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Theories and Finite Elements for Multilayered, Anisotropic, Composite Plates and Shells

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Summary

This work is an overview of available theories and finite elements that have been developed for multilayered, anisotropic, composite plate and shell structures. Although a comprehensive description of several techniques and approaches is given, most of this paper has been devoted to the so called axiomatic theories and related finite element implementations. Most of the theories and finite elements that have been proposed over the last thirty years are in fact based on these types of approaches.
The paper has been divided into three parts.
Part I, has been devoted to the description of possible approaches to plate and shell structures: 3D approaches, continuum based methods, axiomatic and asymptotic two-dimensional theories, classical and mixed formulations, equivalent single layer and layer wise variable descriptions are considered (the number of the unknown variables is considered to be independent of the number of the constitutive layers in the equivalent single layer case). Complicating effects that have been introduced by anisotropic behavior and layered constructions, such as high transverse deformabilty, zig-zag effects and interlaminar continuity, have been discussed and summarized by the acronim C_z^0–Requirements.
Two–dimensional theories have been dealt with in Part II. Contributions based on axiomatic, asymtotic and continuum based approches have been overviewed. Classical theories and their refinements are first considered. Both case of equivalent single-layer and layer-wise variables descriptions are discussed. The so-called zig-zag theories are then discussed. A complete and detailed overview has been conducted for this type of theory which relies on an approach that is entirely originated and devoted to layered constructions. Formulas and contributions related to the three possible zig-zag approaches, i.e. Lekhnitskii–Ren , Ambartsumian–Whitney–Rath-Das , Reissner–Murakami-Carrera ones have been presented and overviewed, taking into account the findings of a recent historical note provided by the author.
Finite Element FE implementations are examined in Part III. The possible developments of finite elements for layered plates and shells are first outlined. FEs based on the theories considered in Part II are discussed along with those approaches which consist of a specific application of finite element techniques, such as hybrid methods and so-called global/local techniques. The extension of finite elements that were originally developed for isotropic one layered structures to multilayerd plates and shells are first discussed. Works based on classical and refined theories as well as on equivalent single layer and layer-wise descriptions have been overviewed. Development of available zig-zag finite elements has been considered for the three cases of zig-zag theories. Finite elements based on other approches are also discussed. Among these, FEs based on asymtotic theories, degenerate continuum approaches, stress resultant methods, asymtotic methods, hierarchy–p, –s global/local techniques as well as mixed and hybrid formulations have been overviewed.

List of Symbols and Acronims

Symbols and acronyms that are used frequently in places distant from their definition are listed below.

Symbols

a, b, h, plate/shell geometrical parameters (length, width and thickness).

k sub/super-script used to denote parameters related to the k-layer.

N, order of the expansions used for transverse stresses and displacements

N_l, Number of constituent layers of multilayered plate/shell

x, y, z, coordinates of Cartesian reference systems used for plates.

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α, β, z, curvilinear coordinates of reference systems used for shells.

Acronyms

2D, two-Dimensional.

3D, three-Dimensional.

AWRD, Ambartsumian–Whitney–Rath-Das theory.

CLT, Classical lamination Theory.

ESLM, Equivalent Single Layer Models.

FEs, Finite Elements.

FEM, Finite Element Method.

FSDT, First Shear Deformation Theory.

HOT, Higher Order Theories.

HTD, High Transverse Deformability.

IC, Interlaminar Continuity.

KR, Koiter's Recommendation.

LR, Lekhnitskii–Ren theory.

LFAT, Love First Approximation Theory

LSAT, Love Second Approximation Theory

LWM, Layer-Wise Models

RMC, Reissner–Murakami-Carrera theory.

RMVT, Reissner's Mixed Variational Theorem

TA, Transverse Anisotropy.

VRT, Vlasov-Reddy Theory.

WFHL, Weak Form of Hooke's Law.

ZZ, Zig-Zag.

1 INTRODUCTION AND CONTENTS OF THIS REVIEW

Layered structures are increasingly used in aerospace, automotive and ship vehicles. The most common and best known examples of multilayered structures are sandwich panels. These consist of a three layered structure made of a soft core (the central layer), and two external stiff layers (skins or faces), see Figure 1. Many other layered materials have been introduced. The so-called advanced composite materials were developed as part of aerospace vehicles during the second part of the last century. Nowadays there are examples of fighter and commercial aircrafts, helicopters and gliders whose structures are entirely made of composite materials. Composite constructions are multilayered made structures (mostly made of flat and curved panels) constituted by several layers or laminae, which are perfectly bonded together. Each lamina is composite of fibers embedded in a matrix. These fibers are produced according to a specific technology process which confers high mechanical properties in the longitudinal direction of the fibers. The matrix mainly has the role of holding the fibers together. Carbon, boron and glass fibers are used above all along with organic products. The matrices are mostly of an epoxy type. Several possible ways of putting the fibers and matrix together can be considered. Uni-directional lamina (Figure 2) are used in most applications related to the construction of aerospace, automotive or ship vehicles. The lamina are placed one over the other, according to a given lay-out. Such a possibility, which is known as 'tailoring', permits one to optimize the use of the material for a given set of design requirements. For instance, the so-called 'aeroelastic tailoring' has permitted the building of stable swept forward wings of supersonic fighter aircrafts.

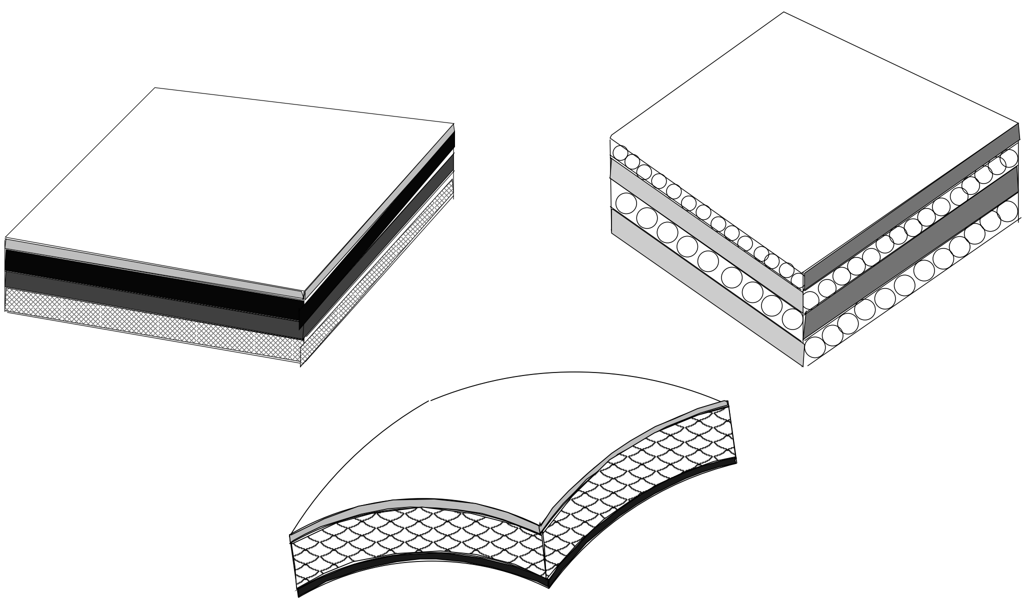

Figure 1. Examples of multilayer structures. Plates (upper part) made of layers of different materials (left) and by unidirectional fibers (right) and sandwich shell (upper part)

Other examples of layered structures are: thermal protected structures in which layers with high thermal properties are used as thermal skins; biomedical retina; advanced optical mirrors; and semiconductor technologies. A more recent example of layered structures i that of intelligent structures that embed piezo-layers and which are used as sensors and/or actuators to build a closed loop controlled 'smart' structure.

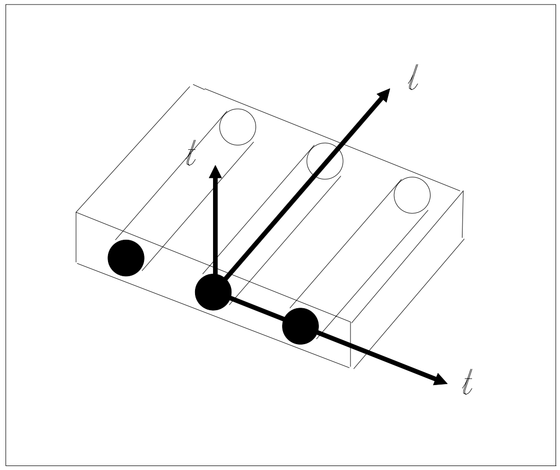

Figure 2. Unidirectional lamina

The analysis, design and construction of layered structures is of a cumbersome subject. So many different, complicated and new problems arise to add to those that are already known for traditional one layered, isotropic structures (herein denoted as 'monocoque' structures). Processing and manufacturing, characterization and properties, mechanical behavior, joints, damage and repair, fatigue etc, are some examples of these topics. A review which gives a description of these topics has recently been provided by Bogdanovich and Sierakowsky [1]. A compendium of available books, review papers, and other sources of information are listed in [1].

Of all the possible topics, this review covers those aspects related to two-dimensional modeling of layered structures and related finite element implementations. It could be considered as a sequel to the preview review articles. Among these, review and assessment various papers [2]-[20] are mentioned. Books [21]–[28] are additional recommended sources for a better understanding of the fundamentals and applications related to layered plate and shell structures.
The paper has been divided into three parts.

Part I has been devoted to the description of both possible approaches to plate and shell structures and the complicating effects that have been introduced by anisotropic behavior and layered constructions, such as high transverse deformabilty, zig-zag effects and interlaminar continuity.

Two–dimensional theories have been dealt with in Part II. Classical theories and their refinements are first considered. Both the case of equivalent single-layer and layer-wise variable descriptions are discussed. Zig-zag theories are then addressed. A comprehensive and detailed overview has been conducted for this type of theory which relies on an approach that is entirely originated and devoted to layered constructions. By referring to findings of a recent historical note which has been provided by the author [20], three possible approaches to the zig-zag theory are discussed and explained with the help of formulas and figures.

Finite Element, FE, implementations have been considered in Part III. The extensions of finite elements that were originally developed for isotropic one-layered structures to multilayerd plates and shells are first discussed. Works based on classical and refined theories, as well as on equivalent single-layer and layer-wise descriptions, have been overviewed. FE developments of zig-zag theories have been considered. Finite elements based on other approaches are also discussed. Among these, FEs based on asymptotic theories, degenerate continuum approaches, stress resultant methods, asymptotic methods, hierarchy–p, –s global/local techniques as well as on mixed and hybrid formulations have been overviewed.

As far as numerical aspects is concerned, it is the author's opinion that FE implementations of multilayered models do not introduce any complicating numerical problems which were not already known for monocoque plate/shell elements. The present overview therefore does not deal with numerical FE mechanisms nor with strategies or techniques known to overcome them. Those readers who are interested can refer to the recent overview paper [18] which has been devoted above all to these topics.

Furthermore, this paper does not report numerical results in the form of tables and figures. A companion paper, which has been scheduled for the near future, will give a unified formulation for layered plate and shell theories and finite elements along with a numerical assessment and benchmarking.

Part I General Aspects

2 POSSIBLE DEVELOPMENTS OF PLATE/SHELL THEORIES

2.1 The 3D Problem

The first, obvious approach to plates and shells can be implemented by solving, in a strong or in a weak sense, the fundamental differential equations of three-dimensional 3D elasticity. These can be summarized in the following compact forms,

- Equilibrium equations,

$$\boldsymbol{E}_{\Sigma}(\boldsymbol{\sigma}, \ddot{\boldsymbol{u}}, \boldsymbol{p}) = 0, \quad \boldsymbol{E}_{\Gamma}(\boldsymbol{\sigma}, \boldsymbol{p}) = 0 \tag{1}$$

- Compatibility equations,

$$\boldsymbol{C}_{\Sigma}(\boldsymbol{u}, \boldsymbol{\epsilon}) = 0, \quad \boldsymbol{C}_{\Gamma}(\boldsymbol{u}) = 0 \tag{2}$$

- Physical 'constitutive' relations or Hooke's law,

$$\boldsymbol{H}(\boldsymbol{\sigma}, \boldsymbol{\epsilon}) = 0 \tag{3}$$

$\boldsymbol{E}_{\Sigma}, \boldsymbol{E}_{\Gamma}, \boldsymbol{C}_{\Sigma}, \boldsymbol{C}_{\Gamma}$ and $\boldsymbol{H}$ are, in the most general case, arrays of differential operators defined in a 3D continuum body with domain Σ and boundary Γ. (bold letters have been used to denote arrays). $\boldsymbol{u}, \boldsymbol{\sigma}$ and $\boldsymbol{\epsilon}$ are the unknown displacements $\boldsymbol{u} = \{u_1, u_2, u_3\}$, stresses $\boldsymbol{\sigma} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}\sigma_{13}, \sigma_{23}\}$ and strains $\boldsymbol{\epsilon} = \{\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, \epsilon_{12}, \epsilon_{13}, \epsilon_{23}\}$ in each point $P(x, y, z)$of a given reference systems (x, y, z or $1, 2, 3$, is a system of thriorthogonal coordinates). $\boldsymbol{p}$ refers to the applied loadings and double dots denote acceleration. Explicit forms of these equations can be found in one of the available books which deals with the theory of elasticity.

Whenever a plate/shell problem is approached by the direct solution of Eqns.(1)–(3), a 3D analysis has been acquired. Examples of 3D solutions for layered structures can be found in articles [29]–[43]. In general, these solutions are difficult to obtain and, moreover these cannot be given in strong form for the most general case of geometry, laminate layout, boundary and loading conditions. Finite element implementation of the 3D approach consists in using so-called 3D 'brick' elements. The computational cost of these analyses can result to be prohibitive for practical problems. However, 3D analyses of layered plate/shell structures is not the subject of this review which is instead devoted to the two-dimensional, 2D, modelings of them.

2.2 Available 2D Approaches

Plates and shells are, by definition, structures in which one dimension, the thickness h, is at least of one order of magnitude lower than to a representative in-plane dimension L (which could coincide to a or b), measured on the reference plate/shell surface Ω, (see Figures 3, 4 and 5). This fact permits one to reduce the 3D problem to a 2D one. Such a reduction 'in a certain sense' transforms a problem which is defined in each point $P_\Sigma(x, y, z)$ of the 3D continuum (occupied by the considered plate/shell structure) into a problem which is defined in each point $P_\Omega(\alpha, \beta)$ of a reference shell/plate surface Ω (α, β and z is a thriorthogonal curvilinear coordinate system defined for the shell reference surface, see Figure 5).

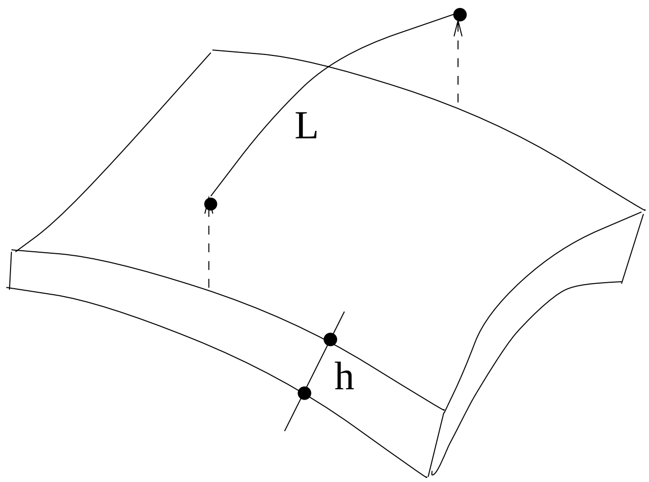

Figure 3. Geometrical parameters in a two-dimensional structure $h/L << 1$

2D modeling of traditional monocoque plates and shells is a classical problem of the theory of structures. The elimination of the thickness coordinate z is usually performed upon integration of Eqns.(1)–(3). Such an elimination can in practice be made according to several methodologies. A significant number of approaches and techniques were proposed, developed and applied over the last century. A comprehensive overview of all these can be made by reading the contributions presented at the IUTAM conferences that were devoted to this subject. In particular, the author recommends the articles printed in proceedings [44],[45]. Among these articles the contributions by Koiter [46], Goldenvaizer [47], John, [48], Green and Naghdi [49] and by Reissner [50] are of particular interest.

It is convenient, even though there is a certain degree of arbitrariety, to group the available formulations that were already known for monocoque plates and shells into the following three groups:

1. Continuum based or stress resultants based models.
2. Asymptotic type approaches.
3. Axiomatic type approaches.

Continuum based or stress resultants based models

According to Green and Naghdi [49], plate and shell theories can be constructed in light of generalized continua which make use of Cosserat surface concept [51]. These authors have made significant contributions to these, showing the 'equivalence' between this last approach

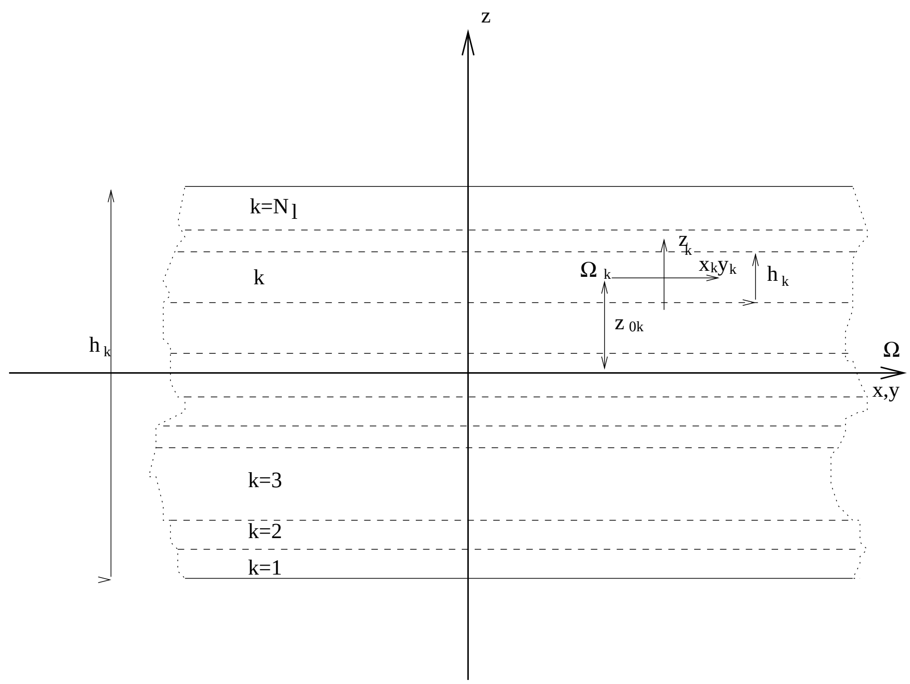

Figure 4. Notations for a multilayered plate

and the classical one based on axiomatic formulations in the case of isotropic monocoque plates and shells. In this kind of approach, a 3D continuum, such as a shell, is seen as a surface on which correspondence stress resultants are defined. The 2D approximations are then introduced at a certain level and integration in the thickness direction is then performed. The most remarkable advantage of this approach is that, being based on a 3D continuum, it does not present any difficulties in the formulation of nonlinear theories in both the case of geometric (large displacement and large rotations) and physical (plasticity, viscoelasticity, etc.) nonlinear behavior.

Asymptotic type approaches

In the framework of asymtotic approaches the 3D governing equations are expanded in terms of a perturbation parameter δ (usually the shell thickness to length ratio is used, e.g. $\delta = h/L$) and theories related to the same order in δ are derived. For instance, the expanded equilibrium equations could appear in the following form,

$$\boldsymbol{E}_\Sigma \approx \boldsymbol{E}^1_\Sigma \delta^{p_1} + \boldsymbol{E}^2_\Sigma \delta^{p_2} + ... + \boldsymbol{E}^N_\Sigma \delta^{p_N} \quad (4)$$

Where $p_i, i = 1, N$ are the exponents of the perturbation parameter δ, these exponents are in general real numbers. In most of the available works this type of expansion is derived using a certain variational statments (see Sec.2.3). Examples of asymptotic approach for monocoque structures can be found in the already mentioned book by Goldenvaizer [21]. This type of approach has mostly been used in the Russian scientific community. A relevant example of the asymtotic treatment of shell structures in the Western scientific community was given in the brilliant monographs by Cicala [52]-[53]. As an advantage, the asymptotic approach furnishes 'consistent' approximations, in the sense that all the terms which have the same order of magnitude as the introduced perturbation parameter δ, are retained in a given asymptotic theory. Furthermore, 3D solutions are approached in the

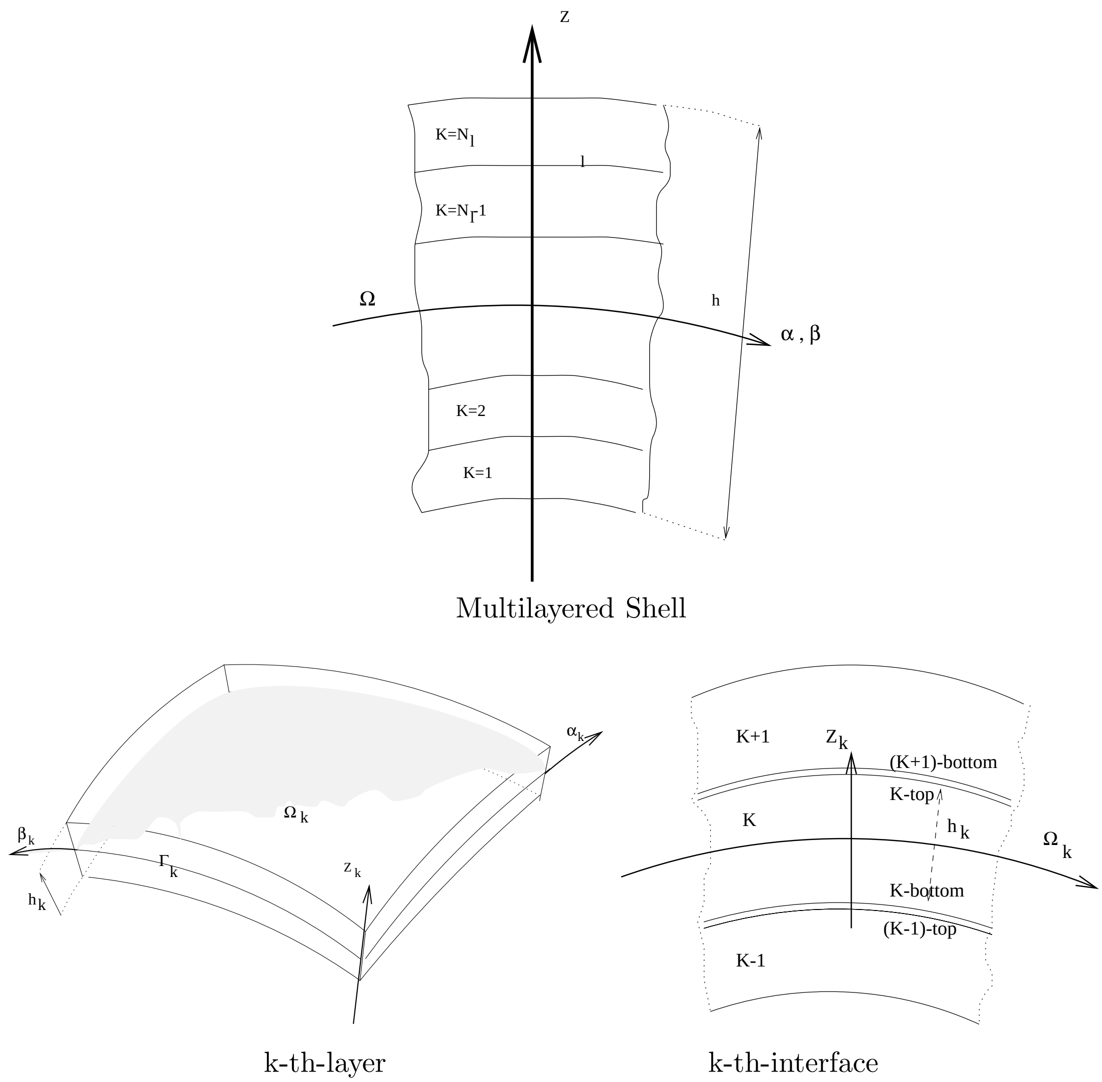

Figure 5. Geometry and notations used for multilayered shells

limit case $\delta \to 0$. However, it is expected that the convergence rate becomes poor when the thickness increases. Furthermore, the extension to multilayered structures would introduce further difficulties: taking into account the anisotropy of composite layers along with the multilayerd construction, the 3D equations would be expanded in terms of both δ and a mechanical layer parameter such as the orthotropic ratio of the lamina E_L/E_T by trough an increase of the number of corresponding formulations (E_L and E_T are Young muduli in the longitudinal and transverse directions of the fibers).

Axiomatic type approaches

In the axiomatic framework, which has been extensively considered in the present paper, a certain displacement and/or stress field is postulated in the thickness z–direction,

$$f(x,y,z) = f_1(x,y)F_1(z) \quad + \ldots + \quad f_N(x,y)F_N(z) \tag{5}$$

where f is a generic component of the unknown variables $\boldsymbol{\sigma}$, $\boldsymbol{\epsilon}$ or $\boldsymbol{u}$. f_i are instead introduced unknowns which are defined on Ω and F_i are the polynomials that have been introduced as the base function for the expansion in z. N is the number of introduced variables.

Axiomatic type approches offer the advantage of permitting to introduce 'intuitive' approximations into the plate/shell behavior while no results can be established as far as the convergence of 'intuitive models' to 3D solutions. An interesting discussion on the implications of the axiomatic character of a given theory was provided by Antona [54].
A possible scenario for a function f, with respect to 3D solutions, is shown in Figure 6 where the two cases of axiomatic and asymptotic approaches are compared.

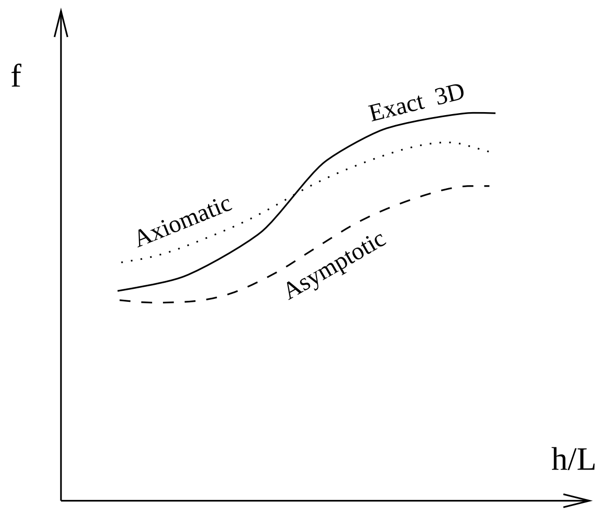

Figure 6. Possible scenarios for a function f calculated by using a 2D approach with respect to its 3D exact solution

2.3 Stress, Displacement and Mixed Formulations and Related Variational Statments

Eqns.(1)–(3) are given in terms of displacements, strains and stresses. Approximated 3D or 2D solutions of these equations are usually established by making a choice for the unknown variables. Normally stresses are used in the so-called 'stress approach' and displacements are used in the so-called 'displacement approach'. In case both stresses and displacements are introduced as unknown variables the so-called 'mixed' formulation is referred to. A choice for the unknown variables therefore introduces a further possibility to formulate a plate/shell theory.

Many variational tools, theorem, equations or principles, have been established in the open literature to derive governing equations consistent to the made choice for the unknown variables. For sake of completeness a brief discussion on that topic is given below.

In solid mechanics, it is well known that the Principle of Virtual Displacement, PVD, involves only a compatible displacement field as a variable and has for its Euler-Lagrange equations the conditions of balance of momenta and traction boundary conditions. Likewise, the dual form of PVD i.e. the Principle of Virtual Forces, PVF, involves a stress field which is equilibrated and satisfies the traction boundary conditions, alone as a variable and has the kinematic compatibility conditions and displacement boundary conditions as its Euler-Lagrange equations. If in PVD kinematic compatibility and displacement boundary conditions are introduced as conditions of constraint through Lagrange multipliers, which turn out to be stresses and surface traction respectively, one then obtain the so-called Hu-

Washizu Variational Principle. Likewise, if the conditions of equilibrium of stresses are introduced as a constrain condition through a Lagrange multipliers field (which turns out to be displacement) into PVF, one is led to the so–called Hellingher–Reissner principles. Thus, the Hu-Washizu and Hellingher-Reissner principles, which involve that one field in the continuum as variables (some of which play the role of Lagrange multipliers to enforce certain constraint conditions), are often referred to as mixed variational principles. See the book by Washizu [58] and the article by Atluri, Tong and Murakawa [59] for a more detailed discussion. A partial mixed formulations very useful for the analysis of layered structures can be made by referring to the variational theorem that was proposed by Reissner [55]–[57]. Such a theorem will be discussed with some details in Part II.

2.4 Layer or Multilayer Variables Description

The multilayered form in which a layered structure appears in most of the applications, introduces a further possibility of its 2D modelings. Such a possibility is related to the choice of the variable description. There are in fact the two following possibilities:

1. Layer-wise description

Each layer k is seen as an independent plate/shell. For instance, the expansion at Eqns.(5) is

$$f^k(x,y,z) = f_1^k(x,y)F_1^k(z) \ + ... + \ f_N^k(x,y)F_N^k(z) \tag{6}$$

The number of the independent variables is dependent on the number of layers in this case. Governing equations are written for each constitutive layer while interface conditions on displacement and transverse stress variables, are introduced as additional constraints. These interface requirements could be more conveniently imposed if the interface values [60]–[63]: $f^k(x,y,h^k/2)$, $\quad f^k(x,y,-h^k/2)$, are used as trasnverse stress end displacement unknowns. According to Reddy [28] these theories are grouped as Layer Wise Models, LWMs. Examples of LW descriptions have been traced in Figure 7. To notice that the expansions are independent in each layer and only the contact condition is requested at the interface between two adjacent layers.

2. Equivalent single layer description

Variables are introduced for the whole plate/shell. These are usually referred to a given reference plate/shell surface Ω. For instance,

$$f(x,y,z) = f_1(x,y)F_1(z) \ + ... + \ f_N(x,y)F_N(z) \tag{7}$$

According to Reddy [28] these theories are grouped as Equivalent Single Layer Models, ESLMs. Examples of ESL descriptions are given in Figure 7.

2.5 Summary

In conclusion, plate/shell theories for composite, multilayered structures could be developed by making an appropriate choice for the following three points.

1. Elimination of the thickness coordinate z:
   (a) Continuum based models or stress resultants based models;
   (b) Asymptotic type approaches;
   (c) Axiomatic type approaches.

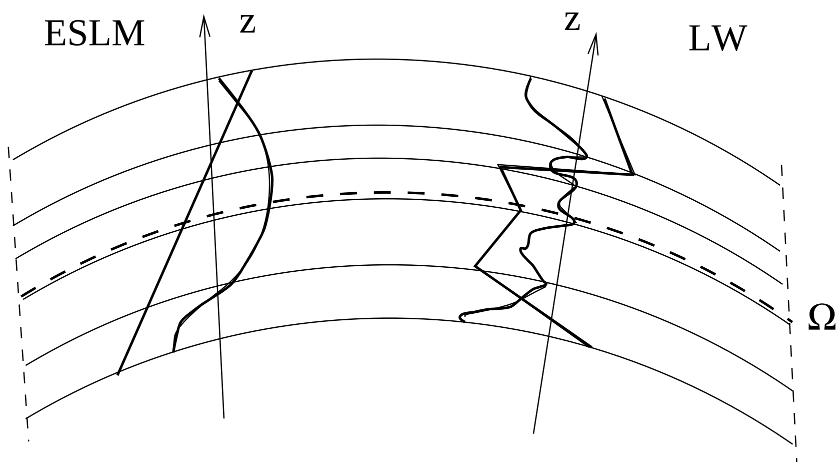

Figure 7. Linear and Higher z-expansion in a layered shell. Comparison between LWM and ESLM assumptions

2. Choice of the unknown variables:
   (a) stress formulation;
   (b) displacement formulation;
   (c) mixed formulation.
3. Choice of the variables description:
   (a) Equivalent Single Layer Models.
   (b) Layer-Wise Models.

For instance, well known Classical Lamination Theory, CLT, and First order Shear Deformation Theory, FSDT, see Part II, are laminated theories in which the axiomatic approach (1a) is employed along with a classical displacement formulation (2a) and an Equivalent Single Layer description (3a). Many other choices can be made as these were already adopted for the traditional metallic structures. Nevertheless it will be clear by reading the next section that the extension 'sic and simpliciter' of those modelings already known for monocoque structures would not lead to satisfactory results. Complicating effects, such as the C_z^0 Requirements, should be conveniently taken into account to this purpose.

3 COMPLICATING EFFECTS OF LAYERED STRUCTURES

The main task of multilayered composite constructions is related to the possibility of exhibiting the so-called,

- In-plane Anisotropy, IA;
- Transverse Anisotropy, TA, Zig-Zag effects and Interlaminar Continuity : C_z^0–Requirements

These two peculiarities of multilayered constructions play a fundamental role in the developments of any plate and shell theories and related finite element implementions.

In-plane Anisotropy, IA

In the case of laminates made by anisotropic layers, a multilayered made structure could show high in-plane anisotropy. That is, the structure has different mechanical-physical properties in different in-plane directions. One of the most relevant consequences of IA, is that the multilayered structure could show higher transverse (both shear and normal components) flexibility with respect to in-plane deformabilty if compared to the traditional isotropic monocoque case. For that reason multilayered structures are often said, High Transversely Deformable, HTD, structures. For instance, laminated structures made of advanced composite materials presently used in aerospace structures, could exhibit high values of Young's moduli orthotropic ratio ($E_L/E_T = E_L/E_z = 5 \div 40$, where L denotes the fiber direction and T and z are two directions orthogonal to L), and low values for the transverse shear muduli ratio ($G_{LT}/E_T \approx G_{TT}/E_T = 1/10 \div 1/200$), leading to higher transverse both shear and normal stress deformabilty than the isotropic cases ($G/E \approx .4$ in this case).

A further relevant consequence of IA is related to the coupling between shear and axial strains. Such a coupling leads to many complications in the solution procedure of any problem related to anisotropic panels [25],[28].

Transverse anisotropy, Zig-Zag effects and Interlaminar Continuity: C^0_z–Requirements

A further task of multilayered constructions is related to the possibility of exhibiting different mechanical-physical properties in the thickness direction. Layered structures are in fact said Transversely Anisotropic, TA, structures. Transverse discontinuous mechanical properties cause displacement fields $\boldsymbol{u}$ in the thickness direction which can exhibit a rapid change of their slopes in correspondence to each layer interface (see Figure 8). This is known as the Zig-Zag, ZZ, form of displacement fields in the thickness direction. In-plane stresses $\boldsymbol{\sigma}_p$=$(\sigma_{11}, \sigma_{22}, \sigma_{12})$ can in general be discontinuous at each layer interface. Nevertheless, transverse stresses $\boldsymbol{\sigma}_n$=$(\sigma_{13}, \sigma_{23}, \sigma_{33})$, for equilibrium reasons, i.e. the Cauchy theorem, must be continuos at each layer interface. This is often referred to in literature as Interlaminar Continuity IC, of transverse stresses.

Figure 8 shows, from a qualitative point of view, what could be the scenarios of displacement $\boldsymbol{u}$ and transverse stress $\boldsymbol{\sigma}_n$ distributions in a multilayered structure as it would come from and exact 3D analysis or from experimental datas. Details of stresses at the interfaces are displayed in Figure 9. In-plane components, which can be discontinuous, are also depicted for comparison purpose. It appears evident that displacements and transverse stresses, due to compatibility and equilibrium reasons, respectively, are C^0–continuous functions in the thickness z direction. It is also evident that, in the most general case, $\boldsymbol{u}$ and $\boldsymbol{\sigma}_n$ have discontinuous first derivatives with correspondence to each interface to which correspondence the mechanical properties change. In [62],[63] ZZ and IC were referred to as C^0_z–Requirements. *The fulfillment of C^0_z–Requirements is a crucial point in the development of any theory suitable for multilayered structures.*

The in-plane anisotropy could introduce a further coupling between in-plane and out-of-plane strains, as it is the case of unsymmetrically laminated panels [25], [28]. The in-plane/out-of-plane coupling could cause large displacements even though low level of the applied loadings are considered. A geometrically nonlinear analysis is required to handle such a phenomena. This aspects has not not fully discussed in this paper. A detailed description can be found in the book by Palazotto and Dennis [27] and in the articles [64]–[69].

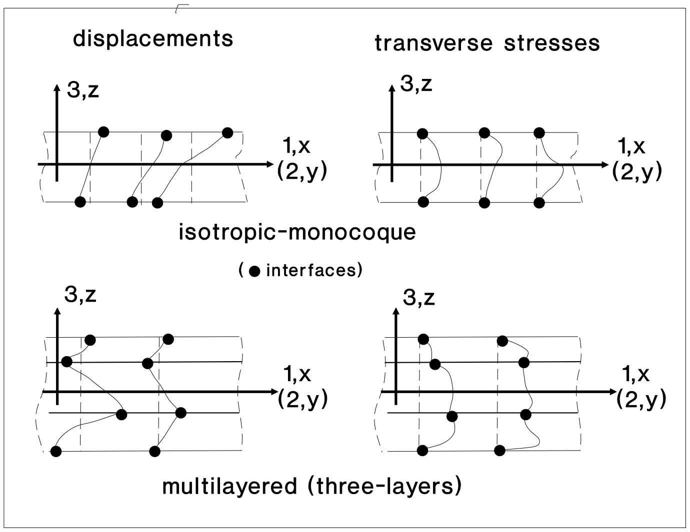

Figure 8. C_z^0–Requirements. Comparison of transverse stress field between a one-layer structure and a three-layer structures

Part II Review of Theories

4 AXIOMATIC THEORIES

Due to the impact that the axiomatic approach has played and continue to play on composite structure modeling most of the attention of the present article has been devoted to the axiomatic type theories.

Sec.4.1 deals with the extension to laminate structures of classical models already known for monocoque structures. Overview of Layer-Wise contributions is given in Sec.4.2. Zig-zag theories are extensively discussed in Secs.4.3 and 4.4. Sec.4.3 introduces in a simple one-dimensional flat case the main ideas related to the development of a zig-zag theory. Sec.4.4 overviews available contributions for ZZ theories. Sec.4.5 deals with full mixed formulations.

4.1 Classical Theories and their Refinements

Following Kraus [22] two-dimensional theories, that were originally developed for single-layer 'monocoque' structures made of traditional isotropic materials, can be conveniently grouped in the two cases: Love First Approximation Theories, LFAT, or Love Second Approximation Theories, LSAT. The well known Cauchy-Poisson-Kirchhoff-Love [70]–[73] thin shell assumptions can be assigned to the first grouping: normals to the reference surface Ω remain normal in the deformed states and do not change in length. That is, transverse shear as well as normal strains are postulated to be negligible with respect to the other strains. Whenever one of these LFAT postulates are totally or partially removed, the obtained theories are grouped into LSAT. It is of interest to emphasize that LSAT should take into account the well known *Koiter's Recommendation KR,* [46]. Such a recommendation, which was reached on the basis of energy considerations, states that: 'a refinement of Love's first approximation theory is indeed meaningless, in general, unless the effects of transverse shear and normal stresses are taken into account at the same time.' More general and systematic substantiation of the Koiter's conclusion can be found in the already mentioned

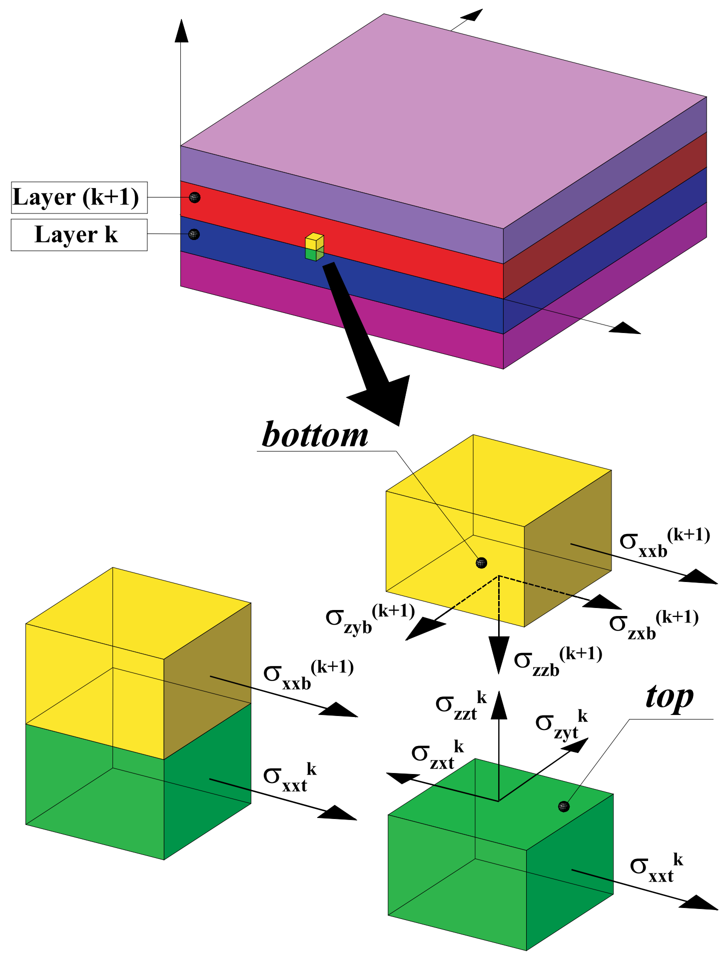

Figure 9. Details and notations of stresses at the interface

works by Goldenvaizer [21] and Cicala [52], [53], in which the method of the asymptotic expansion of the three–dimensional governing equations has been employed.

A large amount of contributions have been published in which LFAT and LSAT are extended to multilayered structures. A discussion on that topic is given in the following.

Classical Lamination Theory, CLT

Applications of LFAT to multilayered structures are often referred to as the *Classical Lamination Theories, CLT,* see Reissner and Stavwsky [74] and the books by Jones [25] and Reddy [28]. The displacement models on the basis of this theory for the case of plate geometries is written,

$$\begin{aligned} u_i(x,y,z) = &\quad u_i^0(x,y) - z u_{i,z}(x,y) \quad i=1,2, \quad or \quad i=x,y \\ u_3(x,y,z) = &\quad u_3^0(x,y) \end{aligned} \tag{8}$$

commas denote partial derivatives while apexes 0 denotes values with correspondence to a given reference surface Ω (usually the middle surface of the plate). A geometric interpretation of this well known model is given in Figure 10.

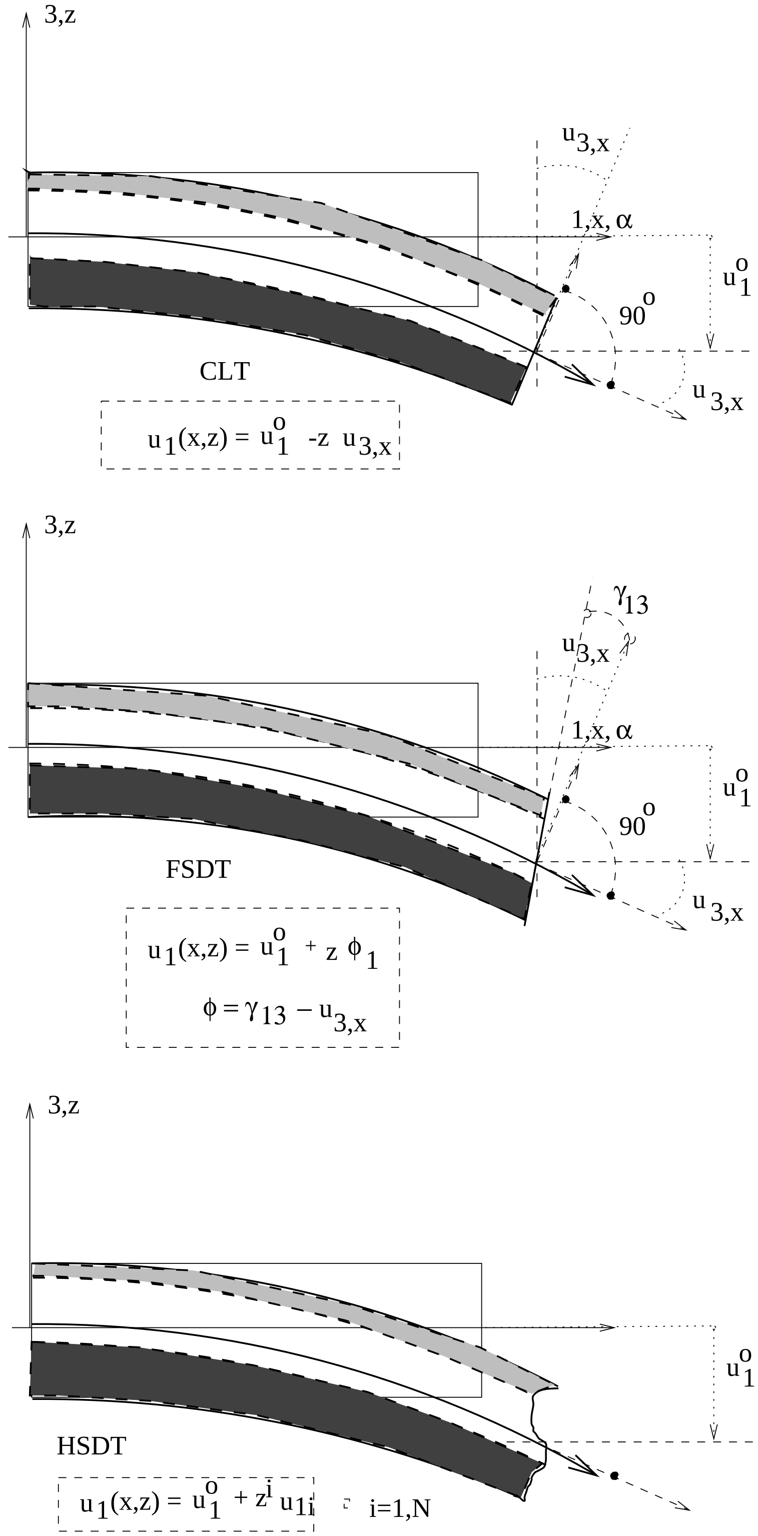

Figure 10. CLT, FSDT and HSDT assumptions in one-dimensional cases

Curvatures terms appear in the case of shells, see [22]. The manners in which such a terms appear is very much subordinate to the hypotheses made on the thickness to radii curvature parameters $(h/R_\alpha, h/R_\beta)$ with respect to the unity, see [75]. A possible form of CLT for double curved shells is

$$\begin{array}{l} u_i(\alpha,\beta,z) = \frac{u_i^0}{R_i}(\alpha,\beta) - z u_{i,z}(\alpha,\beta) \quad i=1,2 \quad or \quad i=\alpha,\beta \\ u_3(\alpha,\beta,z) = u_3^0(\alpha,\beta) \end{array} \tag{9}$$

First order Shear Deformation Theory, FSDT

Extensions of the so-called Reissner–Mindlin (Reissner [76] and Mindlin [77]) LSAT type model, which includes transverse shear strains, to layered structures are known as the *Shear Deformation Theory SDT, (*or First order SDT, FSDT) see Yang, Norris and Stavwsky [78]. The displacement models related to FSDT can be written in a form which is similar for both plate and shell geometries. By using the plate notations the FSDT displacement model is written as follows:

$$\begin{array}{l} u_1(x,y,z) = u_i^0(x,y) + z\phi_i(x,y) \\ u_3(x,y,z) = u_3^0(x,y) \end{array} \tag{10}$$

A geometric interpretation of this diplacement model is also given in Figure 10. Shear strain γ_{3i} distribution vs thickness, is instead given in Figure 11. The relation between the rotations of the two normals ϕ_i and the two transverse shear strains γ_{3i} is: $\phi_i = u_{3,i} - \gamma_{3i}$.

Vlasov-Reddy Theory, VRT

A simply refinements of Reissner-Mindlin theory was accounted for by Vlasov [79] for monocoque structures. Vlasov's FSDT type–theory permits fulfillment of the homogeneous conditions for the transverse shear stresses in correspondence to the top and bottom shell/plate surfaces $\sigma_{i3}(\pm h/2) = 0$. Reddy [80] and Reddy and Phan [81] have shown that such a simple inclusion lead to significant improvements with respect to FSDT analysis to trace the static and dynamic response of thick laminated structures. Further works on these type of theories were documented in [28]. Resulting model, Vlasov-Reddy Theory, VRT, has a third-order z expansion for the in-plane displacement field and preserve the number of the variables to that of FSDT,

$$\begin{array}{ll} u_i(x,y,z) = & u_i^0 + (z + \frac{4}{3h^2}z^3)\phi_{i3} - \frac{4}{3h^2}z^3 u_{3,i} \\ u_3(x,y,z) = & u_3^0 \end{array} \tag{11}$$

Square terms in z are missing due to intrinsic symmetry of the considered laminates. Displacement fields along with transverse shear stresses are depicted in Figure 11.

Higher order Theory, HOT

Koiter's recommendation have not been included in the above mentioned theories. These recommendation can be fulfilled by including both transverse shear and normals strains as done in the milestone work by Hildebrand, Reissner and Thomas [82], whose displacement model is,

$$\begin{array}{ll} u_i(x,y,z) = & u_i^0 + \phi_i \\ u_3(x,y,z) = & u_3^0 + z\phi_3 + z^2\varphi_3 \end{array} \tag{12}$$

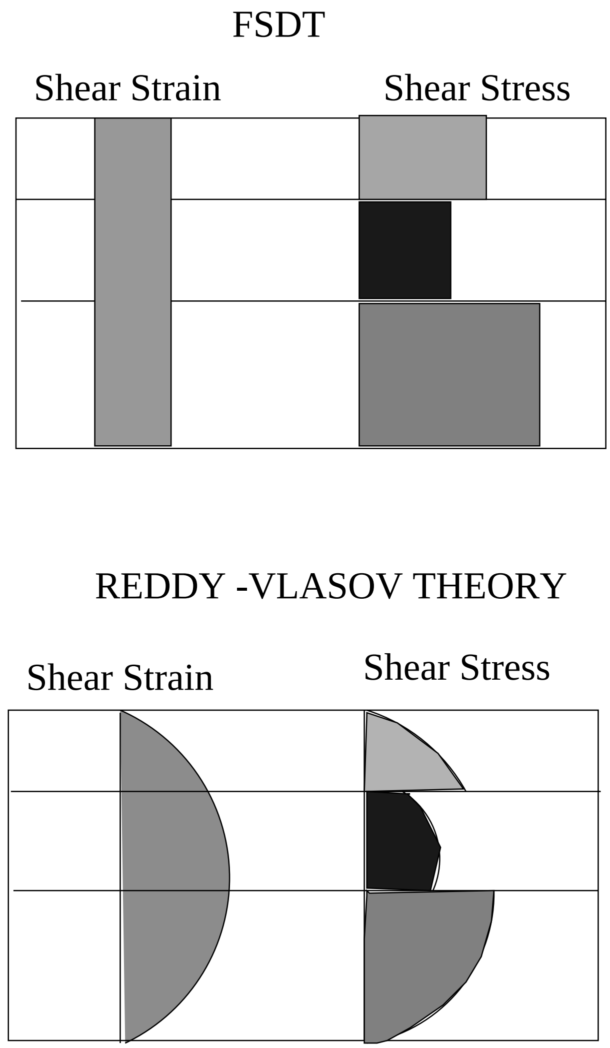

Figure 11. Shear strains and stresses for FSDT and Reddy–Vlasov theory

Additional variables with respect to FSDT case have been introduced in the transverse diplacement expansions. This last type of refinements of FSDT as well as VRT are all known as Higher order Theories HOT. In general higher order theories are based on displacement models of the following type,

$$u_i(x,y,z) = u_1^0 + z u_{i1} + z^2 u_{i2} + .. + z^{N_i} u_{i1}, \quad i = 1,2,3 \tag{13}$$

where N_i are the order of the expansions used for the displacements variables. Displacement models related to HOT have been already traced in Figures 7 and 10. Examples of applications of these types of models to laminated flat and curved structures can be found in [83]—[100]. Exhaustive overview on this type of approches can be read in the already mentioned overview papers.

4.2 Layer-Wise Theories

The theories mentioned above have the number of unknown variables that are independent from the number of constitutive layers N_l. These all are Equivalent Single Layer Models, ESLMs. Although these kinematics theories can describe transverse shear and normal strains, including transverse warping of cross section, the approach is 'kinematically homogeneous' in the sense that the kinematics are insensitive to individual layers. If detailed response of individual layers is required and if significant variations in displacements gradients between layers exist, as it is the case of local phenomena description, this approach will necessitate the use of especially higher order theories in each of the constitutive layers alone with a concomitant increase in the number of unknowns in the solution process as well as complexity of the analysis. In other words, a possible 'natural' manner of including the ZZ effect in the framework of classical model with only diplacement variable, could be implemented by applying CLT, FSDT or HOT at a layer level. That is, each layer is seen as an independent plate and compatibility of displacement components with correspondence to each interface is then imposed as a constraint. In these cases Layer Wise Models, LWM, are obtained. Examples of these types of theories are those found in the articles [101]–[106]. For instance, Cho, Bert and Striz [105] employed the following HOT at layer level to analyze the dynamics of layered plates,

$$u_i^k(x,y,z) = \quad u_i^{k0} - z_k u_{i1}^k + z_k^2 u_{i2}^k + .. + z_k^{N_1} u_{i1}^k \quad i=1,2,3 \quad k=1,N_l \tag{14}$$

Displacement models written in the form above require to include constraint conditions in order to enforce the compatibility conditions at the each interface. Generalizations on these types of theories were given by Nosier, Kapania and Reddy [61] and by Reddy [28] who expressed the displacement variables in the thickness direction in terms of Lagrange polynomials,

$$u_i^k(x,y,z) = L_1(z_k)u_i^k|_{h/2} + L_2(z_k)u_i^k|_{-h/2} + L_3(z_k)u_{i2} + ... + L_{N_1}(z_k)u_{iN_i}^k \qquad i=1,2,3 \quad k=1,N_l \tag{15}$$

Interface values are used as unknown variables (the first two terms of the expansion $u_i^k|_{h/2}$ and $u_i^k|_{-h/2}$) therefore permitting an easy linkage of compatibility conditions at each interface. L_1, L_2 coincide, in fact, to linear Lagrangian polynomials while $L_3, ..L_{N_1}$ should be and independent base of polynomials which start from the parabolic one (L_3).

4.3 Zig-Zag Theories: An Introduction

The extension *sic et simpliciter* of CLT, FSDT and HOT to multilayered plates does not permit the fulfillment of the C_z^0–Requirements, that is, ZZ and IC are not addressed by the mentioned CLT, FSDT and HOT. An exception is given by the already discusses VRT which fulfills homogeneous conditions for the transverse shear stresses with correspondence to the top and bottom shell/plate surfaces.

Literature, see [19], has shown that HOT could provide acceptable global response even though thick structures are analyzed. Nevertheless transverse stresses, as they come by Hooke's law can results very inaccurate. A possible strategy to improve transverse stresses evaluations consists to recovery them by direct integration of 3D equilibrium equations Eqns.(1). In any case the accuracy of the obtained results cannot be guaranteed.

Much better description can be obtained by employing layer-wise approaches. In particular it was shown by the author [107] that a third and four order displacement fields in each layer could provide accurate displacements as well as stress descriptions (both in-plane and transverse components) directly by Hooke's law.

Anyway in both framework of ESL and LW variables descriptions it is of interest to try to developed theories which would fulfill *a priori* (in a complete or partial form), what it has been summarized as the C_z^0–Requirements. Refined theories have therefore been motivated to meet what above. Due to the form of displacement field in the thickness plate direction, see Figures 8, 13, these type of theories are referred to as Zig-Zag theories.

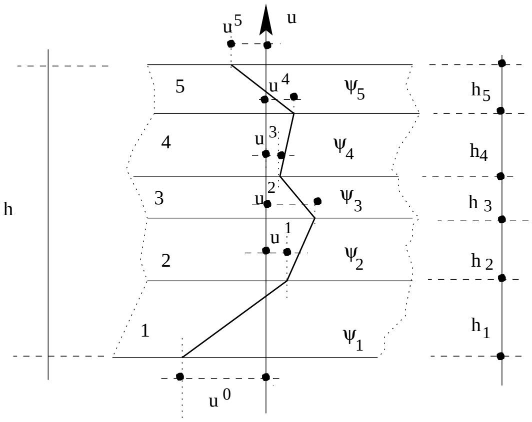

Figure 12. Geometry and notations employed to introduce zig-zag theories

The fundamental ideas in developing zig-zag theories consists to assume a certain displacement and/or stress models in each layer then to use compatibility and equilibrium conditions at the interface to reduce the number of the unknown variables. In order to introduce a ZZ theory a sample problem has been addressed below.

For sake of simplicity a one-dimensional flat case is considered. Extension to two-dimensional case and shell geometries should be immediate. The origin of thickness coordinates is for sake of simplicity put in the bottom layer; attention has been restricted to a piecewise continuos, linear displacement field.
The displacement field u in each layer can be first written by using displacement values at the interfaces (see Figure 12). For instance, for the first and last layers one has,

$$\begin{aligned} u^1(z) &= \quad u^0 + z\psi_1, & 0 \le z \le h_1 \\ u^{N_l}(z) &= \quad u^{N_l-1}(h_{N_l-1}) + (z - h_{N_l-1})\psi_{N_l}, & h_{N_l-1} \le z \le h_{N_l} \end{aligned} \tag{16}$$

where

u^0 is the value of the diplacement u with correspondence to the bottom surface.

$u^k(h_k)$, $k = 1, N_l$ are the interface value of u.

ψ_k, $k = 1, N_l$ are the values which identify the rotations in the layers.

The generic interface value can be re-written,

$$u^k(h_k) = \quad u^0 + \sum_{k=1}^{N_l-1} (h_k - h_{k-1})\psi_k, \quad k = 1, N_l \tag{17}$$

It is intended that h_0=0. It follows that the displacement field in each layer can be written in the following unified form,

$$u^k(z) = \quad u^0 + \sum_{k=1}^{N_l-2} (h_{k-1} - h_{k-2})\psi_{k-1} + (z - h_{k-1})\psi_k, \quad k = 1, N_l \tag{18}$$

The N_l rotations ψ_k can be expressed in terms of one of them (for instance the rotation in the bottom layer) by imposing the $N_l - 1$ interlaminar continuity for transverse shear stresses

$$\sigma_{xz}^k = \sigma_{xz}^{k+1},$$

from which one has,

$$\psi_k = a^k \psi_1, \quad k = 2, N_l - 1$$

where a^k are layer constants defined by interlaminar transverse stresses. To be more precise, ψ_1 often appears as a combination of an in-plane derivative of transverse displacement $u_{3,_x}$, and a transverse shear strains γ_{xz}. The previous relation should be therefore written

$$\psi_k = -u_{3,_x} + a^k \gamma_{xz}$$

For sake of simplicity we take valid the first of these, as a consequences the displacement u is

$$u^k(z) = \quad u^0 + \sum_{k=1}^{N_l-2} (h_{k-1} - h_{k-2}) a^{k-1} \psi_1 + (z - h_{k-1}) a^k \psi_1, \quad k = 1, N_l \tag{19}$$

Eqns.(18) gives the displacement field in each layer. Such a displacement field can be written in a form which is valid for the whole multilayer by using the Heaveside step function. Such a function is defined as follows

$$H(z - zk) \quad = \quad \begin{cases} 0 & z \leq z_k \\ 1 & z \geq z_k \end{cases} \tag{20}$$

By means of H the diplacement u can be written in a form which is formally not affected by k,

$$u(z) = \quad u^0 + \sum_{k=1}^{N_l-1} (z - z_{k-1}) \psi_k H(z - z_k) \tag{21}$$

or

$$u(z) = \quad u^0 + \sum_{k=1}^{N_l-1} (z - z_{k-1}) a_k \psi_1 H(z - z_k) \tag{22}$$

It appears clear that a linear, piece-wise form for u leads to layer continuity stiffnesses a^k which are independent from z. In fact, top–bottom homogeneous transverse shear stress conditions cannot be imposed in this case. This was known to Ambartsumian [109] and Whitney [116] who assumed a^k which were cubic function of z. In fact, the two additional functions related to quadratic and cubic terms of z are determined by the two homogeneous conditions of transverse shear stresses with correspondence to top and bottom plate/shell surfaces. Available zig-zag theories are discussed in the next section.

4.4 Zig-Zag Theories: A Review

Early, most significant contributions to Zig-Zag theories come from Russian School. The first Zig-Zag theory was given by Lekhnitskii [108] for beam geometries. Further, outstanding contributions to plates and shells have been given by Ambartsumian [109]–[113]. An independent manner to formulate zig-zag plate/shell theories has been more recently provided in the Western by Reissner [55]–[57]. Many other theories are known which are based on these three Scientists' original contributions. An historical note, recently provided by the author [20], has proposed to denote the these three independent approches as follows:

- Lekhnitskii–Ren
- Ambartsumian–Whitney–Rath-Das
- Reissner–Murakami-Carrera

Related contributions are briefly discussed and illustrated in the following subsections.

Lekhnitskii–Ren theories

The first name of this type of theory coincides with the author of the original work while the second name coincides with the author who first has extended the work to plate. In [108], Lekhnitskii proposed a splendid method able to describe zig-zag effects (for both in-plane and through the thickness displacements) and interlaminar continuous transverse stresses.

Lekhnitskii wrote first the governing 3D equation in the case of plane stress problems related to a cantilever beam. The problem was formulated in terms of stress function. An approximation for this function 'in axiomatic sense' was then introduced for each layer. Layer constants were computed for each layer by using the give boundary conditions along with C_z^0–Requirements. At the very end explicit formulas for stresses and displacements were given by Lekhnitskii . A more brief treatment can be found in the English translation of the book by the same Lekhnitskii [23] (Sec. 18 of Chapter III, page 74). Although the Lekhnitskii 's theory was published in the middle thirties of last century and reported in a short paragraph of the English edition of his book, it has been systematically forgot in the recent literature.

To the best of the author knowledge, Ren is the only scientist who has used the Lekhnitskii 's work described in the previous section. In the two papers [114],[115] Ren has, in fact, extended the Lekhnitskii 's theory to orthotropic and anisotropic plates. Ren assumed the following distribution of transverse shear stresses in a laminated plate, composed by N_l orthotropic layers,

$$\begin{aligned} \sigma_{xz}^k(x,y,z) = &\quad \xi_x(x,y)a^k(z) + \eta_x(x,y)c^k(z) \\ \sigma_{yz}^k(x,y,z) = &\quad \xi_y(x,y)b^k(z) + \eta_y(x,y)g^k(z) \end{aligned} \tag{23}$$

Four independent function of x, y have been introduced to describe transverse shear stresses. The layer constants are parabolic function of the thickness coordinate z. Formulae for the layer constant can be read in the original work. Eqns.(23) represents an interlaminar continuous transverse shear stress field which is parabolic in each layer. Displacement fields are obtained by integrating strain displacement relation and by imposing compatibility conditions for the displacement at the interface. The displacement field assumes the following Zig-Zag form,

$$\begin{aligned} u^k(x,y,z) &= u_0(x,y) - w_{,x} + \xi_x(x,y)A^k(z) + \eta_x(x,y)C^k(z) \\ v^k(x,y,z) &= u_0(x,y) - w_{,y} + \xi_y(x,y)B^k(z) + \eta_y(x,y)G^k(z) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{24}$$

where $A_z^k(z)$, $B_z^k(z)$,$B_z^k(z)$ and $G_z^k(z)$ are obtained by integrating the corresponding $a_z^k(z)$, $b_z^k(z)$, $c_z^k(z)$ and $g_z^k(z)$. That is the Eqns.(24) represents a piecewise continuous displacement field in the thickness direction z which is cubic in each layers.

No shell applications of Lekhnitskii–Ren theory are known to the author.

The Ambartsumian–Whitney–Rath-Das Theory

The first name of this type of theories coincides with Ambartsumian, the author of the original works [109]–[113]; the second name coincides with Whitney [116], the author who has both extended the theory to anisotropic plates and introduced the theory in the western scientific community; the third name coincides with the two authors, Rath and Das [117], who made the extension of the Whitney's work to shell geometries. The Ambartsumian–Whitney–Rath-Das approach has the peculiarity to preserve the number of the unknown variables as much as those of FSDT, i.e. three displacements and two rotations (or shear strains). Transverse shear stresses are assumed as in the following,

$$\begin{aligned} \sigma^k_{xz}(x,y,z) &= [Q^k_{55}f(z) + a^k_{55}]\phi_x(x,y) + [Q^k_{45}f(z) + a^k_{45}]\phi_y(x,y) \\ \sigma^k_{xz}(x,y,z) &= [Q^k_{45}f(z) + a^k_{55}]\phi_x(x,y) + [Q^k_{44}f(z) + a^k_{44}]\phi_y(x,y) \end{aligned} \tag{25}$$

$f(z)$ is a function of the thickness coordinate which form can be assumed different as far as symmetrical and unsymmetrical laminated cases is concerned. The layer constants $a^k_{44}, a^k_{45}, a^k_{55}$ are determinate by imposing the continuity conditions of transverse shear stresses at the interface while top-bottom homogeneous conditions are used to determined the form of $f(z)$. By assuming transverse displacement constant in the thickness direction, i.e. ϵ_{zz}=0, upon integrating the shear strains, the displacements fields result in the following form,

$$\begin{aligned} u^k(x,y,z) &= -zw_{,x} + [J(z) + g^k_1(z)]\phi_x(x,y) + g^k_2(z)]\phi_y(x,y) \\ v^k_{xz}(x,y,z) &= -zw_{,y} + [J(z) + g^k_3(z)]\phi_y(x,y) + g^k_4(z)]\phi_x(x,y) \\ w(x,y,x) &= w_0(x,y,z) \end{aligned} \tag{26}$$

The explicit expression of layer constants has been here omitted.

Extension to doubly curved shell and dynamic case of the Whitney's work was done by Rath and Das (1973), which displacement fields is,

$$\begin{aligned} u^k(\alpha,\beta,z) &= (1+\tfrac{z}{R_\alpha})u^0_{,\alpha} - \tfrac{z}{A_1}w_{,\alpha} + [z + \tfrac{z^2}{R_\alpha} - 4\tfrac{z^3}{3h^2} - \tfrac{z^4}{3h^2R_\alpha}) + G^k_1(z + \tfrac{z^2}{R_\alpha}) + d^k_1(1+\tfrac{z}{R_\alpha})]\phi_\alpha \\ v^k(\alpha,\beta,z) &= (1+\tfrac{z}{R_\beta})v^0_{,\alpha} - \tfrac{z}{A_2}w_{,\beta} + [z + \tfrac{z^2}{R_\beta} - 4\tfrac{z^3}{3h^2} - \tfrac{z^4}{3h^2R_\beta}) + G^k_3(z + \tfrac{z^2}{R_\beta}) + d^k_3(1+\tfrac{z}{R_\beta})]\phi_\beta \\ w(\alpha,\beta,z) &= w_0(\alpha,\beta) \end{aligned} \tag{27}$$

and G^k_1,d^k_1, G^k_3,d^k_3 are layer constant directly derived by those assumed for the transverse shear stress Eqns.(25). Stress and displacement fields relate to AWRD zig-zag theories are, from qualitative point of view, the same of those traced in Figure 13.

Further to the two papers by Whitney [116] and Rath and Das [117] a third article co-authored by Sun [118] compared Whitney's zig-zag theory to simplified ones which discard interlaminar continuity and/or zig-zag effects. With the exception of this last paper, the author does not known of any further, 'direct' application of the AWRD Theory.

Dozens of papers have instead been presented over the last decades that deal with zig-zag effects and interlaminar continuos transverse shear stresses, and which have stated that new theories were being proposed. The author has instead found out that these articles should be considered as simplified cases of the AWRD theory or the AWRD theory. Unfortunately, the original works and authors (Ambartsumian , Whitney and Rath and Das) are not mentioned, or barely cited, in the literature lists of this large number of articles. In order

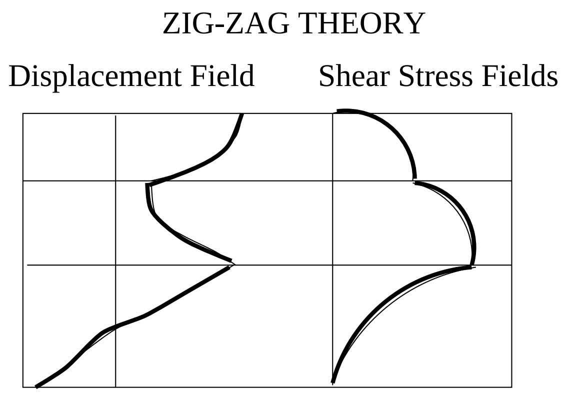

Figure 13. Displacement and transverse stress field for a Zig-Zag type theory

to try to explain such historical unfairness a reconstruction of what happened has here been attempted.

A pioneering article by Yu [119] should first be mentioned in which a zig-zag theory was presented. Because of the short time between the Ambartsumian 's works and Yu's article along with the Cold War, it could be given for sure that the early works by Ambartsumian were unknown to Yu. A sandwich one-dimensional plate made of isotropic core and faces was considered in [119]. The three slopes of the displacement fields in the three layers were derived by imposing transverse shear continuity at the two interfaces. The in-plane displacement fields were assumed linear in each layer. Yu did not start his derivation by direct assumption of transverse stress field as Ambartsumian [109]; Yu, in fact, preferred to start from a Reissner-Mindlin type displacement model (i.e. linear displacement field in the plate thickness direction) and compute the faces and core slopes by imposing transverse shear stress continuity at the interfaces. It was also written by Yu, that the method could easily be extended to displacement fields which were given as cubic functions of z. Details of this last development were not given by Yu. Almost 15 years later Chou and Carleone [120] presented a zig-zag theory of anisotropic plates. As in Yu (even though the original work by Yu was not mentioned) a piece-wise linear displacement field in each layer was considered and a 'zig-zag' theory was proposed. It seems that Chou and Carleone were not aware of the work by Yu [119], neither of the works by Ambartsumian and Whitney. Chou and Carleone as well as Yu's analyses consist in fact of a particular case of the AWRD theory in which only those terms in $a^k_{44}, a^k_{45}, a^k_{55}$ of Eqns.(31) which are independent of z are retained. Dischiuva and co-authors [121]–[124] wrote about Yu and Chou and Carleone's zig-zag displacement field, which was linear in each layer, by employing a Heaveside step function. These two manners of writing a zig-zag displacement field have been detailed in the previous sections for for a simple one-dimensional case. It should be pointed out, once again, that Yu type analysis, deal with of a particular case of what is herein called the AWRD theory. As previously mentioned, the linear piece-wise continuous displacement field used by Yu can in fact be obtained from the AWRD one by simply neglecting the higher order terms (which multiply z^2 and z^3) in Eqns.(27). By doing this, the resulting models are not able to fulfill: (i) – homogeneous conditions for transverse shear stress at the top/bottom plate/shell surface, neither, (ii) –or be suitable for unsymmetrically laminated structures.

Due to such historical unfairness, most the the subsequent works on zig-zag theories did not refer to the work by Ambartsumian nor to those by Whitney and Rath and Das. Almost ten years later on, the original AWRD theory was, re–obtained by Cho and Parmeter [125].

First Bhaskar and Varadan [126] and than Savithri and Varadan [127] and Lee *et al.* [128] introduced the top-bottom homogeneous conditions mentioned at point (i). This was done by extending, in a Yu type theory the Vlasov [79] third order in-plane displacement fields. In particular the contribution by Savithri and Varadan [127] was also directed to include transverse normal strains effects. Further refinement to what in [127] were provided by Li and Liu [129]. The obtained models, which did not show the quadratic terms in the expression of displacement fields, was still not suitable for unsymmetrical laminated plates. A final 'best version', which includes what is mentioned at points (i) and (ii) was proposed by Cho and Parmerter [125] for plates, by Bekou and Touratier [130] for shells, and then (among others) in the articles [131]–[137]. As in [121], the Heaveside step function was used by Cho and Parmerter [125], whose displacement fields for a generally laminated plate were written in the form reported below,

$$\begin{aligned}
u(x,y,z) &= u_0 + \sum_{k=0}^{N_u-1} S_x(z-z_k)H(z-z_k) + \sum_{k=0}^{N_s-1} T_x(z-\zeta_k)H(-z+\zeta_k) \\
&\quad -\frac{z^2}{2h}(\sum_{k=0}^{N_u-1} S_x^k + \sum_{k=0}^{N_s-1} T_x^k) - \frac{z^3}{3h^2}\{w_{,x} + \frac{1}{2}(\sum_{k=0}^{N_u-1} S_x^k + \sum_{k=0}^{N_s-1} T_x^k)\} \\
v(x,y,z) &= v_0 + \sum_{k=0}^{N_u-1} S_y(z-z_k)H(z-z_k) + \sum_{k=0}^{N_s-1} T_y(z-\zeta_k)H(-z+\zeta_k) \\
&\quad -\frac{z^2}{2h}(\sum_{k=0}^{N_u-1} S_x^k + \sum_{k=0}^{N_s-1} T_y^k) - \frac{z^3}{3h^2}\{w_{,x} + \frac{1}{2}(\sum_{k=0}^{N_u-1} S_y^k + \sum_{k=0}^{N_s-1} T_y^k)\} \\
w(x,y,x) &= w_0(x,y)
\end{aligned} \tag{28}$$

N_u and N_s are the number of layers in the upper and lower half, respectively. z and ζ are the two thickness coordinates for the upper and lower half, respectively; z_k and ζ_k are the corresponding interface values. The mid-plane rotations ϕ_x, ϕ_y at z^+ (z^+ and z^- are the top and bottom layer interfaces, respectively) are

$$\phi_x = \frac{\partial u}{\partial z}_{|z=0^+} = S_x^0, \quad \phi_y = \frac{\partial v}{\partial z}_{|z=0^+} = S_y^0,$$

and

$$S_x^k = a_{x\tau}^k(w_{,\tau}^0 + \phi_{,\tau}) + b_{x\tau}^k w_{,\tau}^0, \;\; S_y^k = a_{y\tau}^k(w_{,\tau}^0 + \phi_{,\tau}) + b_{y\tau}^k w_{,\tau}^0, \quad k = 0,1,..,N_u-1, \;\; \tau = x,y$$

$$T_x^k = c_{x\tau}^k(w_{,\tau}^0 + \phi_{,\tau}) + d_{x\tau}^k w_{,\tau}^0, \;\; T_y^k = c_{y\tau}^k(w_{,\tau}^0 + \phi_{,\tau}) + d_{y\tau}^k w_{,\tau}^0, \quad k = 0,1,..,N_s-1, \;\; \tau = x,y$$

where $a_{x\tau}^k, b_{x\tau}^k, c_{x\tau}^k, d_{x\tau}^k$ $a_{y\tau}^k, b_{y\tau}^k, c_{y\tau}^k, d_{y\tau}^k$ are layer stiffnesses, see [125], for further details).

The previous displacement field, even though written in a more tedious form, coincides exactly with that of Eqns.(31). The use of the Heaveside step function is not essential. Its use is, in fact, preferred by some authors and omitted by others. The author's opinion is that the use of the Heaveside function, although it has some formal advantages, is unuseful as far as calculations and/or computer implementations are concerned.

The Reissner–Murakami-Carrera Theory

A third approach to laminated structures was originated by the two papers by Reissner [55]–[57] in which a mixed variational equation, namely Reissner's Mixed Variational Theorem RMVT, was proposed. Displacement $\boldsymbol{u}$ and transverse stress $\boldsymbol{\sigma}_n$ variables are independently assumed in the framework of RMVT. In the dynamic case RMVT states:

$$\sum_{k=1}^{N_l} \int_{\Omega_k} \int_{A_k} \left(\delta\boldsymbol{\epsilon}_{pG}^{k^T} \boldsymbol{\sigma}_{pH}^k + \delta\boldsymbol{\epsilon}_{nG}^{k^T} \boldsymbol{\sigma}_{nM}^k \quad + \quad \delta\boldsymbol{\sigma}_{nM}^{k^T} (\boldsymbol{\epsilon}_{nG}^k - \boldsymbol{\epsilon}_{nH}^k) \right) d\Omega_k dz$$
$$= \sum_{k=1}^{N_l} \int_{\Omega_k} \int_{A_k} \rho^k \, \delta\boldsymbol{u}^k \, \ddot{\boldsymbol{u}}^k \, dV \quad + \quad \delta L^e$$

The superscript T signifies an array transposition and V denotes the 3D multilayered body volume while the subscripts n and p denote transverse (out-of-plane, normal) and in-plane components, respectively. ρ is the mass density and double dots denote accelerations while $\boldsymbol{p}$=(p_1, p_2, p_3) are body forces. The subscript H underlines that stresses are computed via Hooke's law. The variation of the internal work has been split into in-plane and out-of-plane parts and involves stress from Hooke's law and strain from geometrical relations (subscript G). δL_e is the virtual variation of the work made by the external layer-force $\boldsymbol{p}$. the third 'mixed' term variationally enforces the compatibility of the transverse strain components. Subscript M underlines that transverse stresses are those of assumed model, see the discussion reported in [19].

The list of the names used to denote the present approach has been build according to the following items: Reissner proposed the variational theorem and traced the manner in which a mixed theory could be developed; Murakami [139]–[144] scholar of Professor Reissner in San Diego, was the first to develop a plate theories on the basis of RMVT and introduced fundamental ideas on the application of RMVT in the framework of ESLM; Carrera [62], [145]–[153] has presented a systematic manner of using RMVT to develop plate and shell theories along with finite elements for statics and dynamics as well as linear and nonlinear problems. Carrera also introduced [62] a Weak Form of Hooke's Law, WFHL, which permits to reduce mixed theories to classical models with only displacement variables (see the end of this paragraph).

The first application of RMVT to modeling of multilayered flat structures was performed by Murakami [139],[140]. He introduced a first order ESL displacement field in his papers,

$$\begin{aligned} u_i(x,y,z) &= u_i^0 + z\phi_i + (-1)^k \zeta_k \, u_{iZ} \\ u_3(x,y,z) &= u_3^0 \end{aligned} \tag{29}$$

It consists in a refinement of FSDT. Subscript Z refers to the introduced zig-zag term. To be noticed that the unknown variables u_0, ϕ_i, u_Z are k–independent. The geometrical meaning of the zig-zag function is explained in Figure 14. ζ_k=$2z_k/h_k$ is a non–dimensioned layer coordinate (z_k is the physical coordinate of the k-layer whose thickness is h_k). The exponent k changes the sign of the zig-zag term in each layer. Such an artifice permits one to reproduce the discontinuity of the first derivative of the diplacement variables in the z-directions which physically comes from the intrinsic transverse anisotropy TA of multilayer structures (as depicted in Figure 8). The transverse shear stress field was retained independent and parabolic in in each layer (transverse normal stress and strains were discarded).

$$\sigma_{i3}^k(x,y,z) = \quad \sigma_{i3_t}^k\left(-\tfrac{1}{4} + \tfrac{\zeta_k}{2} + \tfrac{3}{4}\zeta_k^2\right) + \sigma_{i3_b}^k\left(-\tfrac{1}{4} - \tfrac{\zeta_k}{2} + \tfrac{3}{4}\zeta_k^2\right) + R_{i3}^k\left(\tfrac{3-12\zeta_k^2}{2h_k}\right), \quad i = 1,2 \tag{30}$$

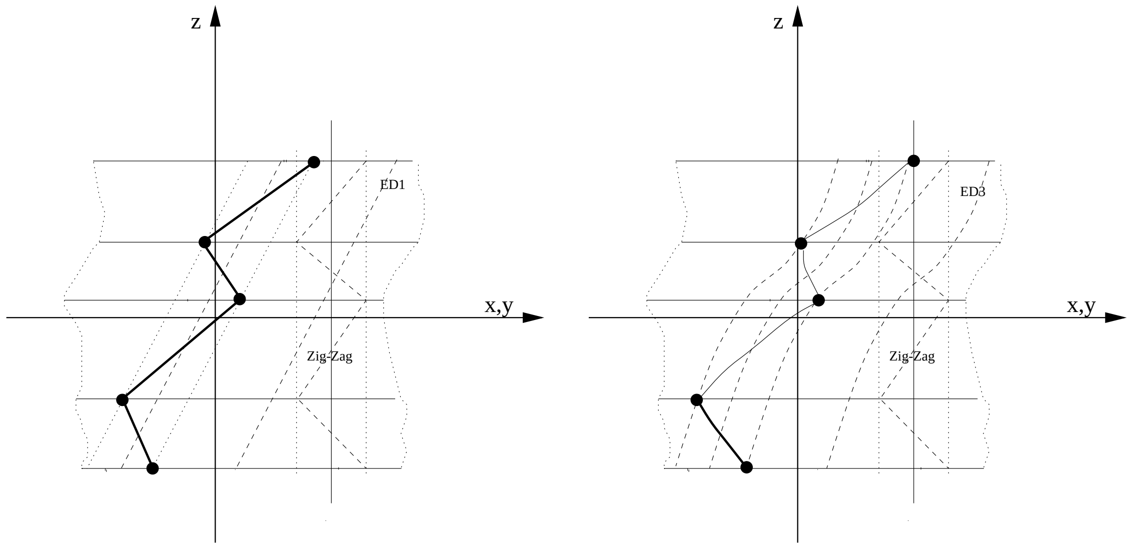

Figure 14. Geometrical meaning of Murakami's zig-zag function. Linear case and higher order cases

An extension to a higher order displacement field was proposed by Toledano and Murakami [141]. Parabolic ψ and cubic φ terms were added to the Murakami's displacement field,

$$u_i(x,y,z) = u_i^0 + z\phi_i + z^2\psi_i + z^3\varphi_i + (-1)^k\zeta_k\, u_{iZ} \quad i = 1,2,3 \tag{31}$$

Geometrical meaning of such a displacement fields is given in Figure 14. The related transverse stress fields, not quoted herein, was assumed of forth and fifth order for trasverse shear and transverse normal stresses, respectively. Toledano and Murakami in a subsequent paper [142] applied RMVT in conjunction with a layer-wise description of both displacement and transverse stress fields. The displacement field was considered linear in each layer for the in-plane components u_x and u_y, while the transverse displacement was kept constant for the whole plate as in classical thin-plate assumptions. Transverse normal stress was discarded and transverse shear stresses were assumed to be parabolic in each layer.

A generalization, proposing a systematic use of RMVT as a tool to furnish a class of two dimensional theories for multilayered plate analysis, was presented by Carrera [62]. Even though this work was directed to the approximate solution techniques, it reported most of the ideas that were used in subsequent works. The ESLM displacement field was written in the following generalized form,

$$\boldsymbol{u} \;=\; \boldsymbol{u}_0 \;+\; (-1)^k\zeta_k\, \boldsymbol{u}_Z \;+\; z^r\, \boldsymbol{u}_r, \quad r = 1,2,..,N \tag{32}$$

Transverse stresses expansion was also generalized according to the following layer-wise expansion,

$$\boldsymbol{\sigma}_{nM}^k = F_t\, \boldsymbol{\sigma}_{nt}^k + F_b\boldsymbol{\sigma}_{nb}^k + F_r\boldsymbol{\sigma}_{nr}^k = F_\tau\boldsymbol{\sigma}_{n\tau}^k, \quad \tau = t,b,r, \;\; r = 2,3,..,N;\; k = 1,2,..,N_l \tag{33}$$

It is now intended that the subscripts t and b denote values related to the top and bottom layer–surface, respectively. These two terms consist of the linear part of the expansion.

The thickness functions $F_\tau(\zeta_k)$ have now been defined at the k–layer level,

$$F_t = \frac{P_0 + P_1}{2}, \quad F_b = \frac{P_0 - P_1}{2}, \quad F_r = P_r - P_{r-2}, \qquad r = 2, 3, .., N \tag{34}$$

in which $P_j = P_j(\zeta_k)$ is the Legendre polynomial of the j-order defined in the ζ_k–domain : -1 $\leq \zeta_k \leq$ 1. For instance, the first five Legendre Polynomials are $P_0 = 1$, $P_1 = \zeta_k$, $P_2 = (3\zeta_k^2 - 1)/2$, $P_3 = \frac{5\zeta_k^3}{2} - \frac{3\zeta_k}{2}$, $P_4 = \frac{35\zeta_k^4}{8} - \frac{15\zeta_k^2}{4} + \frac{3}{8}$. The chosen functions have the following properties:

$$\zeta_k = \begin{cases} 1 & : \; F_t = 1; \quad F_b = 0; \quad F_r = 0 \\ -1 & : \; F_t = 0; \quad F_b = 1; \quad F_r = 0, \end{cases} \tag{35}$$

Layer-wise expansion introduced for stresses can be extended to displacement fields,

$$\boldsymbol{u}^k = F_t\, \boldsymbol{u}_t^k \;+\; F_b\, \boldsymbol{u}_b^k \;+\; F_r\, \boldsymbol{u}_r^k = F_\tau\, \boldsymbol{u}_\tau^k \qquad \tau = t, b, r, \;\; r = 2, 3, ..N, \;\; k = 1, 2, .., N_l \tag{36}$$

Applications of what is reported in [62],[63], to derive governing equations in a strong form have been given in several papers. These are overviewed in the following.

Layer-wise mixed analyses were performed in Carrera [145], [146] for the static case. Governing equations for dynamic cases were given in [147] Transverse normal stress effects in the static and dynamic case were discussed in [150], [151] These two works extended Koiter's recommendations on multilayered structures in the following terms: "Any refinements of classical models are meaningless, unless the effects of interlaminar continuous transverse shear and normal stresses are both taken into account in a multilayered shell theory." Higher order displacement and stress fields (linear, up-to fourth order) were considered in [153] and subsequent works. Different ways of computing transverse stresses were compared in [107]. Stresses from an assumed model, *a priori,* were compared to those calculated *a posteriori,* i.e. from Hooke's law and by integration of three-dimensional indefinite equilibrium equations. ESLM and LWM were implemented in both RMVT and PVD cases. Recently, Messina [154] has compared RMVT results to PVD ones. Both cross-ply and angle-ply plates were analyzed. Transverse normal stresses were however discarded in this work. RMVT has been also applied to trace the response of laminated plates subjected to thermal loadings which vary in the thickness direction [16]. A further interesting work, which was inspired to articles relate to RMVT applications, have been proposed by Ali, Bhaskar and Varadan [157]. These authors uses the zig-zag function introduced by Murakami in a HOT similar to those by [82]. A classical displacement formulation was employed. These theories remain a particular case of those in [16].

All the above discussed papers are related to plate geometries. The first discussion on the application of RMVT to shells was also made by Reissner [57]. A particular case of the Toledano and Murakami [142] type theory was extended to cylindrical shells by Bhaskar and Varadan [155] and by Jing and Tzeng [156]. A systematic extension of RMVT to shells has been proposed by Carrera in recent works [148]–[151].

Before concluding this section it is of interest to give some hints to the so-called Weak Form of Hooke's Law, WFHL.

Full use of Reissner's theorem requires solving a problem in terms of both displacement and transverse stress variables. This can result to be expensive. In order to preserve the advantages of a classical displacement formulation a WFHL was proposed in [62]. The WFHL, which was completely inspired by RMVT, permits one to express, in a weak sense, transverse stress variables in terms of the displacement ones. According to what in [62] the truncated Legendre expansion for displacement and transverse stress variables can be put in the weighted residual form in the thickness direction:

$$\int_{A_k} F_s(\boldsymbol{\epsilon}_{nG}^k - \boldsymbol{\epsilon}_{nH}^k) dz = 0, \quad s = t, b, 2, 3, .., N \tag{37}$$

As for the RMVT, Eq.(37) imposes compatibility of transverse strains. The difference is that the integral is now only introduced in the z–direction.

On substitution of a given displacement and the transverse stress models, as well as a given Hooke's law and strain displacement relation, and by integrating along z, the set of Eqns.(37) leads to a relation between stress and displacement variables that can be formally written in the following array form:

$$\boldsymbol{H}_u^k \, \boldsymbol{u}^k - \boldsymbol{H}_\sigma^k \, \boldsymbol{\sigma}^k = 0 \tag{38}$$

where $\boldsymbol{H}_\sigma^k$ is a square symmetric nonsingular matrix, while $\boldsymbol{H}_u^k$ is rectangular, singular and non symmetric. Examples of these matrices are reported in [62]. Under certain conditions, see [62] the Eqns.(38) can be explicitly written as,

$$\boldsymbol{\sigma}^k = - \; {\boldsymbol{H}_\sigma^k}^{-1} \, \boldsymbol{H}_u^k \, \boldsymbol{u}^k \tag{39}$$

which consists of the wanted relation between stress and displacement variables.

4.5 Full Mixed Theories

A few other axiomatic type theories have been developed and applied to layered structures. As discussed in the case of RMC theory, a natural manners to fulfill C_z^0–requirements consists to assume both displacements and stresses as unknown variables. Full mixed methods have been therefore developed which make use of classical mixed variational statments and assume all six stress components and the three displacement as unknown variables. A detailed discussion with literature has been reported by Pagano [34]. A mirable examples of full mixed theory along with layer-wise description has been presented and documented in [34]. In-plane stress were assumed linear in z while transverse stress expansions were established according to the 3D equilibrium equations Eqn.(1): shear component were assumed parabolic while normal component was assume of third order. A full mixed variational equations in the form developed by Reissner [158] was employed to derive consistent differential equations in each layer. Comparison with full 3D finite element results showed the high accuracy of the proposed technique to evaluate local stress fields for generally laminated structures as well as stress fields with correspondence to free edges.

More recent applications of full mixed theories in the framework of equivalent single layer description and plate and shell geometries have been proposed by Zenkour *et al.* [159]–[161]. A linear FSDT and HOT displacement models were employed along with a stress field similar to that used by Pagano [34].

An interesting discussion on the possible improvement of FSDT type models by using mixed and partially mixed formulation has been given by Auricchio and Sacco [162]. The Hellingher-Reissner mixed principle [58] has been employed in this work for the determination of FSDT governing equations. Two FSDT type models, both describing transverse shear stresses as independent variables, were discussed and implemented.

5 REVIEW OF OTHER APPROACHES

5.1 Asymptotic Theories

Staring by the early works [21], [53] which were originally developed for monocoque structures a few extensions to layered structures have been proposed in the open literature. Due to the increasing number of parameters (thickness, number of layers and mechanical properties such as the value of the orthotropic ratio of the lamina) the applications of asymptotic technique [163]–[173] to layered structures has not been leading to conclusions as much exhaustive as those known for the isotropic one layer cases [53]. In particular we refer to the recent work by Hodges and co–authors [168],[171] which where based on asymptotic correction, an optimum Reissner-like theory was presented. Very recent applications of Cicala's method has been provided by Antona and Frulla [174].

5.2 Continuum Based Theories

Many 'stress resultants', or 'continuum based theories' have been extended to layered structures. Some relevant contributions are outlined in the following. Eriksen and Truesdell [175] initeated the direct approach to the construction of shell theories by considering the shell as a surface with oriented media. In the light of the theory of a deformable surface in [51], if a shell is regarded has a Cosserat Surface, then we have a general nonlinear theory which is based on thermo-dynamical principles of continuum mechanics. Given the basic postulates by Green and Naghdi [49], the theory of a Cosserat surface may be regarded as and exact theory of deformable surfaces which is also applicable to shells. Relevant multilayered implementations were also given by Green and Naghdi [176]. Sun, Achenbach and Herrmann [177] proposed a plate theory for dynamic analysis of laminated media composed of alternating matrix and fiber–reinforced layers. The extension to shells was given by Grot [178]. These two works use a smoothing operation to replace by a homogeneous continuum the original layered medium, which displacements in each layer were expressed by means of a two term expansion. Layer-wise description were introduced in the so–called multi-director theory by Epstein and Glockner [179], [180]. Transverse normal strain/stress effects were retained by Epstein and Glockner who emphasized the fundamental role of such an effects, i.e. Koiter recommendation, for the nonlinear analysis of multilayered shells.

Part III Review of Finite Elements

6 POSSIBLE DEVELOPMENTS

Anisotropy, nonlinear analysis as well as the mentioned complicating effects, such as the C_z^0–Requirements, the couplings between in-plane and out-of-plane strains, make the analysis of layered composite structure complicate in practice. Analytical, closed form solutions are available in very few cases. In most of the practical problems, the solution demand applications of approximated computational methods.

Many computational techniques have been developed and applied to layered constructions. A full mixed 3D finite difference technique was developed by Noor and Rarig [181]. More recently a differential quadrature technique has been proposed by Malik [182], Malik and Bert [183] and applied by Liew *et al.* [184]. A boundary element formulation has been proposed by Daví *et al.* [185]–[187]. A two–phase predictor-corrector computational procedure has also been presented by Noor *et al.* [188]–[190]. Eshelby-Stroh formalism has been used by Vel and Batra [191], [192] to solve 3D problems of anisotropic rectangular plates by giving approximate solutions in terms of infinite series.

Exhaustive overview on several computational techniques and their applications to laminated structures can be read in the already mentioned review articles [2]-[20].

Among the computational techniques implemented for layered plate and shell analyses a predominant role has been played by Finite Element Method, FEM. Both research oriented and commercial FEM codes are, in fact, extensively used as standard tools in both Academic and Industrial Institutions. Implementation of plate and shell elements is a quite cumbersome topic, even though traditional isotropic one-layered structures are considered, see [193]. Last fourthy years literature has continuously proposed improved plate/shell formulations based on several approaches and techniques. Applications to layered structures introduce further complications in the implementation process, such as those described in Part I.

Many finite elements for multilayered structures have been proposed. Many of these were based on the approaches discussed in Part II. Others FEs are instead peculiar of FE implementations. For our convenience we group the main developments on FE for layered plates an shells in the two following categories:

- Two-Dimensional Plates and Shell Elements, 2D-PSE.
  Plate/shell multilayered elements in which FE approximations are introduce with correspondence to a certain reference, plate/shell surface (Figure 15). Is this case one of the axiomatic, asymptotic or stress resultants two dimensional theory overviewed in Part II has been implemented.

- 3D-Dimensional Plates and Shells Elements, 3D-PSE.
  FE approximations are introduced at 3D level (Figure 16) while two-dimensional hypotheses are imposed as constraints equations. This is the case of so-called continuum based, or degenerated plate/shell approach, in which a plate/shell is seen as a 3D continuum while kinematic assumptions are introduced as constraints by means of Lagrange multiplier.

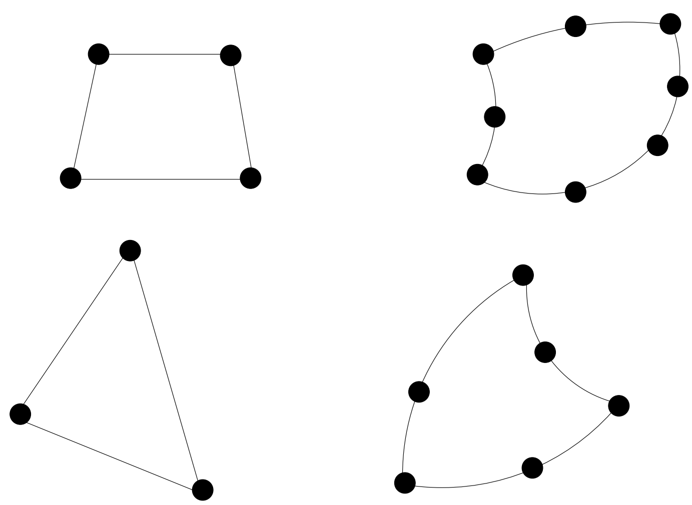

Figure 15. Rectangular and triangular 2D elements

In addition the these two type of grouping two further possibilities are related

- to the treatment of variables at element level, such as *Hybrid Elements*;

- to the discretization technique used in the plane direction *Hierarchy or h-techniques* or in the thickness directions *s-techniques*.

Hybrid methods are typical of any computational technique in which the domain is decomposed in sub-domains (elements) and governing equations are written at sub-domains level. $h-/s-$ techniques belong to the so-called global-local approaches.

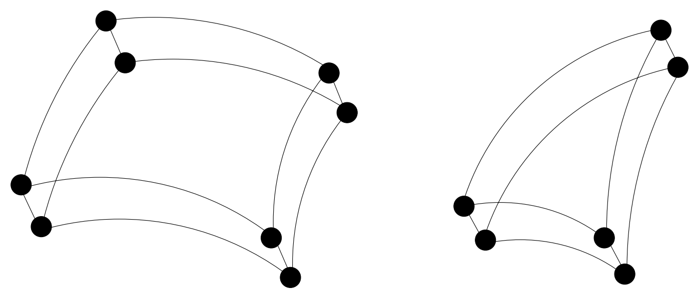

Figure 16. Rectangular and triangular FE degenerated elements

Readers are advised that a discussion of numerical problems, such as locking mechanisms, spurious mode, drilling elements, reduced integration techniques, enhanced strain assumptions has been herein omitted. On the other hand, it is author's experience that no additional significant, numerical problems arise by introducing of both anisotropy and layered construction, with respect to those problems that were already known for traditional monocoque, isotropic cases. Anyway, the latest advanced on these topics have been discussed and largely covered in the recent overview article by Yang *et al.* [18].

As it was for the overview of theories presented in Part II, most of the discussion herein quoted deals with finite elements formulated on the bases of axiomatic type theories. Such a choice has been justified by the fact that most (about 90 per cent) of the available FE literature on layered plate/shell elements refers to such an approach. Two main sections, therefore follows. The first one considers axiomatic type finite elements. The second one deals with others FEs.

7 FE BASED ON AXIOMATIC APPROACH

7.1 Classical and Refined FEs

FEs based on CLT and FSDT

Example of FEs implemenation of Classical Lamination Theory, CLT are given in [194]–[196]. Applications to shell and to nonlinear problems are also given in these works. Papers concerning laminated plate elements including classical transverse shear effects, FSDT, have been developed by Pryor and Barker [197], Noor [198], Mantegazza and Borri [199], Noor and Mathers [200], Panda and Natarayan [201] and Reddy [202]–[204]. However, early FSDT type elements showed severe stiffening in thin plate limits. Such a numerical mechanism, known as shear locking, was first contrasted by implemenation of numerical tricks, such as reduced/selective integration schemes [206]–[208]. Extension to nonlinear problems requires to contrast hourglass mechanisms originated by spurious zero energy modes that are introduced by sub-integration techniques. So-called mixed interpolation of tensorial components was implemented to this purpose. Many articles are available on that topic; examples are the papers [209]–[218].

FEs based on HOT

A large variety of plate/shell finite element implementations of HOT that were discussed in Sec.5.1 have been proposed in the last twenty years literature. Static and dynamic FE matrices as well as linear and non linear applications have been developed. HOT based C^0 finite elements (C^0 means that continuity is required only for the unknown variables and not for their derivatives) were discussed by Kant and co-authors [219]–[222]. Finite element version of the model at Eqns.(12) has been developed by Phan and Reddy [223]. A comprehensive discussion of HOT type theories and related finite element suitability has been provided by Tessler [224]. Inclusion of σ_{zz} along with a specific treatment of transverse shear stresses was proposed in [224]–[226]. The employed HOT was similar to that originally proposed by Hildebrand Reissner and Thomas [82]. A class of quadrilater finite plate elements have been proposed by Barboni and Gaudenzi [227]. The HOT discussed in [92] were employed. Stiffness and dynamic matrixes were derived accounting for linear and third order expansion for the in-plane displacements u_1, u_2 along with constant and quadratic expansion for the transverse component u_3. Both linear and parabolic Lagrangian type shape functions were implemented. Extension of Phan and Reddy [223] plate elements vs the inclusion of transverse normal strains were considered by Singh *et al.* [228]. FEs based on a HOT which use only displacement expansion similar to that by Murakami [140] have been presented by Bhaskar and Varadan [229] and more recently by Ganapathy and Makhecha [230]. Many other papers are available in which HOT have been implement for plates and shells, details can be found in the books by Reddy [28] and Palazotto and Dennis [27].

FEs based on Layer-wise HOT

Finite element implementations of layer-wise theories in the framework of axiomatic type theories have been proposed by many authors. Reddy *et al.* [106], [231],[232] developed plate elements on the basis of generalized layer-wise expansion. Gaudenzi *et al.* [233] extended the developments to extensive comparison between LW and ESLM multilayered finite element with possibilities of including fictitious interfaces and discarding physical ones. Triangular plate elements for plates and shells have been recently proposed by Botello *et al.* [234]. An interesting condensation technique is presented in this last article. Such a technique permits one to switch from layer-wise to equivalent single layer description by condensation of stiffenes matrices from layer to multilayer level.

Multilayerd plates and shells for geometrically nonlinear analysis

As underline at the and of Sec 4, geometrically nonlinear problems play a predominant role in the analyses of composite structures. Recent examples of civil and fighter aircrafts have been designed by requiring that some of their panels can carry loadings in the postbuckling range. Many efforts, have been therefore made by FE literature vs the development of efficient multilayered plate/shell elements suitable for nonlinear problems (describing large rotations and large displacements) and including HOT effects. Consistency between the approximations made on rotations and those typical of FSDT type theories were discussed by Sacco and co-authors [235]–[237]. Reddy and co-authors [238]–[240] have developed a moderate rotation theory for anisotropic shells. Application to plate and shells were given in these works along with comparison to FE formulations based on stress resultant approaches. Treatment of nonlinear problems including environmental loadings such as thermal stresses anf hydrothermal effects have been addressed by Kapania and co-authors [241]–[243]. Triangular shell elements have been mostly implemented in these works.

One of the most remarkable contribution to nonlinear analysis of laminated flat and curved structures comes from the German School. A research project supported by the German National Science Foundation, DFG, was devoted to developments of plate and shell multilayered elements suitable for linear and nonlinear problems. The significant traditions of German School on shell theory as well as on finite element treatment of nonlinear problems have led to important conclusions. The articles [244]–[250] are herein mentioned. Equivalent single layer as well as Layer-wise descriptions were both implemented in the above mentioned works [244]–[250]. It was mainly concluded that layer-wise description is required for accurate description of stress field in nonlinear problems related to laminated plates and shells. Other works made by the same groups and which make use of different approaches with respect to those considered in this section, are overviewed in different paragraphs.

FEs based on mixed theories

The refined HOT by Reddy [80] has been also used in the framework of full mixed formulations by Putcha and Reddy [251], [252]. Authors' aim was to reduce the order of differential equations coming by classical application of the theory in [80] which requires five order polynomials to represent the interpolation functions. Linear and nonlinear analysis of laminated plates was performed in this work.

A recent interesting application of mixed methods direct to introduced a 'best' refinement of FSDT type theory has been proposed by Auricchio and Sacco [253],[254].

7.2 FEs based on Zig-Zag Theories

Lekhnitskii–Ren FEs

As written in Part I, Lekhnitskii–Ren theories although very promising, has been mostly ignored by the open literature. The paper by Ren and Owen [255] is the unique available finite elements which has been formulated on the basis of LR theories.

Ambartsumian–Whitney–Rath-Das FEs

Dozens of finite elements have been proposed based on Ambartsumian-Whitney-Rath-Das type theory. The first FE applications of a particular case of AMRD theories have been done Dischiuva, Cicorello, Dalle Mura [256]. The used linear, zig-zag displacement field was not able to include top/bottom homogenous conditions for the transverse shear stresses and showed high deficiency to describe the response of unsymmetrically laminate plates. Further particular cases of AWRD theory, related to quadrilater and triangular elements were also presented in [257], [258]. The most relevant contributions to plates and shells FE implementations of AWRD have been given by the French group leaded by Professor Touratier. The first full FE implementation of AWRD has been, in fact, developed by Beakoua and Touratier [130]. The shell theory introduced by Rath and Das [117] was implemented with a few modifications concerning the form of the function $f(z)$ in Eqns.(25), which was assumed of trigonometric type in [130]. Further works were proposed by Touratier and co-authors and were related to layered beams, plates and shells as well as linear and nonlinear applications [259]-[267]. An independent FE development with full implemenation of AWRD theories were presented by Cho and Parmeter [268]. Further FE applications of of AWRD are documented in [269]-[271]. Further relevant contributions to AWRD theories as well as to their finite element implementations have been given by Averill and co-authors [272]–[275]. The layer-wise from of AWRD theory including transverse normal stress effects were introduced in these last works.

RMC FEs

A first FEM approach to multilayered structures by means of RMVT was presented by Jing and Liao [276]. RMVT was employed in the framework of a partial hybrid formulation. A self equilibrated stress field was restricted to the three in-plane stresses. As usual in hybrid formulation, stress unknowns were eliminated at the element level in the implemented finite hexaedron element for each layer. The numerical results were restricted to cross-ply plates and showed good accuracy with respect to exact solutions while improvements were shown in comparison to other refined analyses. An application of RMVT to develop standard finite elements was proposed by Rao and Meyer–Piening [277]. Toledano and Murakami's [142] theory was used. Stress unknowns were eliminated before introducing FE approximations by employing a technique which is equivalent the WFHL described at the end of Sec.5.4. That is, only displacements were taken as nodal variables in the considered ESLM framework. Applications were quoted for laminate and sandwich plates and were related to eight noded plate isoparametric element.

A generalization of RMVT as a tool to develop approximate solutions was given by Carrera [62]. Governing weak form equations and related matrix forms, for the general cases of weighted residuals method were derived. The extension of the standard Reissner-Mindlin model to multilayered structures was discussed by Carrera [278]. The obtained finite elements (four, eight and nine nodes) represent the FE implementation of the Murakami theory [140] and were denoted by the acronym RMZC, (Reissner Mindlin Zigzag interlaminar Continuity). The weak form of Hooke's law proposed in [62] was used to eliminate transverse shear stress variables. The numerical efficiency of RMZC models in the nonlinear cases were tested in subsequent works. Large deflection of postbuckling was analyzed by Carrera and Kröplin [279]. Non linear dynamic problems were solved by Carrera and Krause [281]. Applications to linear and nonlinear multilayered plates, embedding piezo layers, were given by Carrera [280]. Applications to sandwich plates were quoted by Carrera [282]. Corrected transverse shear stiffnesses were implemented by Carrera and Parisch [283] for a large displacements, large rotations shell elements. Snap–back problems were also discussed in this paper. Full extensions of RMZC to shell geometries have been done by Brank and Carrera [284]. An assumed shear strain concept has been implemented by Brank and Carrera [284] to eliminate shear locking mechanisms and to prevent spurious modes which are typical of alternative reduced integration techniques. All these applications have demonstrated that the RMZC finite element, formulated on the basis of RMVT, could be considered as the natural extension of well known plate/shell elements, which are normally implemented in most of commercial codes, to the analysis of multilayered structures.

A systematic application of RMVT to developed ESLM, as well as LW plate elements, have been recently provided by Carrera and Demasi [285]–[288].

8 OTHER FINITE ELEMENTS

8.1 FEs Based on Degenerated Continuum Approach

The continuum-based shell element is degenerated from a 3D isoparametric description element by imposing some geometric or static constrains. Displacement variables in a 3D continuum are expressed in terms of shape functions according to the following formula (see figure 16).

$$u_i = \sum_{i=1}^{N} \mathcal{N}_i(x_1, x_1, x_3) U_i \tag{40}$$

U_i are nodal values of displacements locate at the top and bottom shell surfaces. $\mathcal{N}_i$ are shape functions defined in the natural coordinates x_1, x_2, x_3, see [193] for details. 2D hypotheses are introduced in these type of elements as constraint equations. This type of element was introduced by Ahamad, Irons and Zienkiewicz [289] for the linear analysis and extended to nonlinear case by Ramm [290], Kräkeland [291], Bathe and Boulorchi [292], Application to anisotropic, composite shells were developed by Chang and Sawimiphakdi [293] and Chao and Reddy [294]. The convenience to use continuum based shell for geometrically non-linear analysis of composite, layered shells was discussed by Liao and Reddy [295], [296]. CLT and FSDT constraints were there introduced. Stiffened shells were considered in the last article. Constraints, which were equivalent to HOT assumptions (including fundamental role of transverse normal stress σ_{zz}), along with layer-wise kinematics as well as physical nonlinearity such as viscoelastic behaviors, were considered in the article by Pinsky and Kim [297]. Findings by Epstein and Glockner [179], [180], Hsu and Wang [101] and by Epstein and Huttelmeier [298] were employed in the article by Pinsky anf Kim [297].

8.2 FEs Based on Stress Resultant Approach

Epstein and Huttelmeier [298] developed finite elements on the basis of a multidirector surface which is a particular case or Cosserat surface. Layer-wise description was used in [298]. Simo *et al.* [299]–[301] proposed a stress-resultant-based geometrically exact shell theory and FE implemenation which is formulated entirely on stress resultants and is essentially equivalent to one director in-extendible Cosserat surface. In the already mentioned work by Pinsky and Kim a stress resultant shell element with geometrical and physical nonlinear was also proposed and implemented. Argyris and Tenek [302]–[304] developed finite element formulations in which 3D equations were replaced by a set of, in some sense, equivalent shell equations. So-called Natural Mode Method along with the Intelligent Physical Lumping technique were used by Argyris and Tenek to develop efficient multilayerd plate and shell finite elements capable in both linear and nonlinear range.

8.3 FEs Based on Asymtotic Approach

Finite elements based on asymptotic, approaches are quite rare in literature. Herein we mention the recent finite elements by Turn, Wang and Wang [305]. The Hellingher–Reissner functional was employed to write governing FE matrices.

8.4 Hierarchic, p– and s– FEs and Global/Local Approach

A formulation of 'hierarchic' finite elements based on p–extension were discussed by Babuska, Szabo and Actis [306]. p– denotes the order od the expansion of variables in the z thickness direction. Such an order can be fixed according to selection of discretization. A class of hierarchy finite elements were implemented for beams, plates and laminated arches. The implemented procedure was able to solve problems which may concurrently contain thick and thick structures. Analytical derivations and numerical evaluations were restricted to laminated strips. Results showed the capability of proposed hierarchy schemes to account for edges effects. Further analyses can be read in [307]–[308]. Similar techniques were applied and generalized to several shell problems and geometrically nonlinearity by Merk [309]. Further analysis related to axisymmetric laminated shell were provided by Liu nd Surana [310].

Somehow similar approaches, named 's-version' were introduced by Fish and Markolefas [311] who developed a general purpose robust computational tool to accurately resolve the stress field in vicinity of free edges without significant affecting the computational costs. Such a purpose was achieved by combining a set of smooth g–global interpolants in the thickness direction $\boldsymbol{u}^g$ with a displacement field which si l–locally defined within the layer $\boldsymbol{u}^l$,

$$\boldsymbol{u} = \boldsymbol{u}^g + \boldsymbol{u}^l$$

The additional field was superimposed only were high resolution of stress field was required.

All these type of approaches in which different two-dimensional theories are used in different regions of a real structure could be grouped in the so-called global/local approaches. The idea of locally enriching finite element solutions has been pioneered by Mote [312]. Applications to laminated structure were originated by the article by Pagano and Soni [313] who proposed to split the problem in a local and a global region. The stress continuity between the two regions and within the local region was enforced by means of the two–field variational principle. Nonlinear applications was provide by Noor [314]. An interesting and detailed overview of these type of approches has been provided by Reddy and Robbins [106]. Multiple methods, hierarchic FEs as well as mesh superposition technique were illustrated with example in [106].

8.5 Hybrid FEs

Hybrid stress finite elements are based on a modified complementary energy statement in which equilibrating intra-element stresses, and independently, intra-element or element boundary displacements are interpolated in terms of stress parameter and nodal displacement, respectively, see Atluri, Tong, Murakawa [59]. The stress parameters are then eliminate on an element level and a stiffness matrix is obtained. Four-node hybrid stress laminated plate elements, including transverse shear effects, have been originated by Pian [315], Pian and Mau [316] and developed first by Mau, Tong and Pian [317]. A quadrilater, four noded laminated plate element was implemented accounting for a layer-wise variable description. The transverse displacement u_3 was retained constant in the thickness direction while transverse shear strains were retained in the formulation. The same quadrilater element was reformulated by Spilker, Orringer and Witmer [318] and by Spilker, Chou and Orringer [319]. Extension to higher order in-plane displacement fields was given by Spilker [320]. Stress fields were defined for each layer with interlayer traction continuity and upper/lower laminate traction–free conditions enforced exactly. Extension to eight-noded plate element along with the introduction of layer-wise description (FSDT assumption were retained in each layer) was proposed by Spilker [321]. Numerical techniques were introduced in this last paper to contrast dangerous mechanisms related to the four-noded plate element.

A further hybrid stress element has been proposed by Moriya [322], who employed a different separation scheme for transverse shear stresses with respect to the others. Liou and Sun [323] proposed a layer-wise hybrid stress element which was based on trough-thickness linear displacement field for both in-plane and out-of-plane displacement. All stresses were also independently assumed in each layer. Partial hybrid stress plate were developed by Jing and Liao [276]. A twenty-nodes displacement field were employed in each layer. As for the stress field is concerned only two transverse shear stresses were calculated from the assumed stress fields while the others were directly computed from the constitutive stress/strain relations. Hybrid elements for laminated shell analyses were developed by Di and Ramm [324]. A higher order theory for both in-plane and out-of-plane displacement fields were employed and a quadrilater hybrid stress element was developed. Extension to geometrically nonlinear shell problem was presented by Rothert and Di [325].

9 CONCLUSIONS AND FINAL COMMENTS

The present work is a review of two-dimensional theories and finite elements that have been developed for layered plates and shells. Even though contributions that have were formulated on the basis of different approaches have been overviewed, most of the attention has been restricted to those theories and finite elements that were originally developed for layered structures. The following main remarks can be made.

- Multilayered plate and shell structures require appropriate modelings to handle the complicating effects that arise coming from their intrinsic in-plane and out–of–plane anisotropy. C_z^0–requirements should be dealt with for accurate descriptions of the stress and strain fields.

- The literature of the second half of the last century literature displays a large number of analytical and numerical two-dimensional contributions to layered structures. These are all characterized by the fact that a choice has to be made for each of the following points.

1. One of the following approaches can be employed to eliminate the thickness coordinate z: I. Continuum or stress resultants based models; II. Asymptotic type approaches; III. Axiomatic type approaches.
2. One of the following choices can be made for the unknown variables: I. stress formulation; II. displacement formulation; III. mixed formulation.
3. Variables can be considered at layer or multilayer levels according to the so-called: I. Equivalent Single-Layer Models. II. Layer-Wise Models.
4. The application of the finite element method introduces three further possibilities: I. developing degenerated FEs; II. using hybrid methods; III. employing global/local approaches along with p– or s–methods.

- As far as the extension to layered structures of classical models which were already known for monocoque plates and shells, the following remarks can be made.

1. The extension of classical thin plate/shell theories and related finite elements, such as Kirchhoff–Love models, CLT, to layered structures does permit the inclusion of fundamental transverse shear strain effects, and this leads to unsatisfactory results in thick structures analysis.
2. The extension of classical thick plate/shell theory and related finite elements, such as Reissner–Mindlin models, FSDT, to layered structures does not permit the description of both zig-zag effects and interlaminar continuity for transverse shear stresses, this leads to unsatisfactory results as far as very thick structures and local response is concerned.
3. According to Koiter's recommendation, any refinements of classical models could be considered meaningless, unless the effects of interlaminar continuous transverse shear and normal stresses are both taken into account in a multilayered plate/shell theory and related FE implementations.

- It has been outlined that three different approaches are available for a zig-zag theory. These have originated from the works by Lekhnitskii , Ambartsumian and Reissner . Lekhnitskii 's method has rarely been considered in open literature while the developments that are based on Reissner 's method are very suitable for the the treatment of transverse normal stresses/strains effects as well as for FE implementations.

- The use of the Weak Form of Hooke's Law should be considered as a possible method to extend classical theories/elements that were already known for monocoque plates and shells to layered structures.

- The use of a Layer-Wise description becomes mandatory to obtain a complete 3D description of stress states in laminates. Classical theories formulated on the basis of standard displacement formulations can lead to very accurate results if an adequate expansion is used in each layer for the displacement components. In such a case, transverse stresses can be directly computed from Hooke's law.

The present paper does not report any numerical results and assessments for the overviewed theories and finite elements. A companion paper, which has been scheduled for the near future, will provide a unified formulation for layered plate and shell theories and finite elements along with a numerical assessment and benchmarking.

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