omega(x, y) \tag{1}$$

where $T_P(z)$ represents the temperature profile across the plate thickness coordinate z, whereas $T_\Omega(x, y)$ is the temperature distribution over the reference surface domain Ω. Different forms of the temperature profile $T_P(z)$ (see Fig. 2) and their implications on the displacement field are discussed next.

Case 1: *$T_P(z)$ has constant distribution in the thickness direction.* A plate made of isotropic materials heated by constant distribution

$$T_P(z) = T_0 \tag{2}$$

is first considered. The associated in-plane and transverse normal thermal strains are

$$\epsilon_{xx_T} = \epsilon_{yy_T} = \epsilon_{zz_T} = \alpha T_0 \tag{3}$$

where α is the coefficient of thermal expansion and subscript T underlines that the written strains represent the contribution coming from the temperature loadings. In the most general case, in fact, strains are also caused by geometrical (constraints) and/or mechanical (forces) boundary conditions. The following statement (S1) can be made: *The order of magnitude of transverse thermal strains ϵ_{zz_T} is the same as that of the in-plane ϵ_{xx_T} and ϵ_{yy_T} ones*. The field of transverse displacement u_z in the direction of the plate thickness can be calculated from

$$\epsilon_{zz_T} = u_{z_T,z} \tag{4}$$

Received 31 May 2004; revision received 21 April 2005; accepted for publication 22 April 2005.

*Professor, Aerospace Department, Corso Duca degli Abruzzi 24; erasmo.carrera@polito.it.

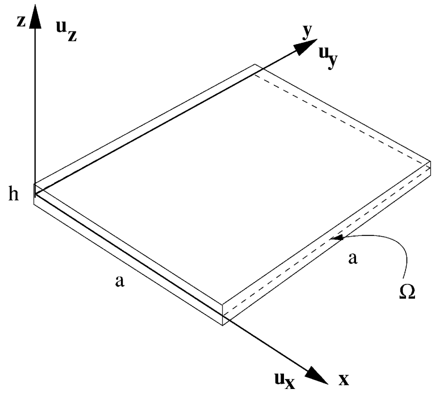


Fig. 1 Plate geometry and notations.

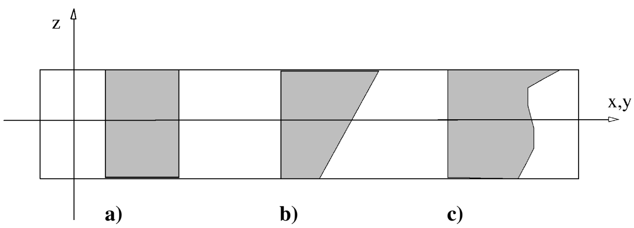


Fig. 2 Possible temperature profile in the plate thickness direction $T_P(z)$: a) constant, b) linear, and c) higher order.

where the comma denotes partial derivatives. Upon integration in z, one obtains

$$u_{z_T} = u_z^0 + \alpha T_0 z \tag{5}$$

The following statement (S2) can be made: *Plate theories with at least a linear transverse displacement field in the z direction are required to capture the transverse normal strain caused by the constant distribution of temperature across the thickness. CLT and FSDT are inadequate for this purpose.* The in-plane displacement could be constant. To capture the bending caused by the constraints and/or to other loadings, the linear form is usually required for u_x and u_y.

Case 2: $T_P(z)$ is linear distribution in the thickness direction. Let us consider a linear temperature profile across the plate thickness,

$$T(z)_P = T_0 + T_1 z \tag{6}$$

According to what has been done for case 1, one obtains

$$\epsilon_{xx_T} = \epsilon_{yy_T} = \epsilon_{zz_T} = \alpha(T_0 + T_1 z) \tag{7}$$

so that

$$u_{z_T} = u_z^0 + \alpha T_0 z + \alpha T_1 z^2 \tag{8}$$

The following statement (S3) follows: *Plate theories with at least a quadratic transverse displacement field in the z direction are required to capture transverse normal strain caused by the linear distribution of temperature across the thickness.* The in-plane displacements u_x and u_y are at least required to be linear in z.

In the most general case, the form of $T_P(z)$ is a result of the solution of a heat-conduction problem. This calculated $T_P(z)$ turns out to be a nonlinear polynomial of z, often of a transcendental form.[8] In the case of a thick multilayered structure, $T_P(z)$ would require a layerwise description (S4),[9] that is: *plate theories with higher-order (or layerwise) displacement fields are required to capture a temperature profile obtained from the solution of a heat-conduction problem.*

A. Brief Literature Overview

Three-dimensional solutions of thermal stress problems related to laminated plates have been given by Bapu Rao.[10] Murakami,[11] and Bhaskar et al.[12] Examples of three-dimensional solutions, in the case of coupled heat-conduction thermal-stress problems, have been considered by Tungikar and Rao[8] and Savoia and Reddy.[13] These three-dimensional works play a fundamental role in the assessment of approximated two-dimensional plate theories. Examples of two-dimensional plate theories are given in the following.

The importance of transverse normal strain on the thermal stress analysis of plates and shells has been known since the work by Hildebrand et al.[14] and before the Koiter recommendation already mentioned. These authors introduced the following displacement field:

$$\begin{aligned} u_x(x,y,z) &= u_x^0(x,y) + z u_x^1(x,y) \\ u_y(x,y,z) &= u_y^0(x,y) + z u_y^1(x,y) \\ u_z(x,y,z) &= u_z^0(x,y) + z u_z^1(x,y) + z^2 u_z^2(x,y) \end{aligned} \tag{9}$$

where u_x^0, u_x^0, u_x^0 are the values of displacements that correspond to the plate-reference surface Ω and u_x^1, u_x^1, u_x^1, and u_z^2 are the additional variables defined on Ω. FSDT is obtained by neglecting the two terms u_z^1, u_z^2. Nevertheless, according to S3, these two terms are necessary to capture thermal loadings caused by linear $T_P(z)$ while the in-plane components can remain linear in z.

The generalization to the third-order expansion for the three displacement components has recently been discussed by Jonnalagadda et al.[15]:

$$\begin{aligned} u_x(x,y,z) &= u_x^0(x,y) + z u_x^1(x,y) + z^2 u_x^2(x,y) + z^3 u_x^3(x,y) \\ u_y(x,y,z) &= u_y^0(x,y) + z u_y^1(x,y) + z^2 u_y^2(x,y) + z^3 u_y^3(x,y) \\ u_z(x,y,z) &= u_z^0(x,y) + z u_z^1(x,y) + z^2 u_z^2(x,y) + z^3 u_z^3(x,y) \end{aligned} \tag{10}$$

Numerical evaluations were given for an orthotropic plate and a linear temperature profile $T_P(z)$ as in case 2.

Cho et al.[16] introduced a higher-order theory (HOT) in the individual layers. A beam-type plate in the x direction was considered, and the displacement field was assumed to be independent in each discrete k layer:

$$\begin{aligned} u_x^k(x,z) &= u_x^{k0}(x) + z u_x^{k1}(x) + z^2 u_x^{k2}(x) + z^3 u_x^{k3}(x) + \alpha_x^k T_0(x) \\ u_z^k(x,z) &= u_z^{k0}(x) + z u_z^{k1}(x) + z^2 u_z^{k2}(x) + \alpha_z^k T_0(x) \end{aligned} \tag{11}$$

where α_x^k and α_z^k are the thermal expansion coefficients of the k layer in the x and z directions, respectively. The displacement field of Eq. (2) is enhanced by adding parabolic and cubic terms to the in-plane displacement expansion and by introducing the displacement components as a result of temperature heating. The latter was considered uniform in z, as in case 1.

The Murakami zig–zag function (MZZF) was introduced by Ali et al.[17] to improve HOT for their application to laminate composite cross-ply plates. The displacement model in Ref. 17 is

$$\begin{aligned} u_x(x,y,z) &= z u_x^1(x,y) + z^3 u_x^3(x,y) + M(z) u_x^4(x,y) \\ u_y(x,y,z) &= z u_y^1(x,y) + z^3 u_x^3(x,y) + M(z) u_y^4(x,y) \\ u_z(x,y,z) &+ u_z^0(x,y) + z^2 u_z^2(x,y) \end{aligned} \tag{12}$$

where $M(z)$ is the MZZF, which will be dealt with in detail in Sec. III. Because it was applied to laminates constituted by layers with the same thermal expansion coefficient in the thickness direction, the use of MZZF was restricted to in-plane displacements. KR was taken into account by Eq. (12). Attention was restricted to symmetrically laminated plates that were only bent by linear $T_P(z)$. As a consequence, the odd terms of the displacement expansions were omitted.

More recent works have been proposed by Gu et al.[18] and Zhao et al.[19] Interest in accurate stress evaluation was clearly declared in the latter articles. A third-order temperature profile was introduced in Ref. 18:

$$T_P(z) = T_0 + zT_1 + z^2T_2 + z^3T_3 \tag{13}$$

The used plate theory referred to a refinement of FSDT, according to the following displacement fields:

$$u_x = u_x^0 - zu_{z,x} + g(z)u_x^1(x, y)$$
$$u_y = u_y^0 - zu_{z,y} + g(z)u_y^1(x, y), \qquad u_z = u_z^0 \tag{14}$$

where $g(z)$ is cubic function of z. Transverse normal strains were discarded in Ref. 18 by violating KR and the S1–S4 statements. The same has been done in many other works that have not been mentioned here (see the review by Tauchert,[20] Noor and Burton,[21] Argyris and Tenek,[22] Carrera,[23] and Carrera and Ciuffreda[24]).

B. Contents of This Work

This aim of this work is to contribute to a better understanding of the effect of transverse normal strain ϵ_{zz} on the response of homogeneous and laminated plates, subjected to heating. Classical theories originally developed for isotropic one-layered plates, refined theories, and advanced zig–zag theories[6] that have been developed for multilayered plates were all considered and compared. Attention was restricted to those theories in which the number of independent variables is independent of the number of the constitutive layers. These are also known as equivalent single-layer models. Examples of these types of theories are those in Eqs. (9), (10), (12), and (14). A discussion on layerwise models, in which the number of the variables is kept dependent on the number of the constitutive layers such as those at Eq. (11), has been omitted. One of these will instead be employed to provide a quasi-three-dimensional solution for the composites plates discussed in Sec. III.

ESL models have been implemented by employing the unified formulation that was already used in Refs. 9, 23, and 24. Up to fourth-order displacement fields in the thickness direction are considered, and the effect of transverse normal strain ϵ_{zz} has been evaluated for each of the implemented models. Both classical theories with only displacement unknowns (formulated on the basis of principle of virtual displacements) and advanced theories with displacement and transverse stress unknowns [formulated on the basis of Reissner mixed variational theorem (RMVT)] have been considered. The theories and results of one-layered (homogeneous) and multilayered (nonhomogeneous) plates are discussed in Secs. II and III, respectively. Section II deals with an isotropic plate. The available theories are described: displacement models related to the CLT, FSDT, and HOT, including and discarding transverse strain effects are written. The most representative results, in terms of displacements and strains, are given in the form of diagrams and tables. Constant and linear forms of $T_P(z)$ are investigated. Theories for multilayered plates are described in Sec. III. Zig–zag theories that make use of the MZZF as well as those based on RMVT are briefly considered. Results are given for a two-layered plate made of two isotropic metallic materials and for a three-layered composite plate for which three-dimensional solutions were provided in Ref. 17. Evaluations are given for constant and linear temperature profiles. In the latter case, a comparison is made between the calculated and assigned temperature profile.

The plate theories have been written according to the unified formulations provided by the author in recent papers[9,23,24] and extensively described in Refs. 25 and 26. For the sake of conciseness, the derivation of governing equations and a detailed description of the employed closed-form solution procedure have been omitted. Those readers who are interested can refer to the previously mentioned papers for details.

II. Homogeneous Plate Constituted by One Layer of Isotropic Material

A. Plate Theories

The plate theories that are applied to analyze the response of a classical one-layered plate made of isotropic materials are detailed next.

CLT

The simplest known plate theory, namely, the classical lamination theory (CLT), is based on Kirchhoff's thin plate assumptions (see Ref. 27). Transverse shear strains as well as transverse normal strains are neglected. The displacement is represented by

$$u_i = u_i^0 - zu_{z,i} \qquad i = x, y$$
$$u_z = u_z^0 \tag{15}$$

where superscript 0 denotes the displacement values on Ω, which is the reference plane of the plate. (In most cases Ω coincides with the middle surface of the plate.)

FSDT

The inclusion of transverse shear strains leads to the following representation of the displacement quantities:

$$u_i = u_i^0 + z\phi_i \qquad i = x, y$$
$$u_z = u_z^0 \tag{16}$$

This first-order shear deformation theory (FSDT) is also known as the Reissner–Mindlin plate theory (see Ref. 27). In this theory, a first-order Taylor-type expansion of displacement unknowns in the neighborhood of the reference surface Ω is considered. And ϕ_x, ϕ_y represent the rotations of the normal to Ω in the two planes x–z and y–z, respectively. These rotations can also be expressed in terms of transverse shear strains:

$$\phi_x = \epsilon_{xz} - u_{3,x}, \qquad \phi_y = \epsilon_{yz} - u_{3,y}$$

For convenience, the FSDT model is rewritten according to the following array form:

$$\boldsymbol{u}(x, y, z) = F_0(z)\boldsymbol{u}_0(x, y) + F_1(z)\boldsymbol{u}_1(x, y) \tag{17}$$

The polynomials used in the expansion take on the following values:

$$F_0(z) = 1, \qquad F_1(z) = z$$

The boldfaced letter denotes arrays:

$$\boldsymbol{u}_0(x, y) = [u_{x_0}(x, y), u_{y_0}(x, y), u_{z_0}(x, y)]$$
$$\boldsymbol{u}_1(x, y) = [u_{x_1}(x, y), u_{y_1}(x, y), u_{z_1}(x, y)]$$

The displacement unknowns are

$$u_{x_0}(x, y) = u_x^0, \qquad u_{y_0} = u_y^0$$
$$u_{z_0}(x, y) = u_z^0, \qquad u_{x_1}(x, y) = \phi_x, \qquad u_{y_1} = \phi_y$$
$$u_{z_1}(x, y) = 0 \tag{18}$$

This last condition underlines that transverse normal strains are discarded in FSDT formulated plate modelings, that is, KR is violated by FSDT. CLT results could be obtained by applying a penalty technique to the shear correction factor in the FSDT case.

Inclusion of Transverse Normal Strain in FSDT

Transverse normal strain can be included by generalizing the representation provided in Eq. (17) and incorporating the transverse normal strain. The explicit form of the displacement field is

$$u_x = F_0(z)u_{0_x} + F_1(z)u_{1_x}, \qquad u_y = F_0(z)u_{0_y} + F_1(z)u_{1_y}$$

$$u_y = F_0(z)u_{0_z} + F_1(z)u_{1_z} \tag{19}$$

The FSDT can be considered as a particular case of the preceding model in which the constraint expressed by Eq. (18) is imposed.

Higher-Order Theories

The displacement model in Eq. (19) can be easily extended to a higher-order Taylor expansions:

$$\boldsymbol{u}(x, y, z) = F_0(z)\boldsymbol{u}_0(x, y) + F_1(z)\boldsymbol{u}_1(x, y) + \cdots$$
$$+ \cdots F_N(z)\boldsymbol{u}_N(x, y) \tag{20}$$

or, by introducing Einstein's convention for repeated indexes,

$$\boldsymbol{u}(x, y, z) = F_\tau(z)\boldsymbol{u}_\tau(x, y) \qquad \tau = 0, N \tag{21}$$

where

$$F_\tau(z) = z^\tau$$

For the sake of convenience, this displacement model is put in the form

$$\boldsymbol{u}(x, y, z) = F_t(z)\boldsymbol{u}_t(x, y) + F_b(z)\boldsymbol{u}_b(x, y)$$
$$+ F_\tau(z)\boldsymbol{u}_\tau(x, y) \qquad \tau = 2, N \tag{22}$$

where

$$F_t(z) = 1, \qquad F_b(z) = z, \qquad F_\tau(z) = z^\tau, \qquad \tau = 2, N$$

With respect to Eq. (20), the constant and linear terms are now denoted with subscripts t and b, respectively.

B. Results

An isotropic plate made of aluminum alloy 5086 is first considered. By referring to known notations, the thermomechanical data are[7] $E = 70{,}300$ MPa, $\nu = 0.33$, and $\alpha = 24.E\text{–}6\ 1/K$. A rectangular plate, simply supported at the edges,[23] subjected to harmonic distribution of in-plane temperature is considered:

$$T_\Omega(x, y) = \sin(m\pi/a)x \sin(n\pi x/a)y \tag{23}$$

where a and b are the plate lengths in the x and y directions, respectively, and m and n are the corresponding wave numbers. The values $a = b$ and $m = n = 1$ are treated in the numerical analysis. For the sake of simplicity, we assign the meaning of temperature heating starting from an environmental temperature (T_e, which can assume any value) to $T_P(z)$. The two cases of temperature profile $T_P(z)$ discussed in the Introduction have been investigated: 1) A constant (z-independent) heating

$$T_P^0(z) = T_0 \tag{24}$$

A unit increment of T_0 is considered in the numerical investigation. 2) A linear increment of amplitude T_0 around T_e is considered in the second case:

$$T_P^1(z) = T_0(2/h)z \tag{25}$$

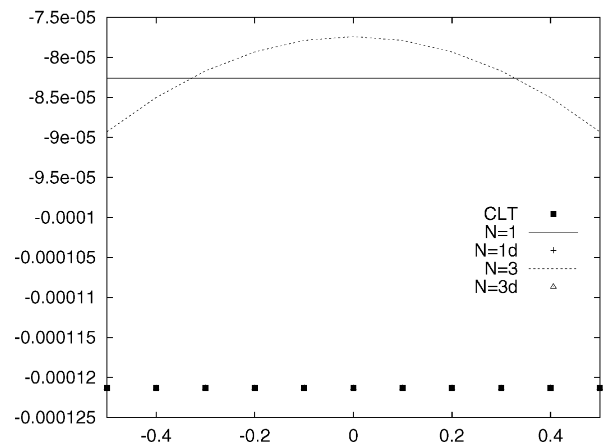
Also in this case, a unit value of T_0 has been used in the numerical investigation, so that it leads to an increment of 1 K and −1 K corresponding to the top and bottom plate surfaces, respectively. Various values of the thickness parameters a/h have been investigated.

The case of constant heating has been dealt with in Table 1. Thick ($a/h = 4$) and thin ($a/h = 100$) plate results are quoted. The results related to CLT and to four HOT (related to linear up to fourth order N values) are compared. The effect of ϵ_{zz} has been evaluated for the latter four theories. FSDT analysis coincides with $N = 1$ and $\epsilon_{zz} = 0$ cases. The following remarks can be made. It has been confirmed that displacements of isotropic plates are not significantly affected by the order N in the case of thin plates and barely evident in thick plate geometries. It has instead been found that transverse normal strain plays a very relevant role in both thin and thick plate cases: results obtained by neglecting ϵ_{zz} can be about 40 to 50% lower than those obtained by retaining ϵ_{zz}. Furthermore, the improvements of N increasing can be appreciated if and only if ϵ_{zz} is retained in the formulation. It can be noticed that while the error vanishes in thin plates subjected to pure mechanical loadings[4] (as well as in free-vibration problems,[5] it has here been found that such an error can be very significant in both thin and thick plates if a thermal stress problem is addressed. This conclusion confirms KR as well as statement S1.

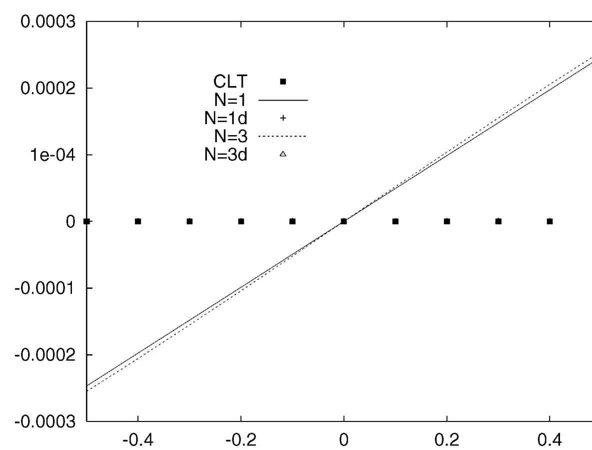
Distribution of in-plane and out-of-plane displacements are given in Figs. 3 and 4 for the T_P^0 case. The following five theories are

Table 1 Transverse normal strain effects in isotropic plates [the analysis of the order of expansion on the accuracy of $u_x(0, b/2, h/2) \times a/h$ for the considered theories: the T_P^0 case]

	$a/h = 4$		$a/h = 100$	
Theory	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$
CLT	——	$-0.1213E-3$	——	$-0.7582E-1$
$N = 1$	$-0.8262E-4$	$-0.1213E-3$	$-0.5080E-1$	$-0.7582E-1$
$N = 2$	$-0.8910E-4$	$-0.1213E-3$	$-0.5081E-1$	$-0.7582E-1$
$N = 3$	$-0.8929E-4$	$-0.1213E-3$	$-0.5081E-1$	$-0.7582E-1$
$N = 4$	$-0.8930E-4$	$-0.1213E-3$	$-0.5081E-1$	$-0.7582E-1$

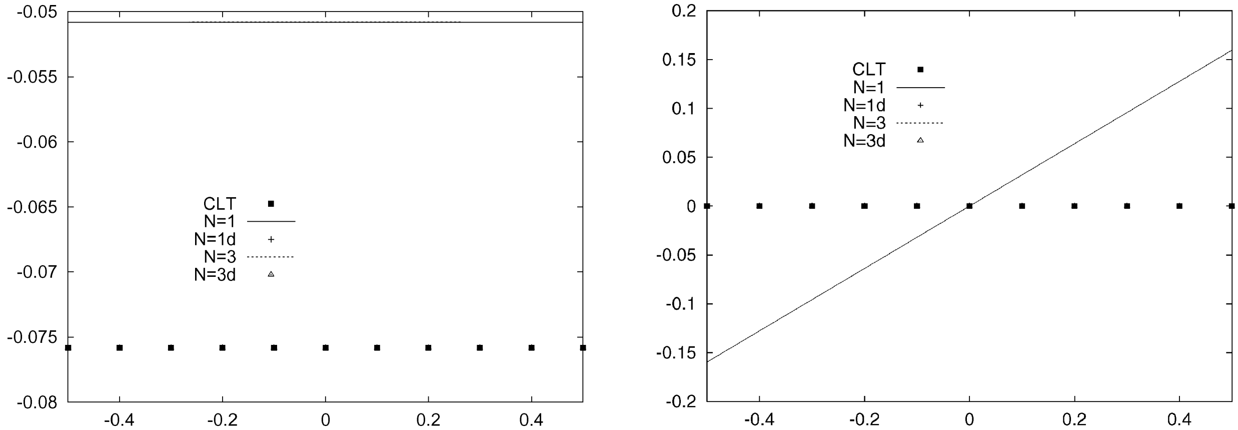


a) u_x (0, b/2, z) × a/h



b) u_z (a/2, b/2, z) × (a/h)^2

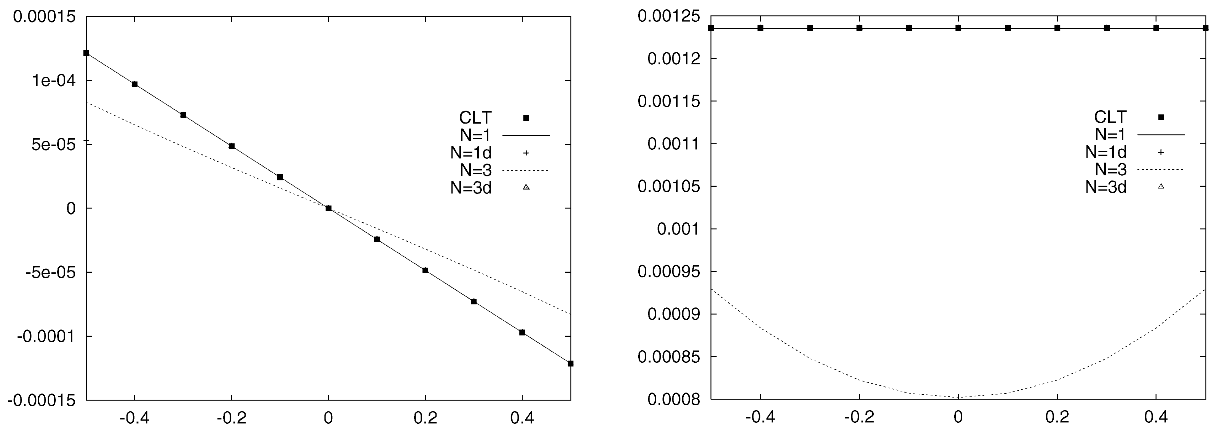
Fig. 3 Through-the-thickness distribution of the in-plane and out-of-plane displacements of an isotropic thick plate ($a/h = 4$): T_P^0 case.



a) u_x $(0, b/2, z) \times a/h$

b) u_z $(a/2, b/2, z) \times (a/h)^2$

Fig. 4 Thin-plate case ($a/h = 100$) of Fig. 3.



a) u_x $(0, b/2, z) \times a/h$

b) u_z $(a/2, b/2, z) \times (a/h)^2$

Fig. 5 Through-the-thickness distribution of the in-plane and out-of-plane displacements of an isotropic thick plate ($a/h = 4$): T_P^1 case.

compared: CLT, the two HOTs related to $N = 1, 3$, and the two corresponding cases related to $\epsilon_{zz} = 0$. The latter have been denoted as $N = 1d$ and $N = 3d$ in the plots. The in-plane displacement distribution varies in the z direction if, and only if, the transverse normal strain is retained in the analysis. The variation of transverse displacements is very important in both thick and thin plates. Almost the same results are obtained by those theories that discard transverse normal strains, for example, CLT, $N = 1d$, and $N = 3d$ analysis.

The case of linear distribution T_P^1 has been addressed in Figs. 5 and 6. The in-plane and out-of-plane displacements have a linear and parabolic distribution in z, respectively. The conclusions made for the T_P^0 case are confirmed. In addition, it is observed that $N = 1$ analysis can be ineffective even though transverse normal strains are retained in the formulation. This fact confirms statement S3.

In-plane and transverse normal strains are directly compared in Fig. 7 for the various cases. A comparison between the in-plane ϵ_{xx} and-out-of-plane ϵ_{zz} normal strains is given for thick and thin plates. Constant and linear $T_P(z)$ distributions are considered. The following should be noticed:

1) The ϵ_{xx} and ϵ_{zz} show the same order of magnitude for both thin and thick plates, as declared in statement S1.

2) The ϵ_{xx} as well as ϵ_{zz} barely change with a variation of the plate thickness. These mostly depend on α and T_0, which are independent of a/h.

The latter conclusion confirms that transverse normal strains remain significant even though thin plates are considered.

III. Multilayered Plates

A. Considered Plate Theories

The discontinuity of thermomechanical properties at the layer interfaces could make the plate theories discussed in the preceding section inadequate for the analysis of multilayered plates. These structures, in fact, exhibit a displacement field in the plate thickness direction that shows discontinuous derivatives at the layer interfaces. This is the so-called zig–zag effect. Furthermore, transverse shear and normal stress must be continuous at the interface for equilibrium reasons. Many contributions have been presented in open literature to introduce both interlaminar continuity (IC) and zig–zag (ZZ) effects in laminated plates. In a recent historical review,[6] these contributions were grouped as Lekhnitskii's multilayered theory,[28] Ambartsumian's multilayered theory,[29] and Reissner's multilayered theory.[30] Those ZZ theories that have been considered in this work are briefly discussed.

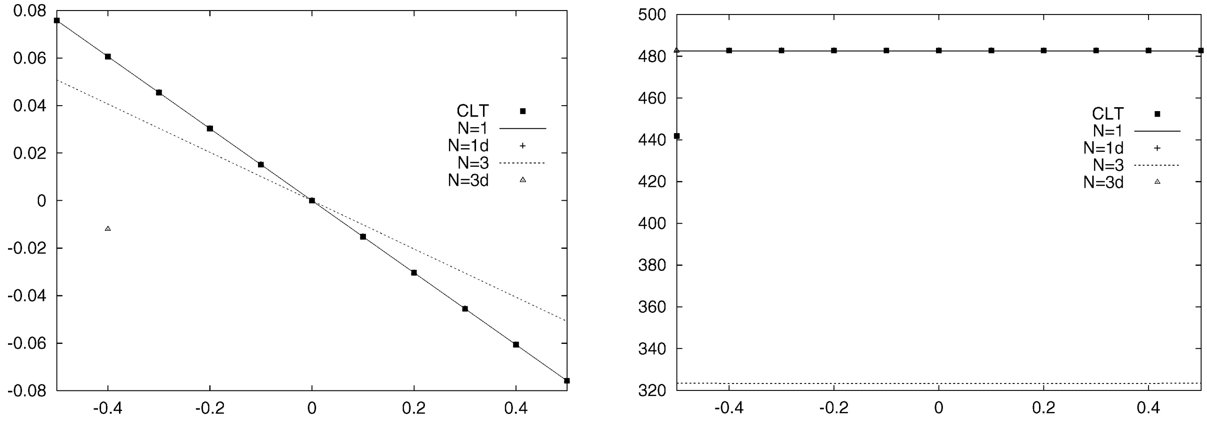
Incorporation of Zig–Zag Effects via MZZF

CLT, FSDT, as well as the HOT considered in the preceding section, are not able to describe the ZZ effect. The discontinuity of the first derivative at the layer interfaces can be incorporated by employing the Murakami zig–zag function. The MZZF was originally proposed in Ref. 31 in the framework of RMVT applications. An assessment on the use of MZZF in the modeling has recently been provided in Ref. 32.

According to Fig. 8, z_k is the layer thickness coordinate. The dimensionless layer coordinate $\zeta_k = z_k/2h_k$ (h_k is the thickness of the kth layer) is considered. MZZF was defined according to the following formula:

$$M(z) = (-1)^k \zeta_k \tag{26}$$

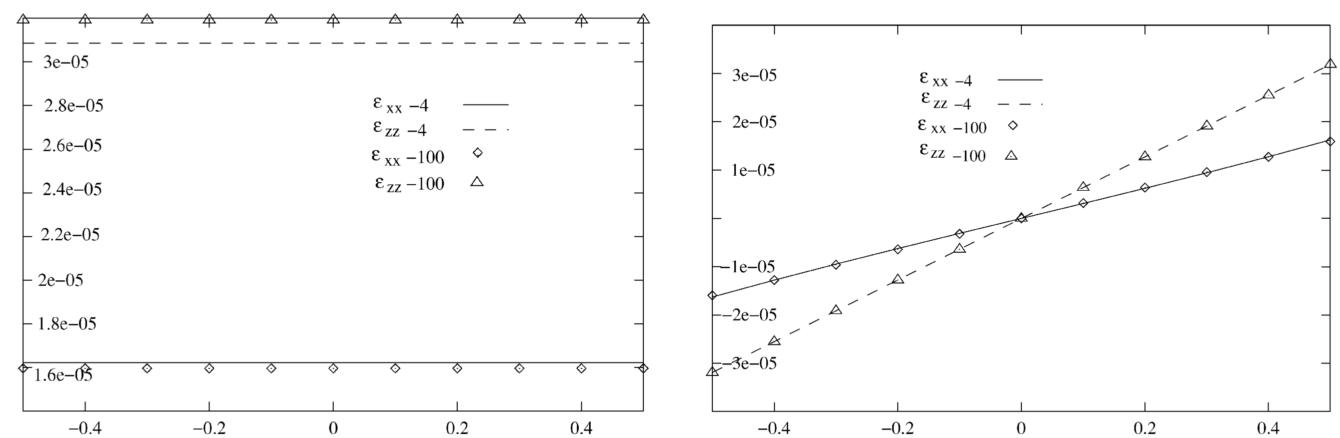
$M(z)$ has the following properties: it is a piecewise linear function of the layer coordinates z_k; $M(z)$ has unit amplitude for the whole layers; and the slope $M'(z) = \mathrm{d}M/\mathrm{d}z$ assumes opposite signs between two adjacent layers. (Its amplitude is layer thickness dependent.)



a) u_x (0, b/2, 0) × a/h

b) u_z (a/2, b/2, z) × $(a/h)^2$

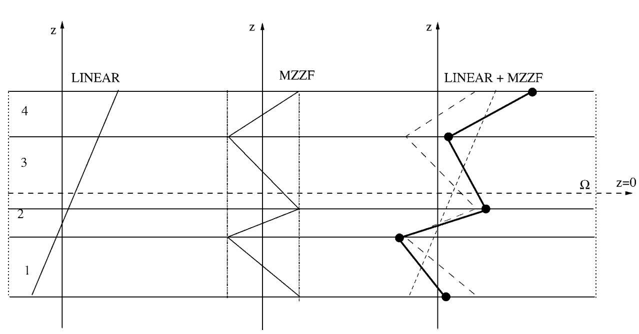
Fig. 6 Thin-plate case ($a/h = 100$) of Fig. 5.



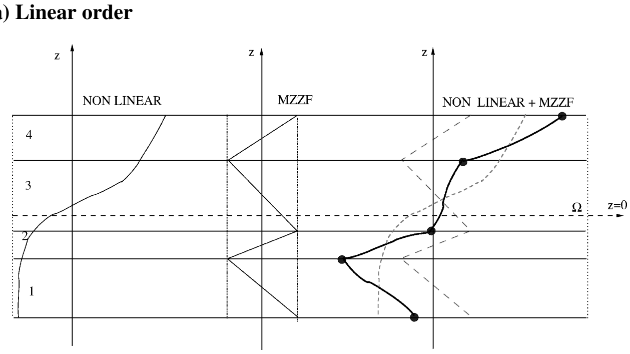
a) T_P^0 case

b) T_P^1 case

Fig. 7 Comparison between the in-plane $\epsilon_{xx}(0, b/2, 0)$ and out-of-plane $\epsilon_{zz}(a/2, b/2, 0)$ normal strains of a thin and thick plate. The results are related to the $N = 3$ analysis for both the T_P^0 and T_P^1 cases.



a) Linear order



b) Higher order

Fig. 8 Meaning of the Murakami zig–zag function.

The geometrical meaning of $M(z)$ is given in Fig. 8. The displacement field of Eq. (21) is enhanced to include the ZZ effect, as in the following:

$$\boldsymbol{u} = \boldsymbol{u}_0 + (-1)^k \zeta_k \boldsymbol{u}_Z + z^r \boldsymbol{u}_r, \qquad r = 1, 2, \ldots, N \tag{27}$$

The subscript $\boldsymbol{u}_z$ refers to the additional variables related to the zig–zag terms. Two cases have been represented in Fig. 8. The displacement model is written in a unified form as

$$\boldsymbol{u} = F_t \boldsymbol{u}_t + F_b \boldsymbol{u}_b + F_r \boldsymbol{u}_r + F_\tau \boldsymbol{u}_\tau, \qquad \tau = t, b, r$$
$$r = 1, 2, \ldots, N \tag{28}$$

The subscript b denotes values related to the plate reference surface $\Omega\,(\boldsymbol{u}_b = \boldsymbol{u}_0)$ while the subscript t refers to the included zig–zag term $(\boldsymbol{u}_t = \boldsymbol{u}_z)$. The F_τ functions therefore assume the following explicit forms:

$$F_b = 1, \qquad F_t = M(z), \qquad F_r = z^r, \qquad r = 1, 2, \ldots, N$$

These theories are denoted as ZZ-$N = 1, \ldots$ ZZ-$N = 3$, ZZ-$N = 1d, \ldots$, ZZ-$N = 3d$; these correspond to the $N = 1, \ldots, N = 3$ and $N1d, \ldots, N3d$ of Sec. II, respectively.

Incorporation of ZZ and IC via RMVT

A possible way of addressing the complete and a priori fulfillment of the zig–zag effect and interlaminar continuity is to refer to Reissner mixed variational theorem (RMVT) applications.[25,30]

According to RMVT statements, displacements $\boldsymbol{u}$ and transverse stresses $\boldsymbol{\sigma}_n = (\sigma_{xz}, \sigma_{yz}, \sigma_{zz})$ can be independently assumed. As discussed in Ref. 25, transverse stresses demand a Legendre layerwise expansion, whereas both Taylor and Legendre expansions could be used for the displacements unknowns. The case in which a Taylor expansion is used for displacement, whereas a Legendre expansion is employed for transverse stresses has here been considered. The zig–zag models in Eq. (26) are used for displacement variables. Transverse stresses instead take the following layerwise form:

$$\boldsymbol{\sigma}_{nM}^k = F_t\boldsymbol{\sigma}_{nt}^k + F_b\boldsymbol{\sigma}_{nb}^k + F_r\boldsymbol{\sigma}_{nr}^k = F_\tau\boldsymbol{\sigma}_{n\tau}^k, \qquad k = 1, 2, \ldots, N_l \tag{29}$$

In contrast to Eq. (28), the subscripts t and b here denote values related to the layer top and bottom surface, respectively. They consist of the linear part of the expansion. The thickness functions $F_\tau(\zeta_k)$ have been defined by

$$F_t = (P_0 + P_1)/2, \qquad F_b = (P_0 - P_1)/2, \qquad F_r = P_r - P_{r-2} \qquad r = 2, 3, \ldots, N \tag{30}$$

in which $P_j = P_j(\zeta_k)$ is the Legendre polynomial of the j order defined in the ζ_k domain: $-1 \le \zeta_k \le 1$. Linear up-to-fourth-order expansions are used in the numerical investigations; the related polynomials are

$$P_0 = 1, \qquad P_1 = \zeta_k, \qquad P_2 = \left(3\zeta_k^2 - 1\right)/2$$

$$P_3 = 5\zeta_k^3/2 - 3\zeta_k/2, \qquad P_4 = 35\zeta_k^4/8 - 15\zeta_k^2/4 + \tfrac{3}{8}$$

The chosen functions have the following properties:

$$\zeta k = \begin{cases} 1 & : \; F_t = 1; \quad F_b = 0; \quad F_r = 0 \\ -1 & : \; F_t = 0; \quad F_b = 1; \quad F_r = 0 \end{cases} \tag{31}$$

The top and bottom values have been used as unknown variables. The interlaminar transverse shear and normal stress continuity can therefore be easily linked:

$$\boldsymbol{\sigma}_{nt}^k = \boldsymbol{\sigma}_{nb}^{(k+1)}, \qquad k = 1, \qquad N_l - 1 \tag{32}$$

In those cases in which top/bottom-plate stress values are prescribed (zero or imposed values), the following additional equilibrium conditions must be accounted for:

$$\boldsymbol{\sigma}_{nb}^1 = \bar{\boldsymbol{\sigma}}_{nb}, \qquad \boldsymbol{\sigma}_{nt}^{N_l} = \bar{\boldsymbol{\sigma}}_{nt} \tag{33}$$

where the overbar is the imposed values in correspondence to the plate boundary surfaces. (These have been assumed zero in what follows.)

Layerwise Mixed Theories LM4

A reference three-dimensional solution is not available in most of the analysis conducted in this paper. A previous work by the author[25] has shown that a layerwise mixed theory with four-order expansion in each layer, which was denoted as LM4, furnishes quasi-three-dimensional solutions for thermal stress and strain analysis of laminated structures. This theory uses a Legendre-based expansion for both the transverse stress and displacement variables,

$$\boldsymbol{u}^k = F_t\boldsymbol{u}_t^k + F_b\boldsymbol{u}_b^k + F_r\boldsymbol{u}_r^k = F_\tau\boldsymbol{u}_\tau^k, \qquad \tau = t, b, r, \qquad r = 2, 3, 4$$

$$\boldsymbol{\sigma}_{nM}^k = F_t\boldsymbol{\sigma}_{nt}^k + F_b\boldsymbol{\sigma}_{nb}^k + F_r\boldsymbol{\sigma}_{nr}^k = F_\tau\boldsymbol{\sigma}_{nT}^k, \qquad k = 1, 2, \ldots, N_l \tag{34}$$

LM4 results are used in this paper as the reference solution.

Table 2 Transverse normal strain effects on the displacement and transverse shear stress of a multilayered plate made of two layers of isotropic material [analysis of the order of expansion on the accuracy of the considered theories: the square-plate problem; thick-plate results ($a/h = 4$) related to the T_P^1 case]

	$u_x(0, b/2, 0) \times a/h$		$\sigma_{xz}(0, b/2, h/4) \times (a/h)^2$	
Theory	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$
Three-dimensional LM4	$0.1103E-4$	——	0.1782	——
Classical theories				
CLT	——	$0.1854E-4$	——	0.3853
$N = 1$	$0.1543E-4$	$0.1854E-4$	0.3847	0.3853
$N = 2$	$0.1069E-4$	$0.1768E-4$	0.3348	0.3583
$N = 3$	$0.1127E-4$	$0.1767E-4$	0.1927	0.3583
$N = 4$	$0.1101E-4$	$0.1698E-4$	0.1882	0.3414
Classical theories including MZZF				
$N = 1$	$0.1047E-4$	$0.1682E-4$	0.2355	0.3495
$N = 2$	$0.9673E-5$	$0.1682E-4$	0.3135	0.3495
$N = 3$	$0.1087E-4$	$0.1681E-4$	0.1886	0.3492
Advanced theories including ZZ and IC				
$N = 1$	$0.9889E-5$	$0.1653E-4$	0.1871	0.3447
$N = 2$	$0.9407E-5$	$0.1653E-4$	0.3085	0.3442
$N = 3$	$0.1080E-4$	$0.1671E-4$	0.1889	0.3475

B. Results

Multilayered Plates Made of Two Layers of Different Isotropic Materials

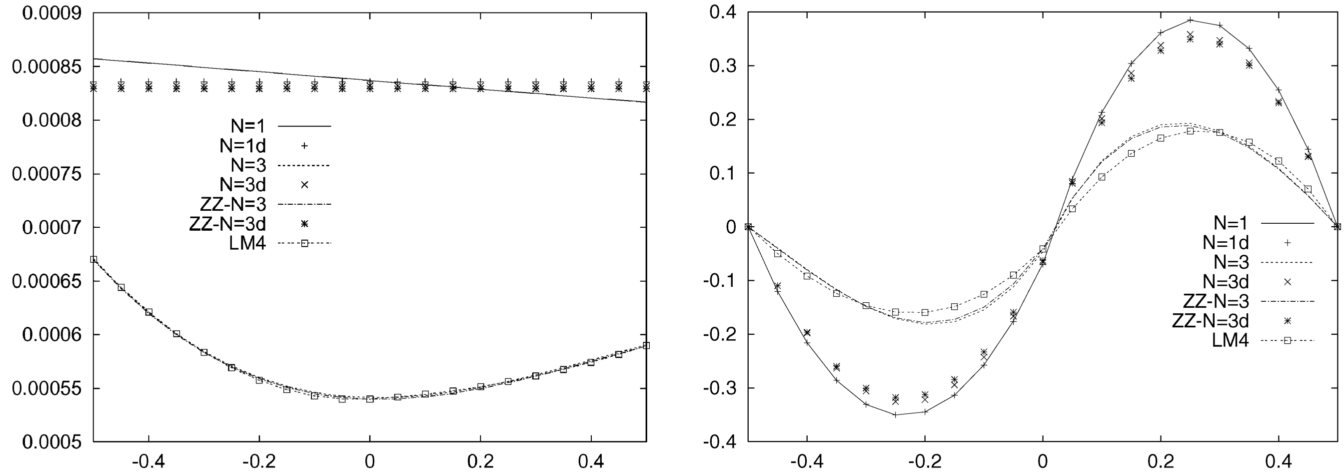
The plate is constituted by two layers (with equal thickness) of different isotropic materials. The first material is the same as Sec. II.B, the second one is a titanium MIL-T-9047 whose thermomechanical data are taken from Ref. 7: $E = 107{,}000$ MPa, $\nu = 0.32$, and $\alpha = 9.4.E$–6 T/K.

A thick plate has been considered in Table 2. In-plane displacements and transverse shear stress are considered. Attention has been restricted to the T_P^1-case. A quasi-three-dimensional solution, obtained from the layerwise mixed theory with fourth-order expansion (LM4), is quoted for comparison purposes. The transverse normal strain effect is evaluated for the whole multilayered plate modeling that was described earlier. The following comments can be made:

1) More benefits are obtained by increasing the order N with respect to the one layered plate of Sec. II for both stress and displacement evaluations. However, these benefits are not effective if ϵ_{zz} is discarded in the calculations.

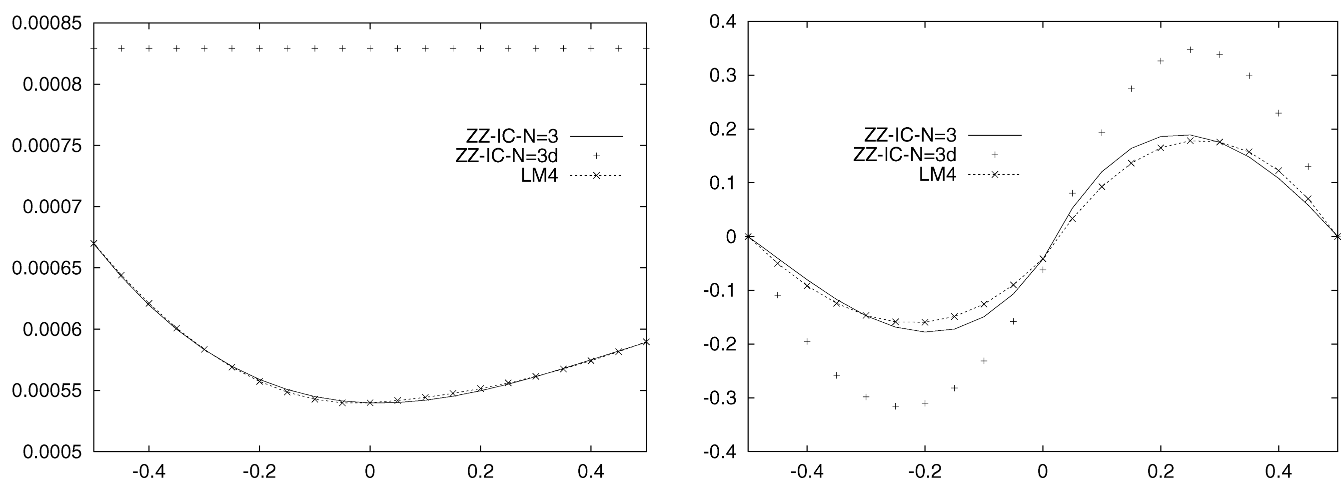
2) Refined and advanced mixed theories improve the results related to classical and HOT theories. N increasing the LM4 results are approached. However, advanced mixed theories that discard ϵ_{zz} can lead to poorer results than classical plate theories which account for ϵ_{zz}. For instance, the value $\bar{\sigma}_{zz} = 0.3475$ is obtained if the advanced mixed theory corresponding to $N = 3$ is employed and ϵ_{zz} is discarded, while the value $\bar{\sigma}_{zz} = 0.1882$ (which is much closer to the LM4 value $\bar{\sigma}_{xz} = 0.1782$) is obtained if a HOT corresponding to $N = 3$ and including ϵ_{zz} is employed. This fact clearly confirms KR for thermal stress analysis of laminated plates.

Through-the-thickness distribution of displacements and stresses related to the various plate theories are compared in Figs. 9 and 10. The suffix ZZ denotes an improved HOT that includes MZZF, whereas the suffix ZZ-IC denotes advanced mixed theories that describe both the ZZ effect and fulfill interlaminar continuity of both the transverse shear and normal stresses. Asymmetrical distributions are caused by asymmetrical layouts. The $N = 1$ analysis is ineffective even though ϵ_{zz} is accounted for. This is because of the linear form of T_p^1, as stated in S2. Third-order theories accounting for ϵ_{zz} lead to a quite accurate description of stress and displacements fields. The best results are those obtained using the advanced mixed theories. Larger differences among the different theories should be registered for stress evaluations with respect to the displacement ones. It has been confirmed that very inaccurate results are obtained in both cases of classic and refined theories when transverse normal strains are discarded.



a) u_z $(0, b/2, z) \times (a/h)^2$ b) σ_{xz} $(a/2, b/2, z) \times (a/h)^2$

Fig. 9 Table 2 problem: analysis of the ϵ_{zz} effect on the accuracy of classical and refined theories with only displacement unknowns.



a) u_z $(a/2, b/2, z) \times (a/h)^2$ b) σ_{xz} $(a/2, b/2, z) \times (a/h)^2$

Fig. 10 Table 2 problem: analysis of the ϵ_{zz} effect on the accuracy of advanced theories describing both ZZ and IC.

Multilayered Plates Constituted by Composite Materials

The multilayered plates used by Bhaskar et al.[12] are considered in this section. The thermomechanical properties of the lamina are[12]

$$E_L/E_T = 25, \qquad G_{LT}/E_T = 0.5, \qquad G_{TT}/E_T = 0.2$$

$$\nu_{LT} = \nu_{TT} = 0.25, \qquad \alpha_T/\alpha_L = 1125$$

$$K_L = 36.42 \text{ W/mC}^{-1}, \qquad K_T = 0.96 \text{ W/mC}^{-1}$$

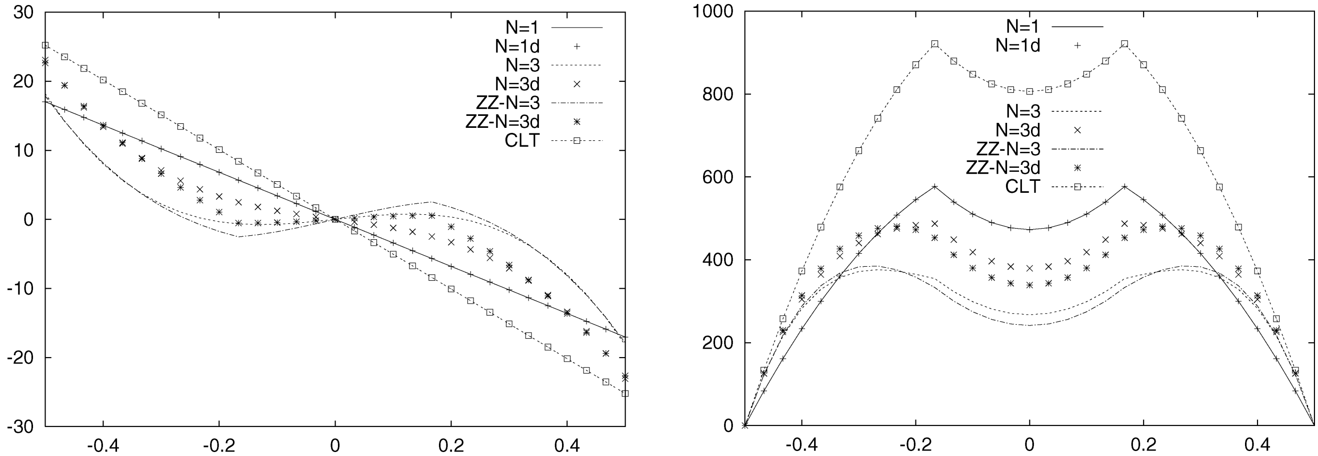
where L and T refer to directions parallel and perpendicular, respectively, to the fibers (see Ref. 27 for notations). K_L and K_T are the thermal conductivity coefficients that were used in Refs. 8 and 9 to compute the temperature profile that is considered at the end of this paragraph. A cross-ply, symmetrically laminated case 0/90/0 was considered for all of the treated problems. The deflection and stresses are presented in terms of the following dimensionless parameters:

$$\bar{u}_x = \frac{u_x(0, a/2, h/2)}{(\alpha_L T_0 a)}, \qquad \bar{u}_z = \frac{u_z(0, a/2, h/2)}{\left(\alpha_L T_0 a^2\right)}$$

$$\bar{\sigma}_{xz} = \frac{\sigma_{xz}(0, a/2, h/6)}{(E_T \alpha_L T_0)}$$

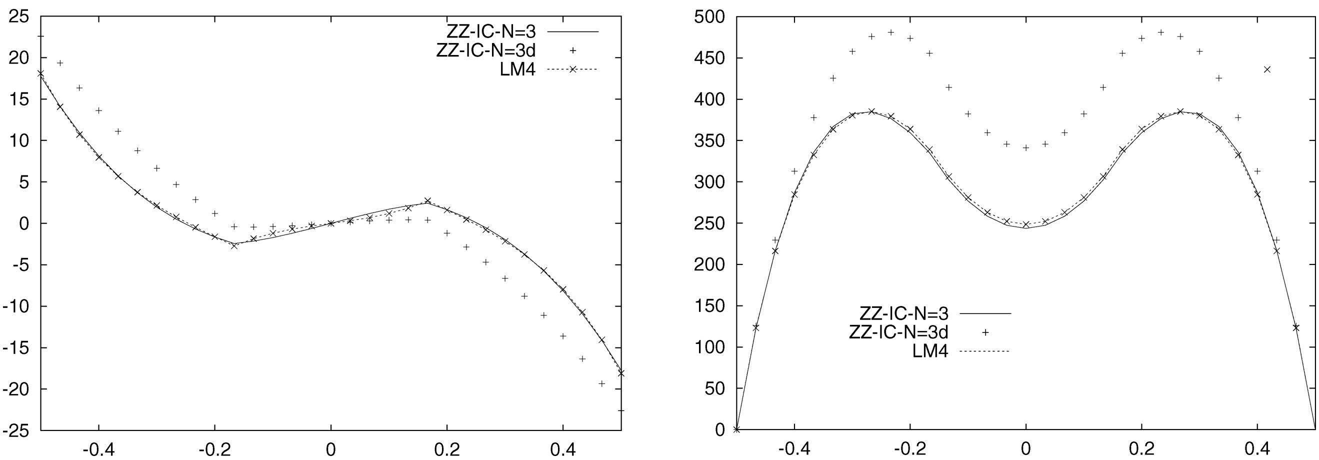
The theories are compared in Tables 3 and 4 for thick and thin plates, respectively. The inaccuracy of plate theories that discard ϵ_{zz} is again confirmed for the analysis of both thick and thin plates. The displacement and stress distributions are compared in Figs. 11 and 12. The case of linear T_P is considered in these latter figures, whereas the case of constant temperature profile T_P^0 is considered in Fig. 13. This last case was not addressed in Ref. 12. The higher transverse shear and normal flexibility of the lamina lead to larger differences among the different theories compared to the isotropic materials that were considered in the preceding analysis.

However as discussed in the Introduction, the temperature profile is a result of a heat-conduction problem.[8] As show in Fig. 14, which is taken from Ref. 9, the calculated temperature profile T_P^{1c} can differ considerably from the assumed linear T_P^1 ones. The two profiles merge if, and only if, thin plates are analyzed. Large differences instead arise in thick ($a/h = 4$) and very thick ($a/h = 2$) plate geometries. The transverse normal strain effects for the two T_P^{1c} and T_P^{1c} cases have also been evaluated in Fig. 15. Transverse shear-stress values were compared. Attention has been restricted to classical theories with a cubic displacement field ($N = 3$). The $N3$ results referring to T_P^{1c} should be considered the most realistic analysis. Lower stresses are obtained for this case. The fundamental role of ϵ_{zz} is also confirmed for this problem. Transverse displacement evaluations of various classical theories are given in Table 5. It can be concluded that the accurate evaluation of



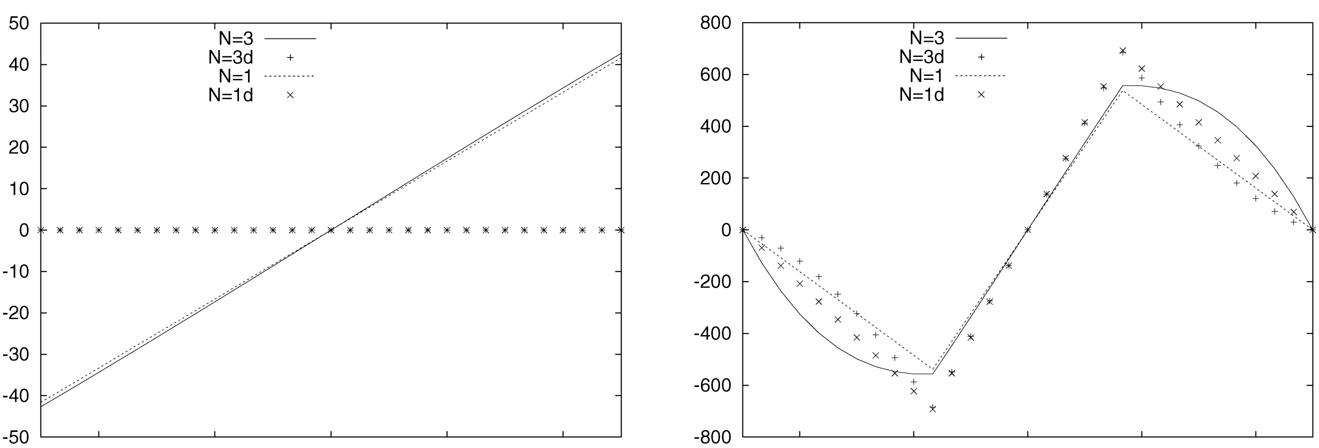
a) $\bar{u}_x$ b) $\bar{\sigma}_{xz}$

Fig. 11 Multilayered composite plates: effects of ϵ_{zz} on the displacement and stress of classical and refined theories; thick ($a/h = 4$) plate for the T_P^1 case.



a) $\bar{u}_x$ b) $\bar{\sigma}_{xz}$

Fig. 12 As in Fig. 11 but for the case of advanced mixed theories.



a) $\bar{u}_z$ b) $\bar{\sigma}_{xz}$

Fig. 13 Multilayered composite plates: effects of ϵ_{zz} on the displacement and stress of classical and refined theories; thick ($a/h = 4$) plate for the T_P^0 case.

Table 3 Multilayered composite plates: effects of ϵ_{zz} on the various theories; thick plate ($a/h = 4$); T_P^1 case

	$\bar{u}_x$		$\bar{\sigma}_{xz}$	
Theory	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$
Three dimensional[12]	18.11	——	84.81	——
Three-dimensional LM4	18.11	——	84.81	——
Classical theories				
CLT	——	25.22	——	230.33
$N = 1$	17.03	17.03	144.23	144.23
$N = 2$	10.35	17.03	116.24	144.23
$N = 3$	18.19	23.08	88.62	121.84
$N = 4$	18.20	23.08	88.64	121.84
Classical theories including MZZF				
$N = 1$	21.36	21.36	118.60	118.60
$N = 2$	14.92	21.36	89.09	118.60
$N = 3$	17.87	22.65	83.27	113.30
Advanced theories including ZZ and IC				
$N = 1$	20.76	20.76	116.28	116.28
$N = 2$	14.53	20.09	89.04	118.54
$N = 3$	17.75	22.57	83.81	113.90

Table 4 Thin-plate case ($a/h = 100$)

	$\bar{u}_x$		$\bar{\sigma}_{xz}$	
Theory	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$
Three dimensional[12]	16.00	——	7.073	——
Three-dimensional LM4	16.00	——	7.073	——
Classical theories				
CLT	——	25.22	——	9.213
$N = 1$	25.20	25.20	9.204	9.204
$N = 2$	15.98	25.20	7.079	9.204
$N = 3$	15.60	25.21	7.075	9.20
$N = 4$	15.60	25.22	7.075	9.20
Classical theories including MZZF				
$N = 1$	25.21	25.21	9.20	9.20
$N = 2$	15.99	25.21	7.074	9.20
$N = 3$	16.00	25.22	7.073	9.20
Classical theories including MZZF				
$N = 1$	25.21	25.21	9.20	9.20
$N = 2$	15.60	25.21	7.074	9.20
$N = 3$	15.60	25.22	7.073	9.20

Table 5 Multilayered plate problems: effect of ϵ_{zz} on the transverse displacement $\bar{u}_z$ of various theories and plate thickness for the two cases of the assigned T_P^1 and calculated T_P^{1c} temperature profiles

	$N = 1$		$N = 2$		$N = 3$		
a/h	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$	$\epsilon_{zz} \neq 0$	$\epsilon_{zz} = 0$	CLT
Case of T_P^{1c} assigned							
2	98.215	50.544	98.150	50.544	83.471	42.714	42.714
4	42.048	35.830	42.040	35.830	34.740	30.417	30.417
10	16.901	21.181	16.901	21.181	10.958	19.452	19.451
100	10.253	16.114	10.253	16.114	10.230	16.093	16.093
Case of T_P^1 calculated							
2	26.087	3.7680	25.786	13.754	6.223	3.184	3.184
4	5.8251	15.388	29.333	25.079	14.920	13.063	13.063
10	13.871	18.166	15.906	19.926	12.828	16.682	16.682
100	10.237	16.114	10.246	16.104	10.214	16.068	16.068

a temperature profile can be meaningless unless transverse normal strains are discarded in the formulation and vice versa. As far as this is concerned, the following should be remarked: although the differences between the results related to calculated and assigned temperature profiles disappear in thin plate analysis, the differences between the results discarding and including ϵ_{zz} preserve the same order of magnitude for thin and thick plates.

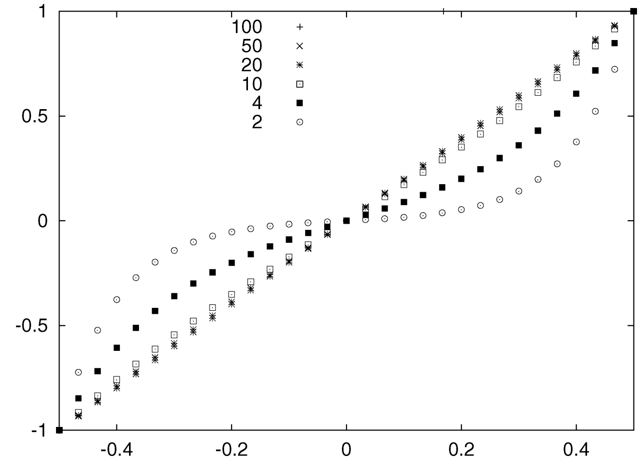


Fig. 14 Multilayered composite plates: calculated temperature profile for the various thickness ratios.

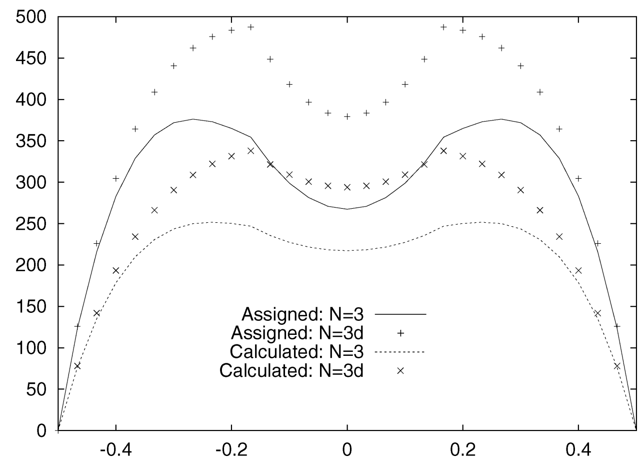


Fig. 15 Multilayered composite plates: effects of ϵ_{zz} on $\bar{\sigma}_{xz}$ as a result of the calculated and assigned temperature profiles; thick plate ($a/h = 4$), $N = 3$ case.

IV. Conclusions

The paper has analyzed the effect of transverse normal strain in homogeneous and multilayered plates. Classical, refined, and advanced mixed theories have been considered. Three sample problems have been investigated: a one-layered plate consisting of isotropic materials, a two-layered plate made of two different isotropic materials, and a three-layered cross-ply laminated composites plate. It has been confirmed that any refinements of classical models are meaningless, in general, unless the effects of transverse shear and normal strains are both taken into account in a plate theory. Furthermore, 1) transverse normal strains cannot be discarded even though thin plates are considered, and 2) enhancements direct toward improving a description of temperature profile in the plate thickness direction are meaningless, unless transverse normal strains are taken into account in homogeneous and multilayered plates and vice versa.

The present work has only directed toward pure thermal stress evaluations. In practical applications, panels are subjected to combined thermomechanical loadings. For those cases in which most of the stresses and strains are as a result of the mechanical fields, the conclusions already mentioned could be omitted. For those cases in which thermal stresses play a predominant role, the conclusions already mentioned should be taken into account.

References

[1] Koiter, W. T., "A Consistent First Approximations in the General Theory of Thin Elastic Shells," *Proceedings of Symposium on the Theory of Thin Elastic Shells*, North-Holland, Amsterdam, 1959, pp. 12–23.

[2]Goldenvaizer, A. N., *Theory of Thin Elastic Shells*, International Series of Monograph in Aeronautics and Astronautics, Pergamon, New York, 1961.

[3]Cicala, P., *Systematic Approach to Linear Shell Theory*, Levrotto and Bella, Turin, Italy, 1965.

[4]Carrera, E., "Transverse Normal Stress Effects in Multilayered Plates," *Journal of Applied Mechanics*, Vol. 66, 1999, pp. 1004–1012.

[5]Carrera, E., "A Study of Transverse Normal Stress Effects on Vibration of Multilayered Plates and Shells," *Journal of Sound and Vibration*, Vol. 225, 1999, pp. 803–829.

[6]Carrera, E., "Historical Review of Zig-Zag Theories for Multilayered Plates and Shells," *Applied Mechanics Review*, Vol. 56, 2003, pp. 287–308.

[7]Thornton, E. A., *Thermal Structures for Aerospace Applications*, AIAA Education Series, AIAA, Reston, VA, 1996.

[8]Tungikar, V. B., and Rao, K. M., "Three Dimensional Exact Solution of Thermal Stresses in Rectangular Composite Laminates," *Composite Structures*, Vol. 27, 1994, pp. 419–427.

[9]Carrera, E., "Temperature Profile Influence on Layered Plates Response Considering Classical and Advanced Theories," *AIAA Journal*, Vol. 40, 2002, pp. 1885–1896.

[10]Bapu Rao, M. N., "3D Analysis of Thermally Loaded Thick Plates," *Nuclear Engineering and Design*, Vol. 55, 1979, pp. 353–361.

[11]Murakami, H., "Assessment of Plate Theories for Treating the Thermo-Mechanical Response of Layered Plates," *Composite Engineering*, Vol. 3, No. 2, 1993, pp. 137–149.

[12]Bhaskar, K., Varadan, T. K., and Ali, J. S. M., "Thermoelastic Solution for Orthotropic and Anisotropic Composites Laminates," *Composites Part B*, Vol. 27B, 1996, pp. 415–420.

[13]Savoia, M., and Reddy, J. N. K., "Three-Dimensional Thermal Analysis of Laminated Composite Plates," *Journal of Thermal Stress*, Vol. 32, 1995, pp. 593–608.

[14]Hildebrand, F. B., Reissner, E., and Thomas, G. B., "Notes on the Foundations of the Theory of Small Displacement of Orthotropic Shells," NACA TN-1833, 1940.

[15]Jonnalagadda, K. D., Tauchert, T. R., and Blandford, G. E., "Higher Order Thermoelastic Composite Plates Theories: an Analytical Comparison," *Journal of Thermal Stress*, Vol. 12, 1993, pp. 321–332.

[16]Cho, K. N., Striz, A. G., and Bert, C. W., "Thermal Stress Analyses of Laminate Using Higher Order Theory in Each Layer," *Journal of Thermal Stress*, Vol. 12, 1989, pp. 321–332.

[17]Ali, J. S. M., Bhaskar, K., and Varadan, T. K., "A New Theory for Accurate Thermal/Mechanical Flexural Analysis of Symmetric Laminated Plates," *Composite Structures*, Vol. 45, 1996, pp. 227–232.

[18]Gu, H., Chattopadhyay, A., Li, J., and Zhou, X., "Higher Order Temperature Theory for Coupled Thermo-Piezoelectric-Mechanical Modeling of Smart Composites," *International Journal of Solids and Structures*, Vol. 37, 2000, pp. 6479–6497.

[19]Zhou, X., Chattopadhyay, A., and Gu, H., "Dynamic Response of Smart Composites Using a Coupled Thermo-Piezoelectric-Mechanical Model," *AIAA Journal*, Vol. 38, 2000, pp. 1939–1948.

[20]Tauchert, T. R., "Thermally Induced Flexure, Buckling and Vibration of Plates," *Applied Mechanics Review*, Vol. 44, No. 8, 1991, pp. 347–360.

[21]Noor, A. K., and Burton, W. S., "Computational Models for High-Temperature Multilayered Composite Plates and Shells," *Applied Mechanics Review*, Vol. 45, No. 10, 1992, pp. 419–446.

[22]Argyris, J., and Tenek, L., "Recent Advances in Computational Thermo-Structural Analysis of Composite Plates and Shells with Strong Nonlinearities," *Applied Mechanics Review*, Vol. 50, No. 5, 1998, pp. 285–306.

[23]Carrera, E., "An Assessment of Mixed and Classical Theories for the Thermal Stress Analysis of Orthotropic Plate Multilayered Plates," *Journal of Thermal Stress,* Vol. 23, 2000, pp. 797–831.

[24]Carrera, E., and Ciuffreda, A., "Closed Form Solutions to Assess Multilayered Plate Theories for Various Thermal Stress Problems," *Journal of Thermal Stress*, Vol. 27, 2004, pp. 1001–1031.

[25]Carrera, E., "Developments, Ideas and Evaluations Based upon the Reissner's Mixed Variational Theorem in the Modeling of Multilayered Plates and Shells," *Applied Mechanics Review*, Vol. 54, 2001, pp. 301–329.

[26]Carrera, E., "Theories and Finite Elements for Multilayered Plates and Shells: A Unified Compact Formulation with Numerical Assessment and Benchmarking," *Archives of Computational Methods in Engineering. State of the Art Reviews*, Vol. 10, 2003, pp. 215–296.

[27]Reddy, J. N., *Mechanics of Laminated Composite Plates, Theory and Analysis*, CRC Press, 1997.

[28]Lekhnitskii, S. G., "Strength Calculation of Composite Beams," *Vestnik Inzhen. i Tekhnikov*, No. 9, 1935.

[29]Ambartsumian, S. A., "On a Theory of Bending of Anisotropic Plates," *Investiia Akademii Nauk SSSR*, No. 4, 1958.

[30]Reissner, E., "On a Certain Mixed Variational Theory and a Proposed Applications," *International Journal of Numerical Methods in Engineering,* Vol. 20, 1984, pp. 1366–1368.

[31]Murakami, H., "Laminated Composite Plate Theory with Improved In-Plane Response," *Journal of Applied Mechanics*, Vol. 53, 1986, pp. 661–666.

[32]Carrera, E., "On the Use of Murakami's Zig–Zag Function in the Modeling of Layered Plates and Shell," *Computer and Structures*, Vol. 82, 2004, pp. 541–554.

D. Mook
Associate Editor