



# **Hierarchical component-wise models for enhanced stress analysis and health monitoring of composite structures**

PhD candidate: **Alberto García de Miguel**

Supervisors: **Prof. Erasmo Carrera, Dr. Alfonso Pagani**

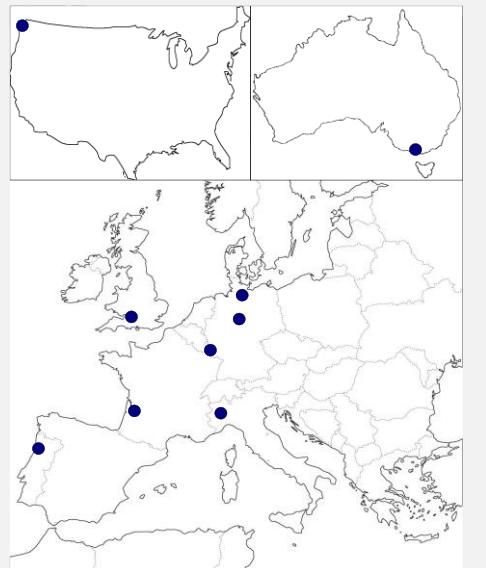
Politecnico di Torino  
Turin (Italy), March 11 2019

# Intro

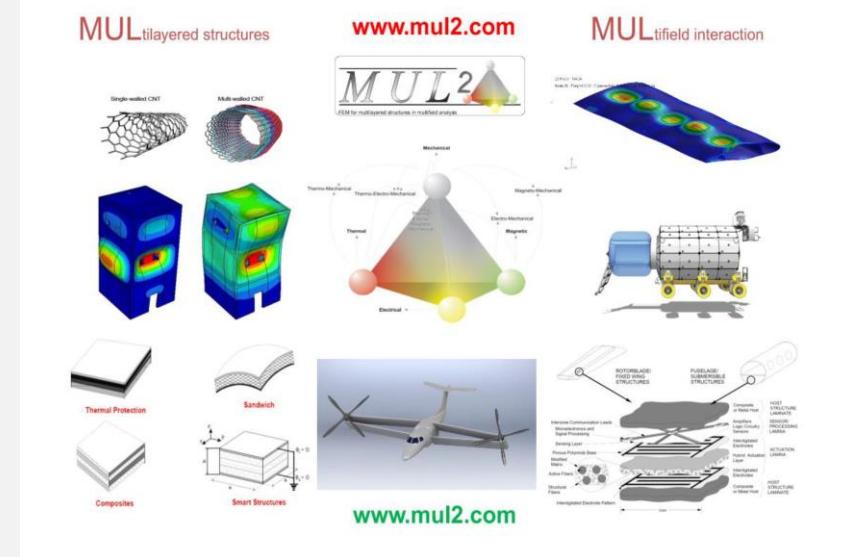
## Project

### FULLCOMP

FULLy integrated analysis, design, manufacturing, and health monitoring of COMPosite structures



## Mul2 group at PoliTO



### Numbers:

- 6 academic staff
- 2 Post-Docs
- 9 PhD students

# Outline

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- 01 Motivation and scope**
- 02 Hierarchical one-dimensional finite elements**
- 03 Analysis of composite structures via hierarchical beams**
- 04 Simulation of Lamb waves for structural health monitoring**
- 05 Conclusions**

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- 01 Motivation and scope**
  - 02 Hierarchical one-dimensional finite elements**
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# Motivation



Composite materials

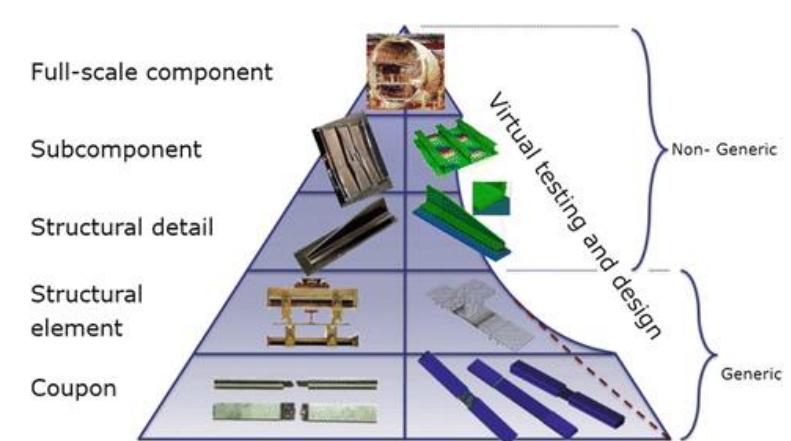
Highly heterogeneous



Complex mechanical response

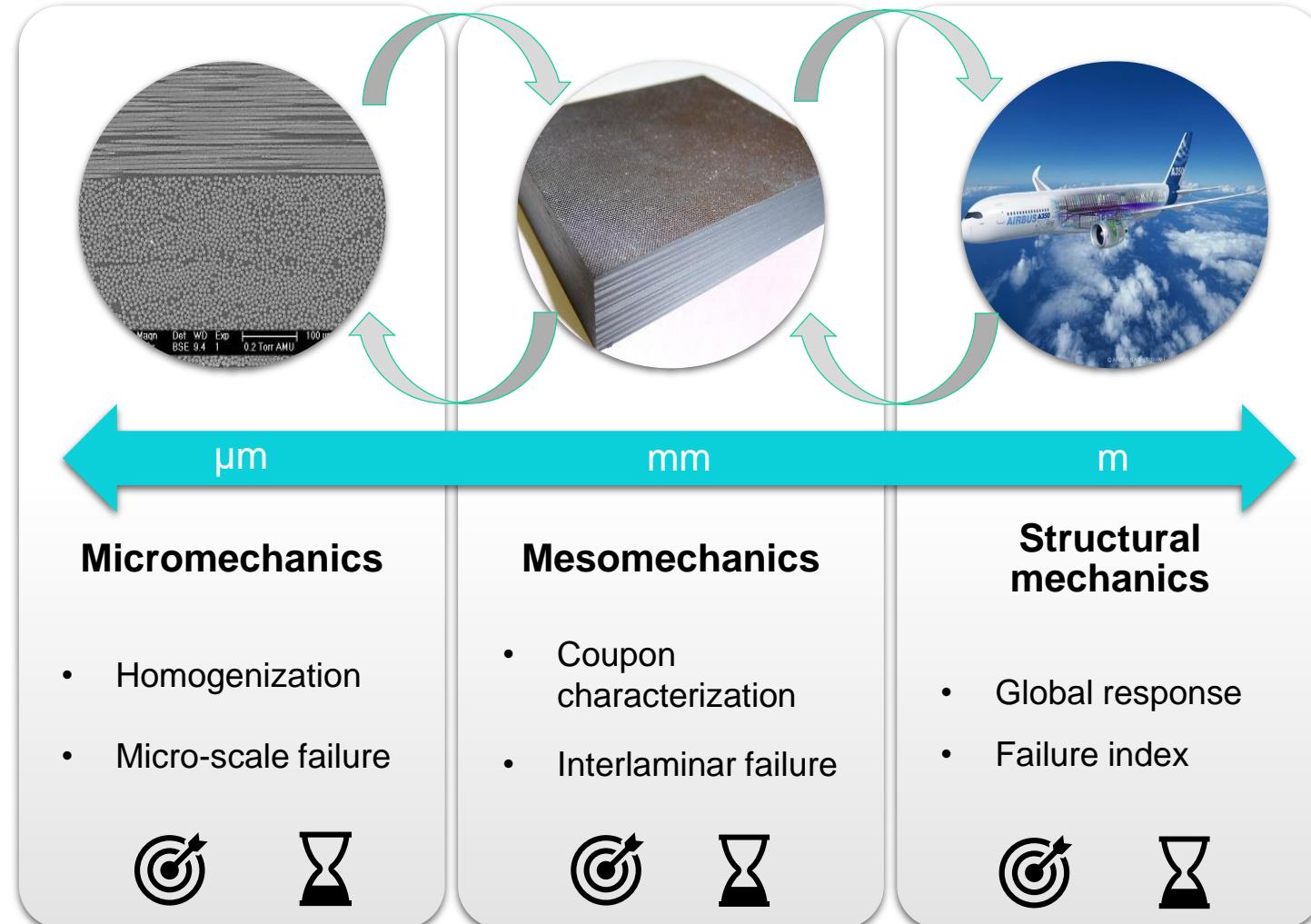


Need for reliable prediction tools

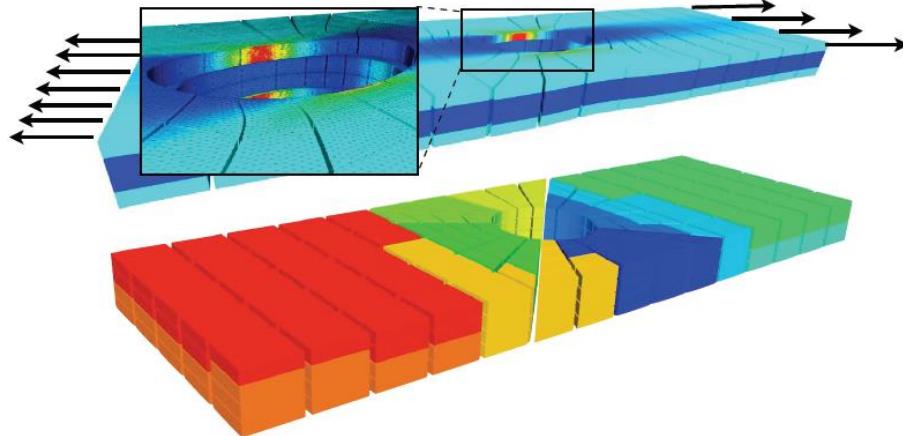


Falzon, B. and Tang, W. "Virtual Testing of Composite Structures: Progress and Challenges in Predicting Damage, Residual Strength and Crashworthiness", Springer, Cham, 2017.

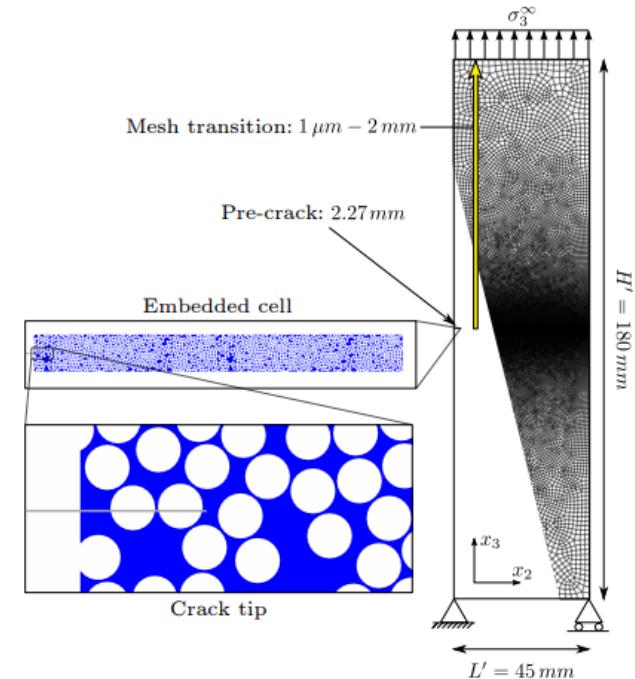
# Motivation



# Motivation



Allix, O. "Virtual delamination testing through non-linear multi-scale computational methods: some recent progress", [Computers, Materials and Continua 2013](#)

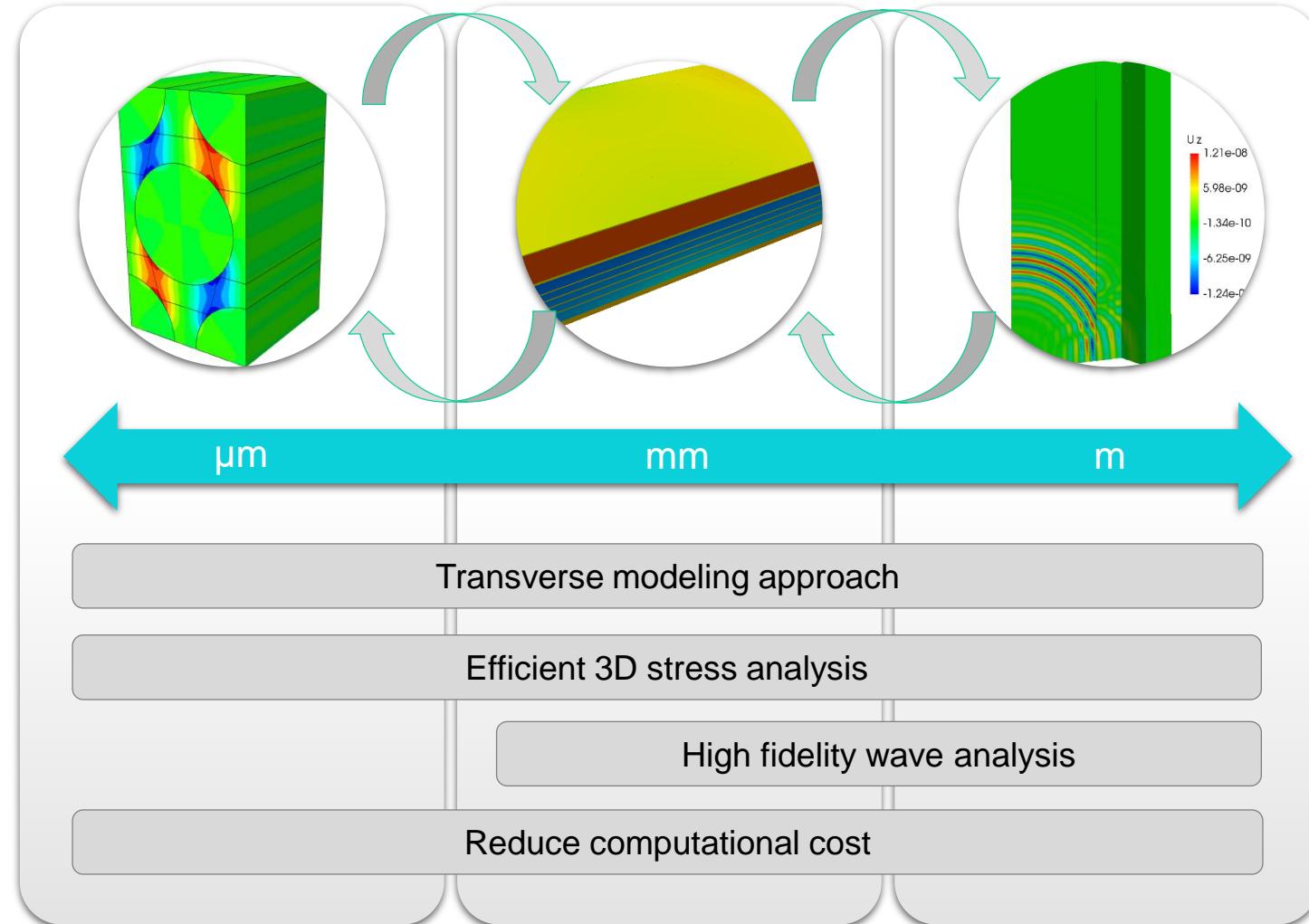


Herraez, M. "A numerical framework to analyze fracture in composite materials: From R-curves to homogenized softening laws", [IJSS 2018, 134:216:228](#)

**12,000,000 DOF**

**500,000 elements**

# Scope



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- 01 Motivation and scope**
  - 02 Hierarchical one-dimensional finite elements**
  - 03 Analysis of composite structures via hierarchical beams**
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  - 05 Conclusions**

# From classical to refined theories

## Beams



kinematic refinement

### Euler-Bernoulli

$$\begin{aligned} u_x(x, y, z, t) &= u_{x_1}(y, t) \\ u_y(x, y, z, t) &= u_{y_1}(y, t) - x u_{x_{1,y}}(y, t) - \\ &\quad z u_{z_{1,y}}(y, t) \\ u_z(x, y, z, t) &= u_{z_1}(y, t) \end{aligned}$$

### Timoshenko

$$\begin{aligned} u_x(x, y, z, t) &= u_{x_1}(y, t) \\ u_y(x, y, z, t) &= u_{y_1}(y, t) - \cancel{x} u_{y_2}(y, t) - \\ &\quad \cancel{z} u_{y_3}(y, t) \\ u_z(x, y, z, t) &= u_{z_1}(y, t) \end{aligned}$$

### Saint Venant

$$\begin{aligned} u_x(x, y, z, t) &= u_{x_1}(y, t) - \cancel{z} u_{x_2}(y, t) \\ u_y(x, y, z, t) &= u_{y_1}(y, t) - x u_{y_2}(y, t) - z u_{y_3}(y, t) \\ &\quad + \cancel{\psi(x, z)} u_{y_3}(y, t) \\ u_z(x, y, z, t) &= u_{z_1}(y, t) + \cancel{x} u_{z_2}(y, t) \end{aligned}$$

## Plates



kinematic refinement

### Kirchhoff–Love

$$\begin{aligned} u_x(x, y, z, t) &= u_{x_1}(x, y, t) - z u_{z_{1,x}}(x, y, t) \\ u_y(x, y, z, t) &= u_{y_1}(x, y, t) - z u_{z_{1,y}}(x, y, t) \\ u_z(x, y, z, t) &= u_{z_1}(x, y, t) \end{aligned}$$

### Reissner–Mindlin

$$\begin{aligned} u_x(x, y, z, t) &= u_{x_1}(x, y, t) + \cancel{z} u_{x_2}(x, y, t) \\ u_y(x, y, z, t) &= u_{y_1}(x, y) + \cancel{z} u_{y_2}(x, y, t) \\ u_z(x, y, z, t) &= u_{z_1}(x, y, t) \end{aligned}$$

### Reddy's Third-order theory

$$\begin{aligned} u_x(x, y, z, t) &= u_{x_1}(x, y, t) + \cancel{z} u_{x_2}(x, y, t) + z^3 u_{x_3}(x, y, t) \\ &\quad + z^3 u_{x_4}(x, y, t) \\ u_y(x, y, z, t) &= u_{y_1}(x, y, t) + \cancel{z} u_{y_2}(x, y, t) + z^3 u_{y_3}(x, y, t) \\ &\quad + z^3 u_{y_4}(x, y, t) \\ u_z(x, y, z, t) &= u_{z_1}(x, y, t) \end{aligned}$$

# The Carrera unified formulation

## Beams



$$u_x(x, y, z) = F_1(x, z) u_{x_1}(y) + F_2(x, z) u_{x_2}(y) + F_3(x, z) u_{x_3}(y) + \dots + F_M(x, z) u_{x_M}(y)$$

$$u_y(x, y, z) = F_1(x, z) u_{y_1}(y) + F_2(x, z) u_{y_2}(y) + F_3(x, z) u_{y_3}(y) + \dots + F_M(x, z) u_{y_M}(y)$$

$$u_z(x, y, z) = F_1(x, z) u_{z_1}(y) + F_2(x, z) u_{z_2}(y) + F_3(x, z) u_{z_3}(y) + \dots + F_M(x, z) u_{z_M}(y)$$

$$\mathbf{u}(x, y, z, t) = F_\tau(x, z) \mathbf{u}_\tau(y, t) \quad \tau = 1, \dots, M$$

## Plates



$$u_x(x, y, z) = F_1(z) u_{x_1}(x, y) + F_2(z) u_{x_2}(x, y) + F_3(z) u_{x_3}(x, y) + \dots + F_M(z) u_{x_M}(x, y)$$

$$u_y(x, y, z) = F_1(z) u_{y_1}(x, y) + F_2(z) u_{y_2}(x, y) + F_3(z) u_{y_3}(x, y) + \dots + F_M(z) u_{y_M}(x, y)$$

$$u_z(x, y, z) = F_1(z) u_{z_1}(x, y) + F_2(z) u_{z_2}(x, y) + F_3(z) u_{z_3}(x, y) + \dots + F_M(z) u_{z_M}(x, y)$$

$$\mathbf{u}(x, y, z, t) = F_\tau(z) \mathbf{u}_\tau(x, y, t) \quad \tau = 1, \dots, M$$



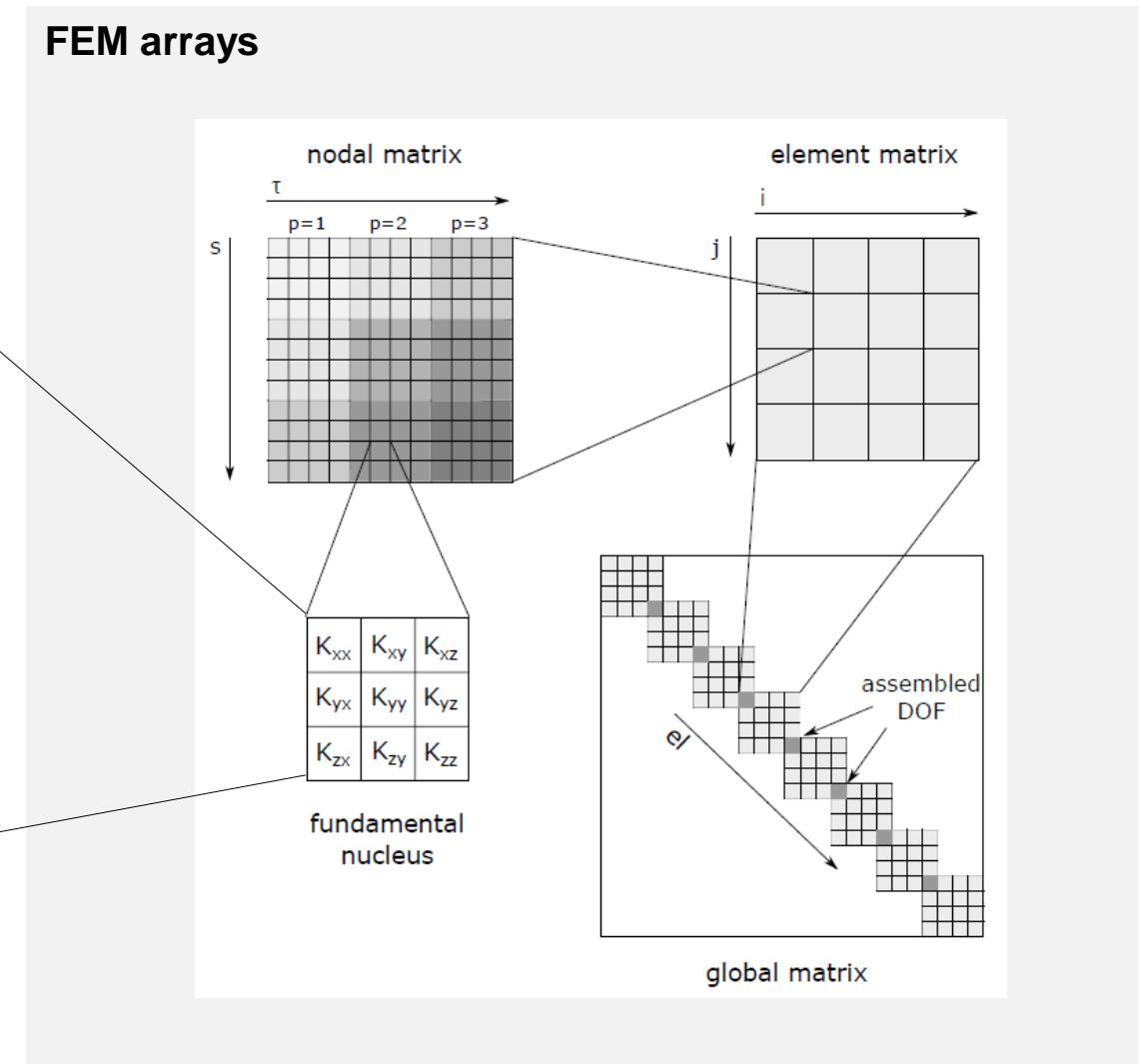
Carrera et. al. "Finite Element Analysis of Structures Through Unified Formulation", John Wiley and Sons, Ltd, 2014

# The Carrera unified formulation

## Fundamental nucleus

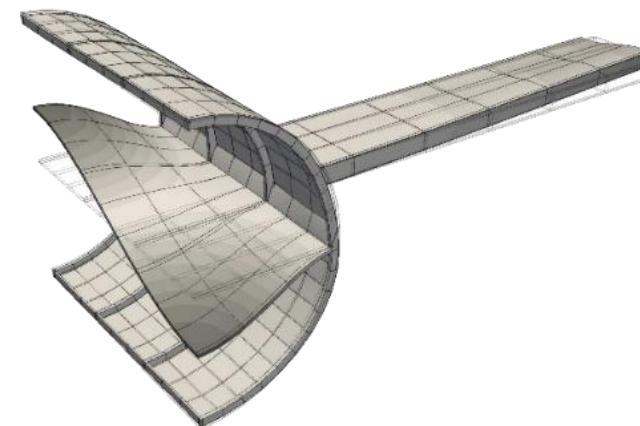
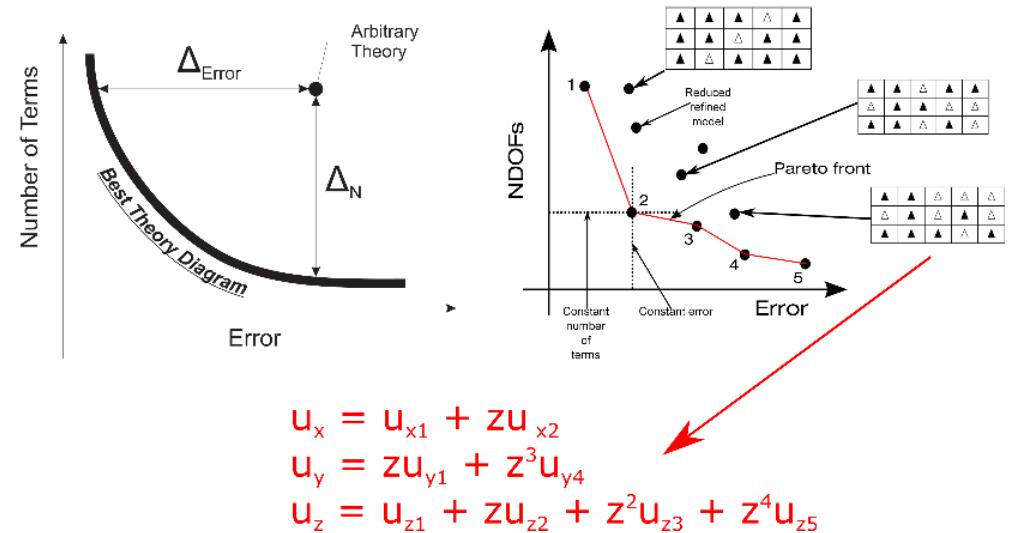
$$\begin{aligned}
 K_{xx}^{\tau sij} &= \tilde{C}_{22} I_{ij} E_{\tau,x s,x} + \tilde{C}_{44} I_{ij} E_{\tau,z s,z} + \tilde{C}_{26} I_{ij,y} E_{\tau,x s} + \tilde{C}_{26} I_{i,y j} E_{\tau s,x} + \tilde{C}_{66} I_{i,y j,y} E_{\tau s} \\
 K_{xy}^{\tau sij} &= \tilde{C}_{23} I_{ij,y} E_{\tau,x s} + \tilde{C}_{45} I_{ij} E_{\tau,z s,z} + \tilde{C}_{26} I_{ij} E_{\tau,x s,x} + \tilde{C}_{36} I_{i,y j,y} E_{\tau s} + \tilde{C}_{66} I_{i,y j} E_{\tau s,x} \\
 K_{xz}^{\tau sij} &= \tilde{C}_{12} I_{ij} E_{\tau,x s,z} + \tilde{C}_{44} I_{ij} E_{\tau,z s,x} + \tilde{C}_{45} I_{ij,y} E_{\tau,z s} + \tilde{C}_{16} I_{i,y j} E_{\tau s,z} \\
 K_{yx}^{\tau sij} &= \tilde{C}_{23} I_{i,y j} E_{\tau s,x} + \tilde{C}_{45} I_{ij} E_{\tau,z s,z} + \tilde{C}_{26} I_{ij} E_{\tau,x s,x} + \tilde{C}_{36} I_{i,y j,y} E_{\tau s} + \tilde{C}_{66} I_{ij,y} E_{\tau,x s} \\
 K_{yy}^{\tau sij} &= \tilde{C}_{33} I_{i,y j,y} E_{\tau s} + \tilde{C}_{55} I_{ij} E_{\tau,z s,z} + \tilde{C}_{36} I_{ij,y} E_{\tau,x s} + \tilde{C}_{36} I_{i,y j} E_{\tau s,x} + \tilde{C}_{66} I_{ij} E_{\tau,x s,x} \\
 K_{yz}^{\tau sij} &= \tilde{C}_{13} I_{i,y j} E_{\tau s,z} + \tilde{C}_{55} I_{ij,y} E_{\tau,z s} + \tilde{C}_{45} I_{ij} E_{\tau,z s,x} + \tilde{C}_{16} I_{ij} E_{\tau,x s,z} \\
 K_{zx}^{\tau sij} &= \tilde{C}_{12} I_{ij} E_{\tau,z s,x} + \tilde{C}_{44} I_{ij} E_{\tau,x s,z} + \tilde{C}_{45} I_{i,y j} E_{\tau s,z} + \tilde{C}_{16} I_{ij,y} E_{\tau,z s} \\
 K_{zy}^{\tau sij} &= \tilde{C}_{13} I_{ij,y} E_{\tau,z s} + \tilde{C}_{55} I_{i,y j} E_{\tau s,z} + \tilde{C}_{45} I_{ij} E_{\tau,x s,z} + \tilde{C}_{16} I_{ij} E_{\tau,z s,x} \\
 K_{zz}^{\tau sij} &= \tilde{C}_{11} I_{ij} E_{\tau,z s,z} + \tilde{C}_{44} I_{ij} E_{\tau,x s,x} + \tilde{C}_{55} I_{i,y j,y} E_{\tau s} + \tilde{C}_{45} I_{ij,y} E_{\tau,x s} + \tilde{C}_{45} I_{i,y j} E_{\tau s,x}
 \end{aligned}$$

## FEM arrays



# The Carrera unified formulation

**The right model at the right cost**



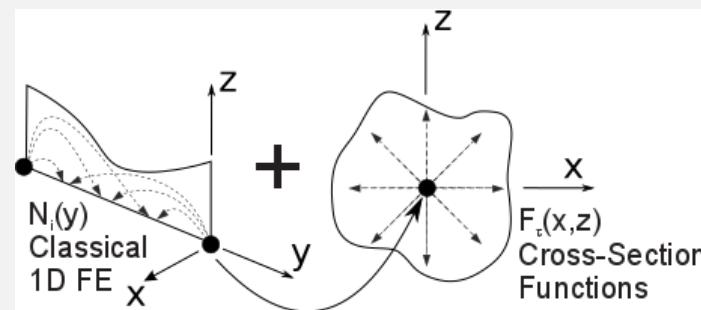
# The Carrera unified formulation

## Taylor Expansion

- Local (centered) expansion
- Taylor polynomials

$$\begin{aligned} u_x &= u_{x_1} + x u_{x_2} + z u_{x_3} + x^2 u_{x_4} + xz u_{x_5} + z^2 u_{x_6} \\ u_y &= u_{y_1} + x u_{y_2} + z u_{y_3} + x^2 u_{y_4} + xz u_{y_5} + z^2 u_{y_6} \\ u_z &= u_{z_1} + x u_{z_2} + z u_{z_3} + x^2 u_{z_4} + xz u_{z_5} + z^2 u_{z_6} \end{aligned}$$

- p-refinement



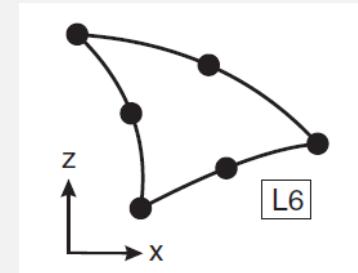
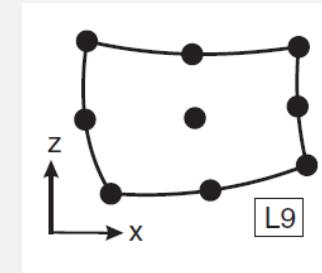
Carrera, E. and Giunta, G. "Refined beam theories based on Carrera's unified formulation.", *International Journal of Applied Mechanics*, 2010, 2(1):117–143

## Lagrange Expansion

- Non-local expansion (Jacobian transformation)
- Lagrange interpolation polynomials

$$\begin{aligned} F_\tau &= \frac{1}{4}(r^2 + r r_\tau)(s^2 + s s_\tau), & \tau = 1, 3, 5, 7, \\ F_\tau &= \frac{1}{2}s_\tau^2(s^2 + s s_\tau)(1 - r^2) + \frac{1}{2}r_\tau^2(r^2 + r r_\tau)(1 - s^2), & \tau = 2, 4, 6, 8, \\ F_\tau &= (1 - r^2)(1 - s^2) & \tau = 9, \end{aligned}$$

- h-refinement

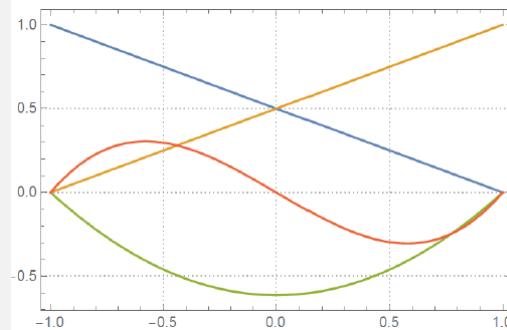


Carrera, E. and Petrolo, M. "Refined beam elements with only displacement variables and plate/shell capabilities.", *Meccanica*, 2012, 47(3):537–556.

# Hierarchical Legendre Expansion

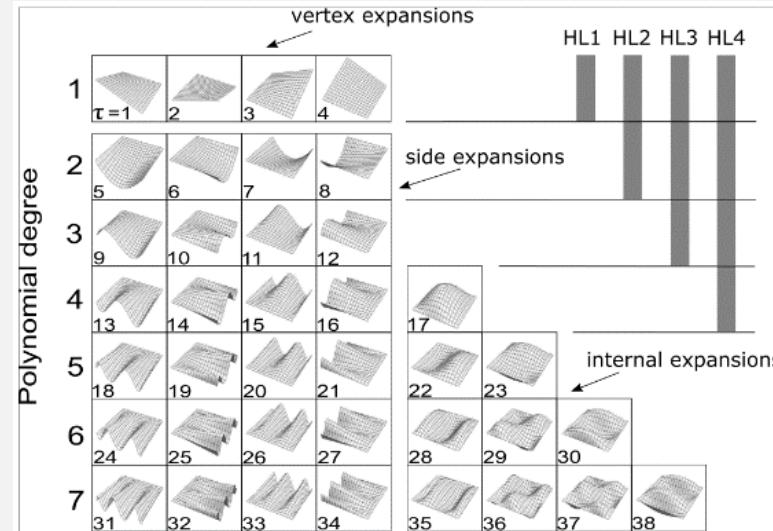
## HLE

- Non-local expansion
- Legendre modes
- h-, p-refinement



B. Szabò and I. Babuska. "Finite Element Spaces", John Wiley and Sons, Ltd, 2011

$$F_\tau =$$



➤ Vertex expansions

$$F_\tau = \frac{1}{4}(1 - r_\tau r)(1 - s_\tau s)$$

➤ Side expansions

$$F_\tau(r, s) = \frac{1}{2}(1 - s)\phi_{p_b}(r) \quad \tau = 5, 9, 13, 18, \dots$$

$$F_\tau(r, s) = \frac{1}{2}(1 + r)\phi_{p_b}(s) \quad \tau = 6, 10, 14, 19, \dots$$

$$F_\tau(r, s) = \frac{1}{2}(1 + s)\phi_{p_b}(r) \quad \tau = 7, 11, 15, 20, \dots$$

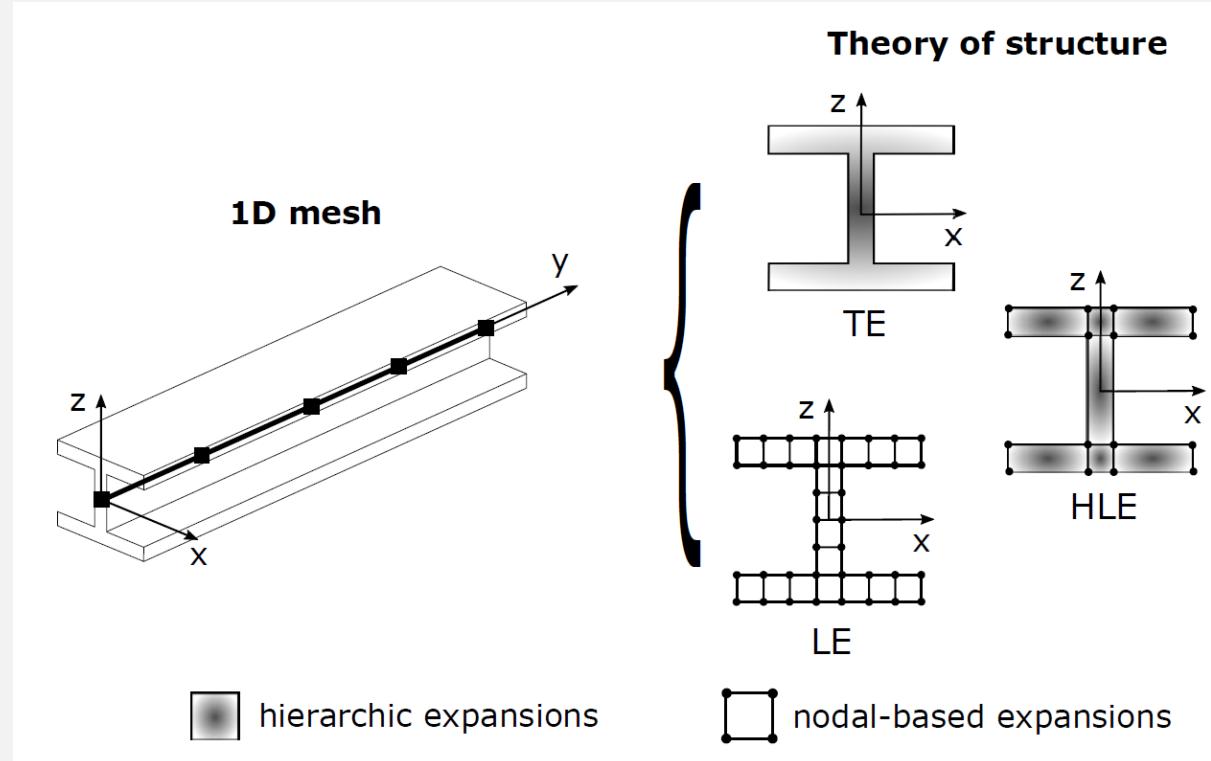
$$F_\tau(r, s) = \frac{1}{2}(1 - r)\phi_{p_b}(s) \quad \tau = 8, 14, 16, 21, \dots$$

➤ Internal expansions

$$F_\tau(r, s) = \phi_{p_1}(r)\phi_{p_2}(s)$$

[1] Carrera, E., de Miguel, A., and Pagani, A., *Hierarchical theories of structures based on Legendre polynomial expansions with finite element applications*, IJMS, Vol. 120, 2017, pp. 286 – 300  
 [2] Pagani, A., de Miguel, A., Petrolo, M., and Carrera, E., *Analysis of laminated beams via Unified Formulation and Legendre polynomial expansions*, Composite Structures, 2016, Vol 156, pp 78 - 92

# Hierarchical Legendre Expansion



# HLE: exact mapping

## Blending function method

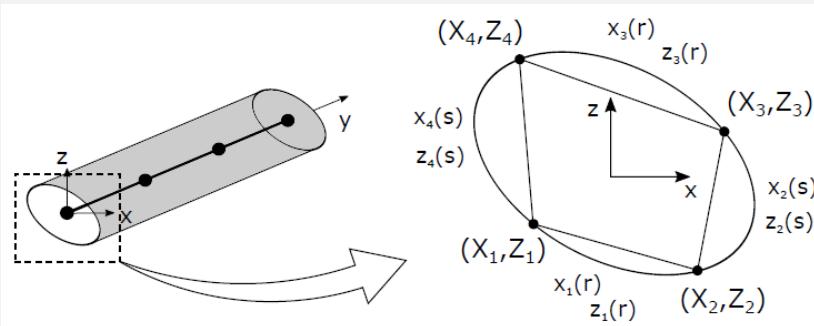


Gordon, W. and Hall, C. "Transfinite element methods: Blending-function interpolation over arbitrary curved element domains", *Numerische Mathematik*, 1973, 21(2):109–129

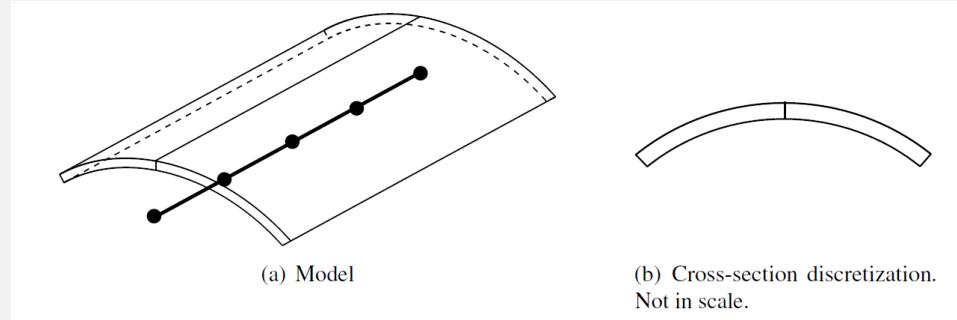
- p-schemes: coarse discretizations -> large domains
- HLE: modal unknowns
- Solution: non-isoparametric transformation over the section

$$x = Q_x(r, s) = F_\tau(r, s)X_\tau + \left(x_2(s) - \left(\frac{1-s}{2}X_2 + \frac{1+s}{2}X_3\right)\right)\frac{1+r}{2}$$

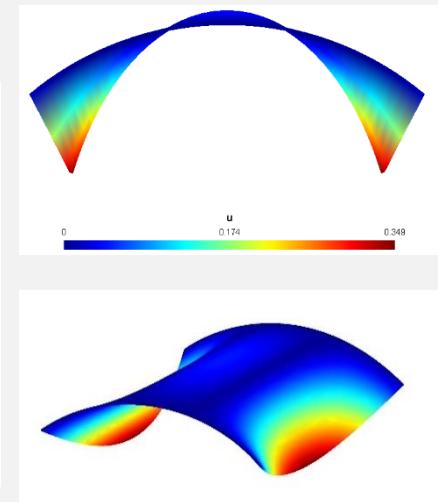
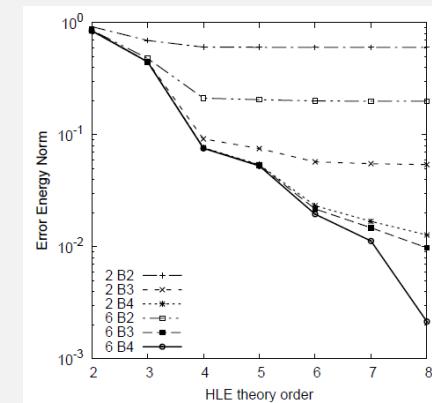
$$z = Q_z(r, s) = F_\tau(r, s)Z_\tau + \left(z_2(s) - \left(\frac{1-s}{2}Z_2 + \frac{1+s}{2}Z_3\right)\right)\frac{1+r}{2},$$



## Example: Scodelis-Lo roof



Numerical convergence:



# Locking-free beam elements

## Shear locking

Sudden increase of the shear stiffness in thin structures loaded under bending

Mitigation methods:

- Reduced integration

 Zienkiewicz, O. C. et al. "Reduced integration technique in general analysis of plates and shells", *International Journal for Numerical Methods in Engineering* 1971, 3(2):275–290.

- Mixed interpolation of tensorial components (MITC)

 Dvorkin, E. and Bathe, K. "A continuum mechanics based four-node shell element for general non-linear analysis", *Engineering Computations* 1984, 1(1):77–88.

 MacNeal, R. "Derivation of element stiffness matrices by assumed strain distributions.", *Nuclear Engineering and Design* 1982, 70(1):3 – 12.

- MITC for beam elements

 Lee, P.-S. "Geometry-dependent MITC method for a 2-node iso-beam element.", *Structural Engineering and Mechanics* 2008, 29(2):2203–221.

 Carrera, E. and Pagani, A. "Evaluation of the accuracy of classical beam FE models via locking-free hierarchically refined elements", *International Journal of Mechanical Sciences* 2015, 100:169–179

## MITC for higher-order elements

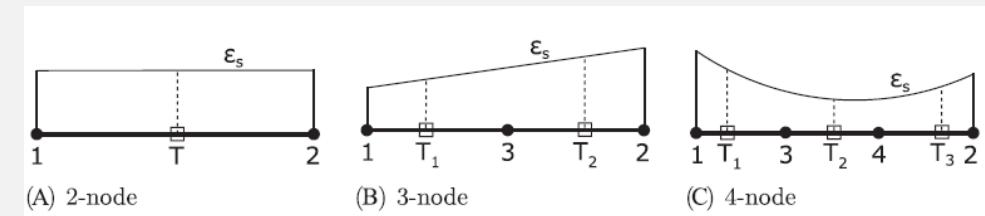
- Energy decoupled in shear and bending/membrane terms:

$$\delta L_{int} = \iint_{\Omega} \delta \boldsymbol{\epsilon}^T \boldsymbol{\sigma} \, d\Omega \, dy = \iint_{\Omega} (\delta \boldsymbol{\epsilon}_B^T \bar{\boldsymbol{\sigma}}_B + \delta \bar{\boldsymbol{\epsilon}}_S^T \bar{\boldsymbol{\sigma}}_S) \, d\Omega \, dy,$$

- Use of assumed functions of lower order for the shear strains

$$\bar{\boldsymbol{\epsilon}}_S = \bar{N}_m \boldsymbol{\epsilon}_{S_m} \quad m = 1, \dots, n_{node} - 1.$$

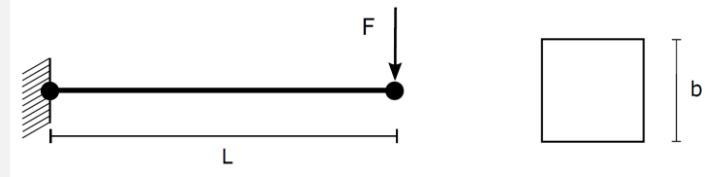
$$\bar{\boldsymbol{\epsilon}}_S = \bar{N}_m F_\tau (\mathbf{D}_{S_y} N_i \mathbf{I})_m \mathbf{u}_{\tau i} + \bar{N}_m (\mathbf{D}_{S_\Omega} F_\tau \mathbf{I}) N_{i_m} \mathbf{u}_{\tau i}$$



- MITC is not a selective integration, is a full integration of a mixed interpolated element

# Locking-free beam elements

## Example: cantilever beam



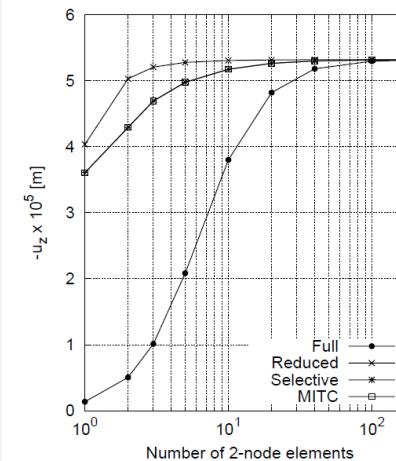
- $L=1$  m
- $b=0.1$  m
- $F=100$  N
- Aluminum

Analytical:

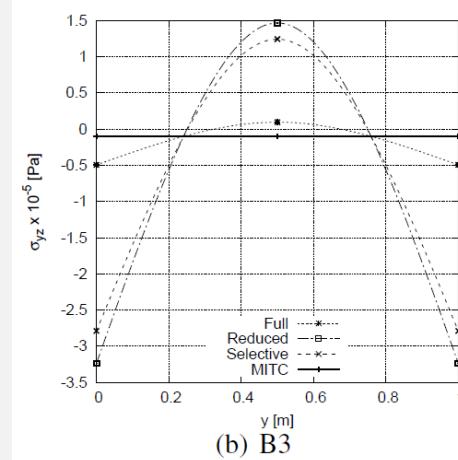
$$u = u_{zb} + u_{zs} = \frac{F_z L^3}{3EI} + \frac{F_z L}{AG} = -5.369 \times 10^{-5} \text{ m}$$

$$\sigma_{yz} = -\frac{F_z}{\Omega} = -1.0 \times 10^4 \text{ Pa.}$$

Numerical convergence:



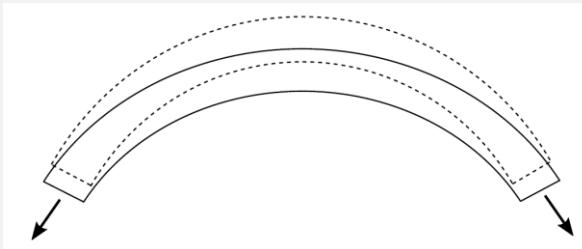
Shear stresses:



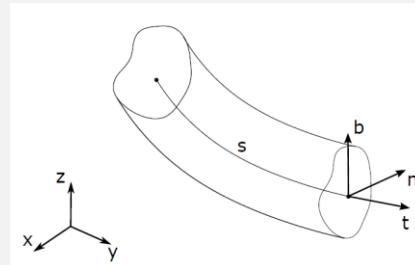
# Curved beams

## Membrane locking

Numerical stiffening due to the coupling of membrane and bending effects in curved structures



- Frenet-Serret system

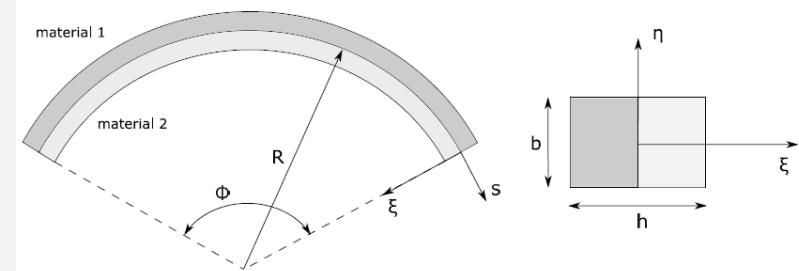


- Assumed membrane and shear strains

$$\bar{\boldsymbol{\varepsilon}}_C = \bar{N}_m F_\tau (\mathbf{D}_M N_i \mathbf{I}_3)_m \mathbf{u}_{\tau i} + \bar{N}_m F_\tau (\mathbf{D}_{S_{\parallel}} N_i \mathbf{I}_3)_m \mathbf{u}_{\tau i} + \bar{N}_m (\mathbf{D}_{S_{\perp}} F_\tau \mathbf{I}_3) N_{i_m} \mathbf{u}_{\tau i} .$$

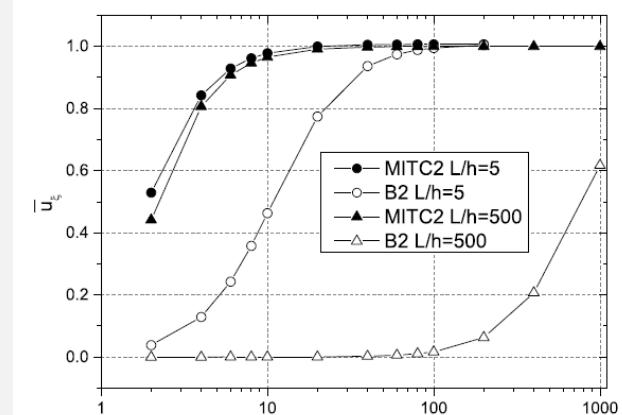
$$\begin{aligned}\varepsilon_{ss} &= \frac{1}{H} \left( \frac{\partial u_s}{\partial s} - \kappa u_{\xi} \right), \\ \varepsilon_{\xi\xi} &= \frac{\partial u_{\xi}}{\partial \xi}, \\ \varepsilon_{\eta\eta} &= \frac{\partial u_{\eta}}{\partial \eta}, \\ \varepsilon_{\xi\eta} &= \frac{\partial u_{\xi}}{\partial \eta} + \frac{\partial u_{\eta}}{\partial \xi}, \\ \varepsilon_{s\eta} &= \frac{1}{H} \left( \frac{\partial u_{\eta}}{\partial s} \right) + \frac{\partial u_s}{\partial \eta}, \\ \varepsilon_{s\xi} &= \frac{1}{H} \left( \frac{\partial u_{\xi}}{\partial s} + \kappa u_s \right) + \frac{\partial u_s}{\partial \xi},\end{aligned}$$

## Example: two-layer arch



- Simply sup.
- $h=0.6 \text{ m}$
- $F=1000 \text{ N}$
- $E_1/E_2 = 30$

Numerical convergence:

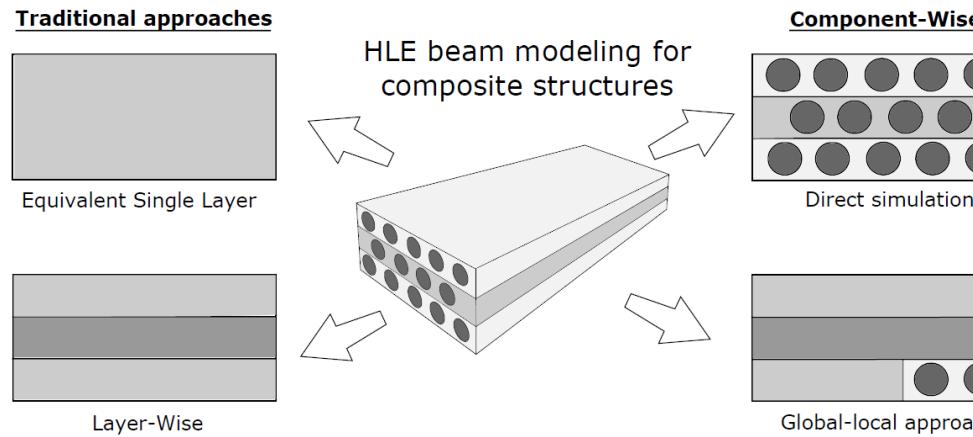


\*error relative to Navier solution

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# HLE for composite analysis

$$\Omega = \sum_{k=1}^{n_l} \Omega_k$$



$$\Omega = \sum_{k=1}^{n_c} \Omega_k$$

## Macro-scale

- Equivalent single layer (ESL)

$$E_{\tau_{(x)(z)} s_{(x)(z)}} = \sum_{k=1}^{n_l} \tilde{C}_{\alpha\beta}^k \int_a \int_{z_b^k}^{z_t^k} F_{\tau_{(x)(z)}} F_{s_{(x)(z)}} dz dx$$

## Meso-scale

- Layer-wise (LW)

$$E_{\tau_{(x)(z)} s_{(x)(z)}}^k = \tilde{C}_{\alpha\beta}^k \int_{-1}^1 \int_{-1}^1 F_{\tau_{(r)(s)}} F_{s_{(r)(s)}} |\mathbf{J}_{\Omega_k}| dr ds.$$

+  
assembly over  $n_l$

## Micro-scale

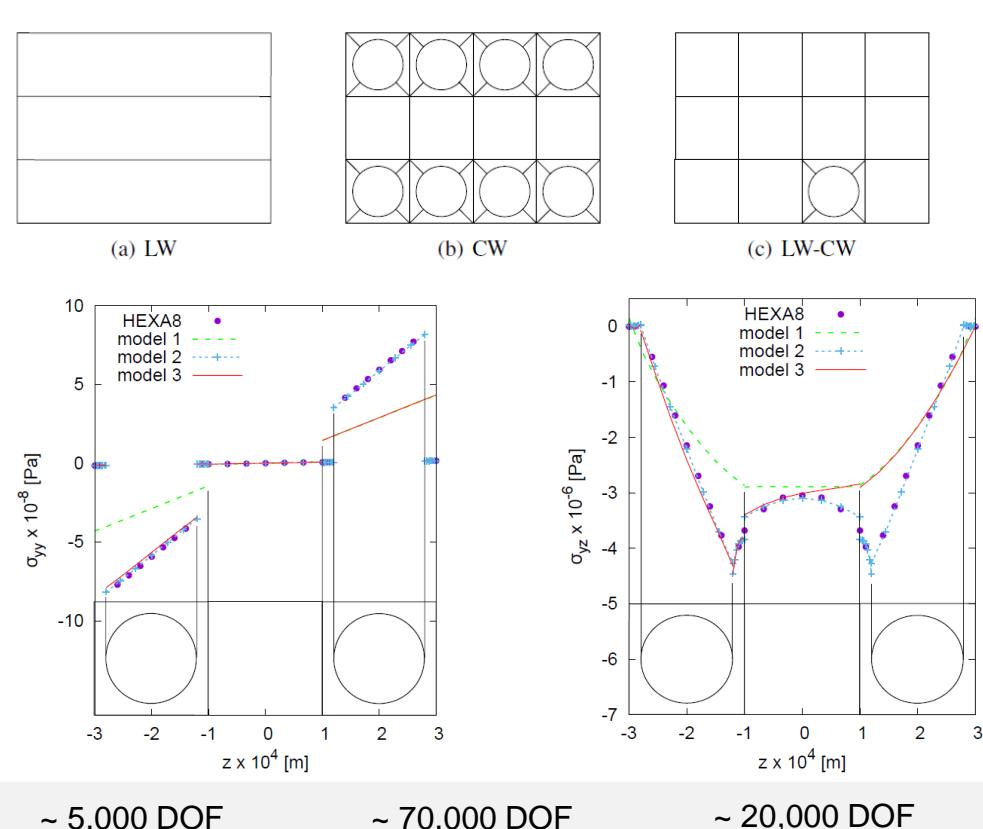
- Component-wise (CW)

$$E_{\tau_{(x)(z)} s_{(x)(z)}}^k = \tilde{C}_{\alpha\beta}^k \int_{-1}^1 \int_{-1}^1 F_{\tau_{(r)(s)}} F_{s_{(r)(s)}} |\mathbf{J}_{\Omega_k}| dr ds.$$

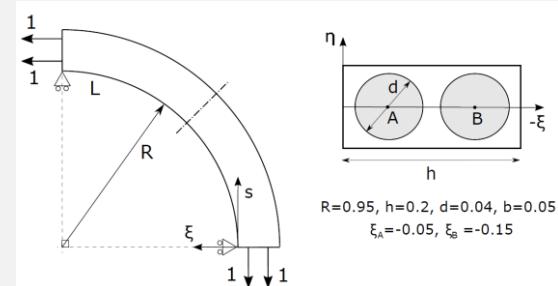
+  
assembly over  $n_c$

# HLE for composite analysis

## Fiber reinforced composites: stress analysis



## Curved microstructure

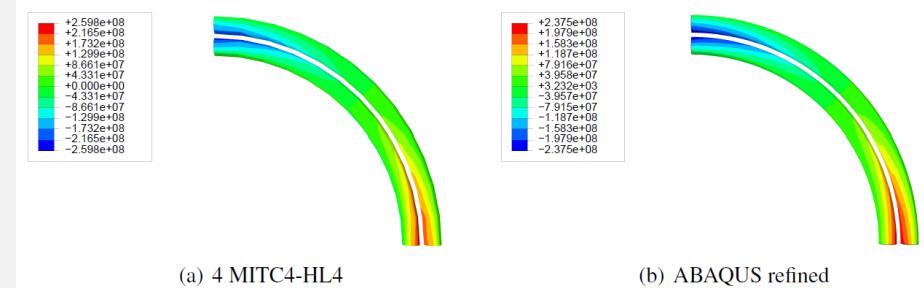


- Simply sup.
  - IM7 fiber
- 8551-7 matrix

## HLE vs 3D

	MITC beam - H4				
mesh	2 B2 (1,413)	2 B3 (2,355)	2 B4 (3,297)	4 B4 (6,123)	14 B4 (20,253)
$u_{max} \times 10^5$	2.775	4.918	4.940	4.942	4.942
	ABAQUS - C3D8				
mesh	(54,417)	(245,979)	(575,667)		
$u_{max} \times 10^5$	5.150	5.115	4.965		

## Stress solutions

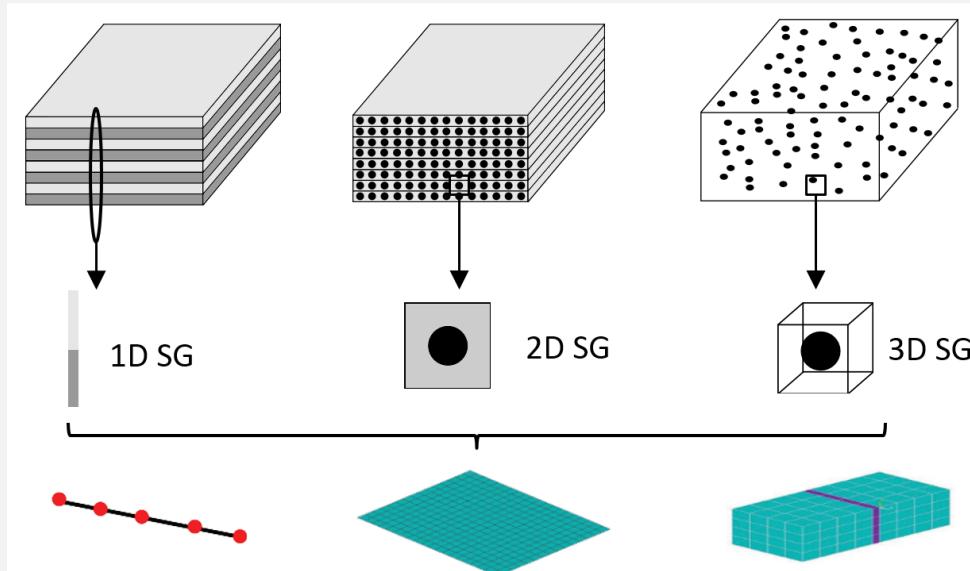


# Micromechanics

## Mechanics of structure genome (MSG)



Yu W. "A unified theory for constitutive modeling of composites", *J Mech Mater Struct* (2016);11(4): pp 379–411



## The unit cell problem

1. Express the kinematics as a sum of the global displacements and the local fluctuations

$$u_i = \bar{u}_i + \delta\chi_i$$

$$\varepsilon_{ij} = \bar{\varepsilon}_{ij} + \chi_{(i,j)}$$

2. Express the energy of the original model as

$$U(\varepsilon_{ij}) = U(\bar{\varepsilon}_{ij}, \chi_{(i,j)})$$

3. Impose the periodic constraints

$$\chi_i^- = \chi_i^+$$

4. Employ the Variational Asymptotic Method to compute the fluctuation unknowns

$$\min_{\chi} U(\bar{\varepsilon}_{ij}, \chi_{(i,j)}) - U(\bar{\varepsilon}_{ij})$$

## Advantages

- No ad-hoc assumptions
- Complete set of properties and local solutions with a single run of the code

# Micromechanics

## Competition of ASC 2017 (West Lafayette, USA)

**MUL2-UC**

Micromechanics code: beam modeling of periodically heterogeneous composites

June 9<sup>th</sup>, 2017

A.G. de Miguel<sup>1</sup>



<sup>1</sup> Politecnico di Torino, 24 Corso Duca degli Abruzzi, 10129 Turin (TO), Italy  
Email: alberto.garcia@polito.it

**Inputs**

Geometry:  
 - fiber-reinforced  
 - particle inclusions  
 - layups, etc...

Shape of constituents  
 - cross-sectional mapping

Materials

Boundary conditions

**Refined beam model**

**p-order**

**Effective properties**

```
E1 = 0.1033523844E+12
E2 = 0.1205393276E+11
E3 = 0.1205393189E+11
G12 = 0.3580559131E+10
G13 = 0.3580558080E+10
G23 = 0.2620250272E+10
nu12 = 0.2740129259E+00
nu13 = 0.2740129255E+00
nu23 = 0.3318178779E+00
```

**p-order<sub>j</sub>**

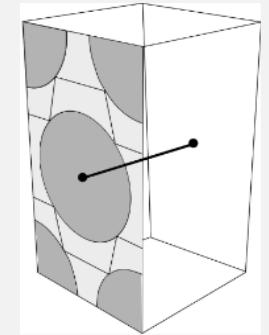
**( $\bar{\epsilon}$ ,  $\bar{v}$ )<sub>i</sub>**

**3D local fields**

\*Code available in <https://cdmhub.org/resources/downloads>

## Example: hexagonal pack

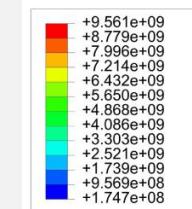
 Sertse, H. et al. "Challenge problems for the benchmarking of micromechanics analysis: Level I initial results.", Journal of Composite Materials, 2017, 1-20



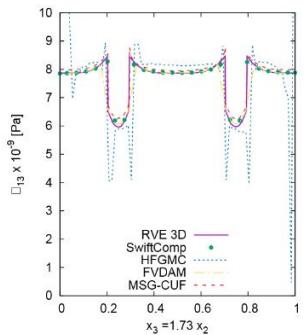
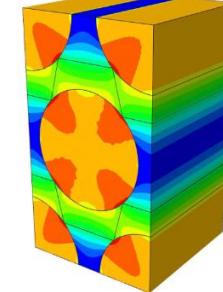
### Effective properties

Model	E <sub>1</sub>	E <sub>2</sub>	E <sub>3</sub>	G <sub>12</sub>	G <sub>13</sub>	G <sub>23</sub>	ν <sub>12</sub>	ν <sub>13</sub>	ν <sub>23</sub>
Literature									
FVDAM	167.30	10.67	10.67	6.38	6.39	3.33	0.310	0.310	0.600
HFGMC	167.40	10.71	10.69	6.58	6.54	3.36	0.312	0.312	0.603
SwiftComp	167.33	10.67	10.67	6.38	6.39	3.33	0.312	0.312	0.600
FEA RVE	167.33	10.67	10.67	6.38	6.39	3.33	0.312	0.312	0.600
MSG-HLE									
HL7	167.65	10.68	10.68	6.40	6.61	3.34	0.312	0.312	0.600

### Local fields



σ<sub>13</sub>

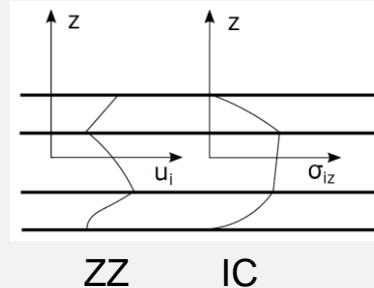


# Mixed beam elements

## Laminate modeling



Source: wikipedia.com



- Compatibility: sudden change in the displacement solutions at the interfaces
- Zig-zag functions: [Lekhniskii, S. \(1968\)](#), [Ambartsumian \(1969\)](#)
- Layer-wise: [Reddy, J.N. \(1989\)](#), [Carrera, E. and Petrolo, M. \(2012\)](#)
- Equilibrium: continuity of transverse stresses
- Stress recovery: [Whitney, J. \(1972\)](#)
- Hu-Washizu principle: [Washizu, K. \(1968\)](#)
- $C^0_z$  requirements
- [Carrera, E. \(1997, 2003\)](#)

## Reissner mixed variational theorem (RMVT)



**Reissner.** "A unified theory for constitutive modeling of composites", [J Mech Mater Struct \(2016\);11\(4\): pp 379–411](#)

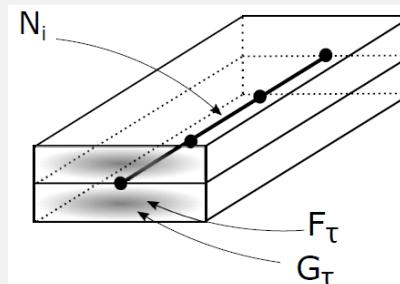
- Stress assumptions are restricted to the transverse stresses
- Variational principle:

$$\int_V (\delta \varepsilon_{pG}^T \boldsymbol{\sigma}_{pH} + \delta \varepsilon_{nG}^T \boldsymbol{\sigma}_{nM} + \delta \boldsymbol{\sigma}_{nM}^T (\boldsymbol{\varepsilon}_{nG} - \boldsymbol{\varepsilon}_{nH})) dV = \delta L_e$$

- Governing equations:

$$\begin{aligned} \delta \boldsymbol{u}_{\tau i}^{kT} : \mathbf{K}_{uu}^{k\tau sij} \boldsymbol{u}_{sj}^k + \mathbf{K}_{u\sigma}^{k\tau sij} \boldsymbol{\sigma}_{sj}^k &= \mathbf{P}_{\tau i}^k \\ \delta \boldsymbol{\sigma}_{\tau i}^{kT} : \mathbf{K}_{\sigma u}^{k\tau sij} \boldsymbol{u}_{sj}^k + \mathbf{K}_{\sigma\sigma}^{k\tau sij} \boldsymbol{\sigma}_{sj}^k &= 0. \end{aligned}$$

- Beam model:



LW beam kinematics:

$$\boldsymbol{u}^k(x, y, z) = N_i(y) F_\tau(x, z) \boldsymbol{u}_{\tau i}^k$$

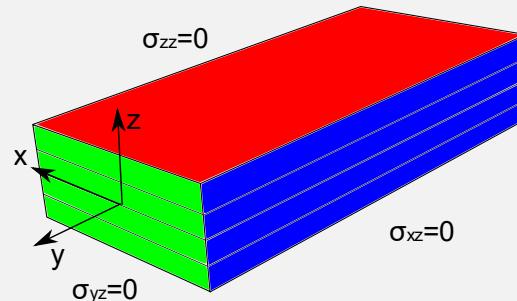
LW stress assumptions:

$$\boldsymbol{\sigma}_n^k(x, y, z) = N_i(y) G_\tau(x, z) \boldsymbol{\sigma}_{n\tau i}^k$$

# Mixed beam elements

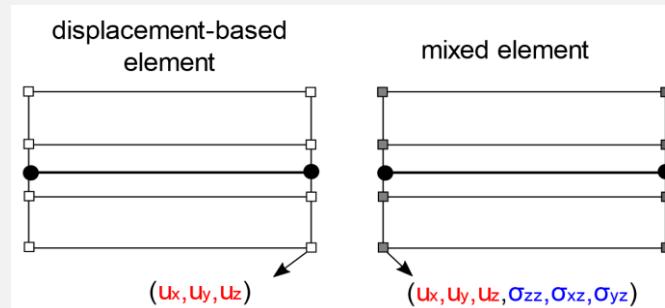
## Stress boundary conditions

- RMVT elements include stress DOF which can be prescribed in the numerical model



## RMVT-PVD coupling

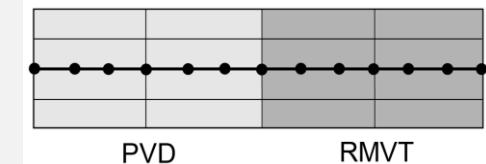
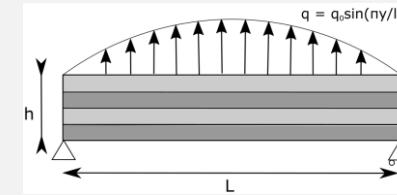
- Efficient stress analysis using RMVT elements only in zones of interest



## Example: Pagano's thick laminate



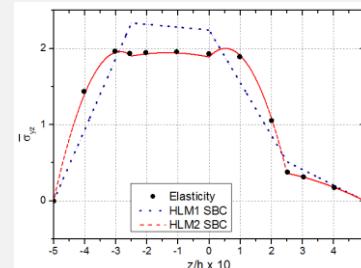
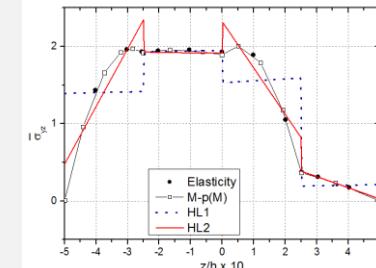
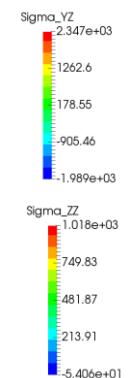
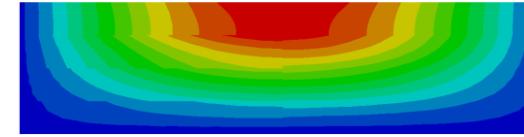
Pagano, N. "Exact solutions for composite laminates in cylindrical bending.", *Journal of Composite Materials*, 3(3):398–411



Shear stress



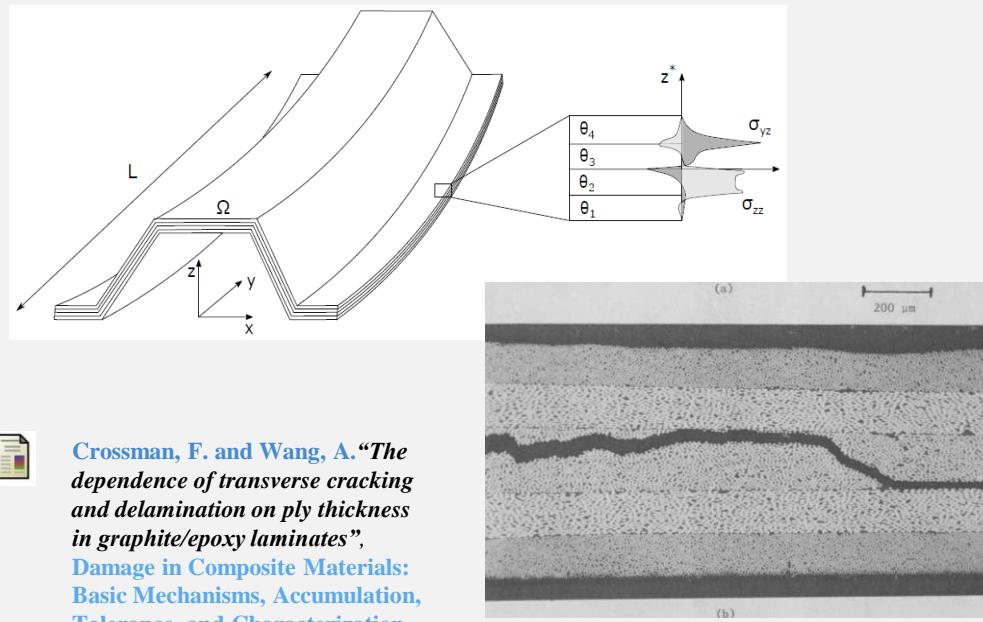
Normal stress



# Stress concentrations

## Free-edge effects

- Singular stress distributions due to the mismatch of the properties between layers
- Delamination governed by transverse stresses



 Crossman, F. and Wang, A. "The dependence of transverse cracking and delamination on ply thickness in graphite/epoxy laminates", *Damage in Composite Materials: Basic Mechanisms, Accumulation, Tolerance, and Characterization* (1982) pages 118 – 139

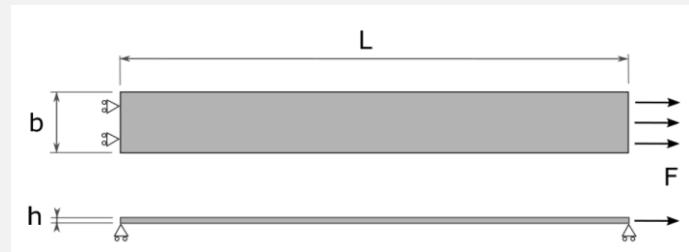
## Literature

- First studies
  -  Hayashi (1967), Puppo and Evensen (1970), Pipes and Pagano (1970, 1974)
- Analytical and semi-analytical approaches
  -  Kassapoglou and Laglace (1987), Becker (1993), Yin (1994), Flanagan (1994), Cho and Kim (2000), Lagunegrand et al. (2006), Dhanesh et al. (2017), Lorriot et al. (2003)
- Numerical approaches
  -  Wang and Crossman (1977), Raju and Crews (1981), Whitcomb et al. (1982), Martin et al. (2010)
- Non-traditional FEM models
  -  Robbins and Reddy (1993), D'Ottavio et al. (2013), Vidal et al. (2015), Peng et al. (2016)

# Stress concentrations

## Tensile test

- BC:



- Material: IM7/8552 [45<sub>2</sub>/-45<sub>2</sub>/90<sub>2</sub>/0<sub>2</sub>]<sub>s</sub>
- Dimensions (ASTM D3039):  
 $L=200\text{mm}$ ,  $b=25\text{mm}$ ,  $h_{\text{ply}}=0.16\text{mm}$

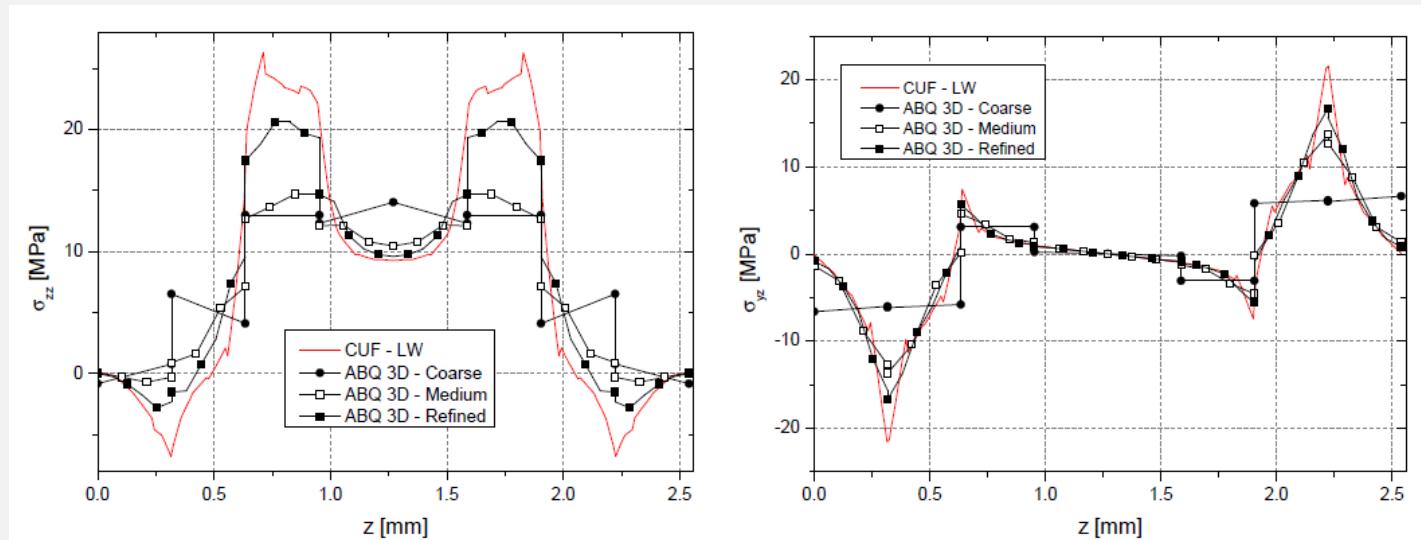
Model information for the tensile specimen.

Model	Discretization <sup>a</sup>	DOF	CPU Time [s]
CUF-LW	320 L9 over the cross-section, with 6 B4 along $y$ .	77,805	82
ABQ3D-Coarse	Linear brick elements (C3D8) with a mesh of $30 \times 8 \times 200$ elements. One element per layer.	168,237	27
ABQ3D-Medium	Linear brick elements (C3D8) with a mesh of $30 \times 24 \times 200$ elements. Three elements per layer.	467,325	261
ABQ3D-Refined	Linear brick elements (C3D8) with a mesh of $70 \times 40 \times 400$ elements. Five elements per layer.	3,501,933	3526

<sup>a</sup> All discretizations are graded towards the free-edges.

Values of the tensile load corresponding to the onset of failure of each mode considered.

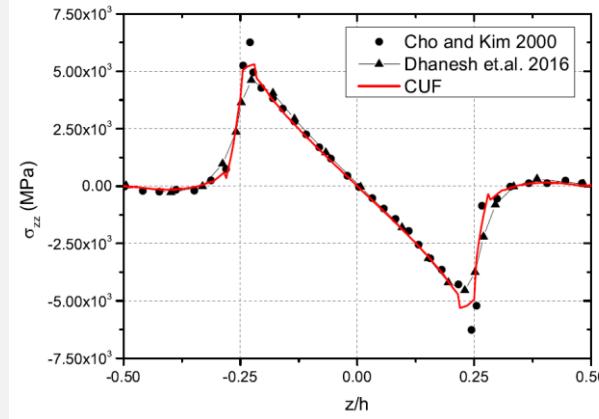
Mode	DOF	Delamination	Matrix Tension	Fibre Tension	Fiber Tension <sup>a</sup>
ABQ3D coarse	168,237	27,178.0	18,796.0	32,385.0	33,401.0
ABQ3D medium	467,325	25,717.5	17,843.5	32,258.0	35,433.0
ABQ3D refined	3,501,933	18,478.5	14,033.5	30,226.0	33,655.0
CUF-LW	77,805	14,287.5	11,938.0	27,178.0	34,417.0



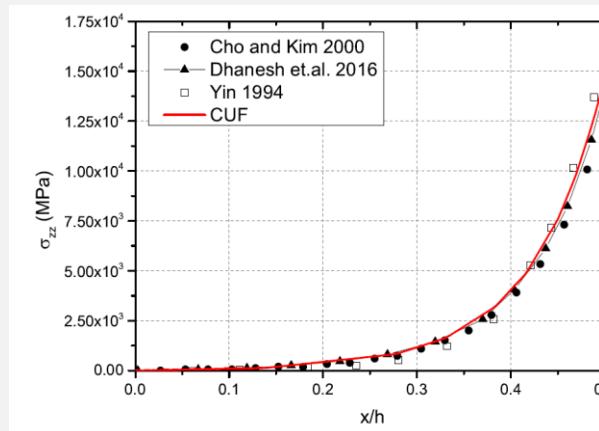
# Stress concentrations

## Generic BCs

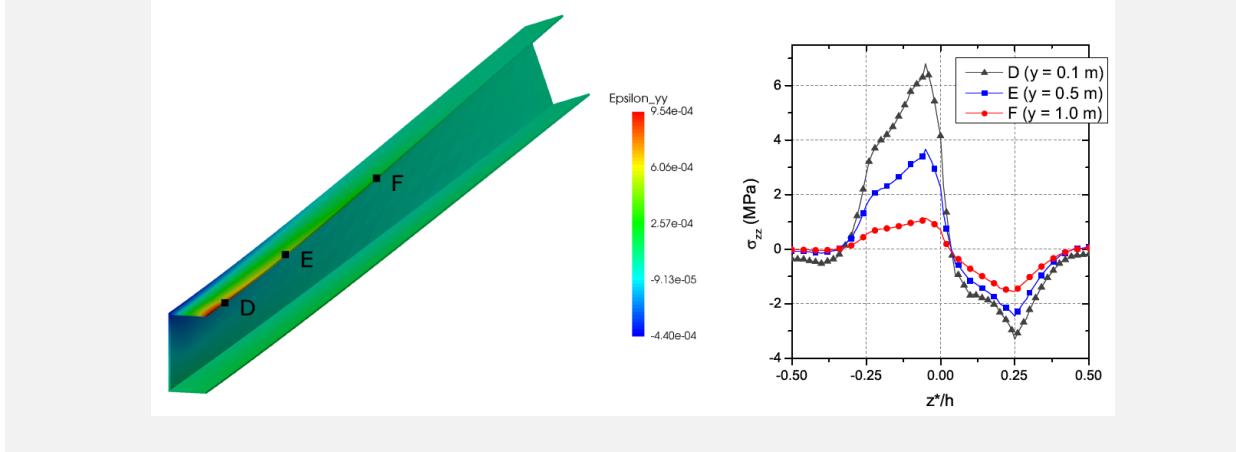
Bending



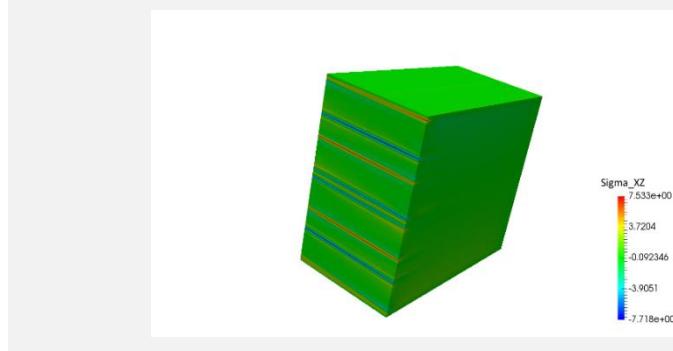
Twisting



## Complex structures



## Thick laminates



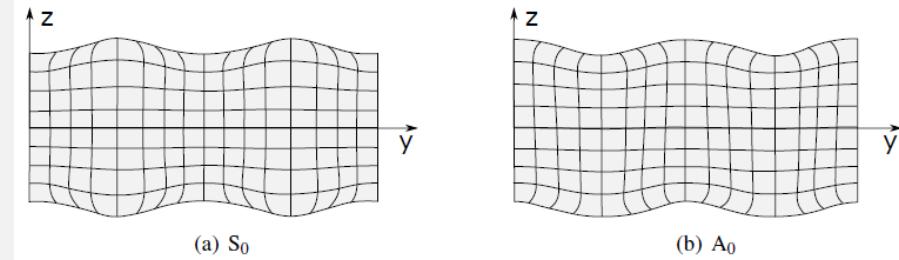
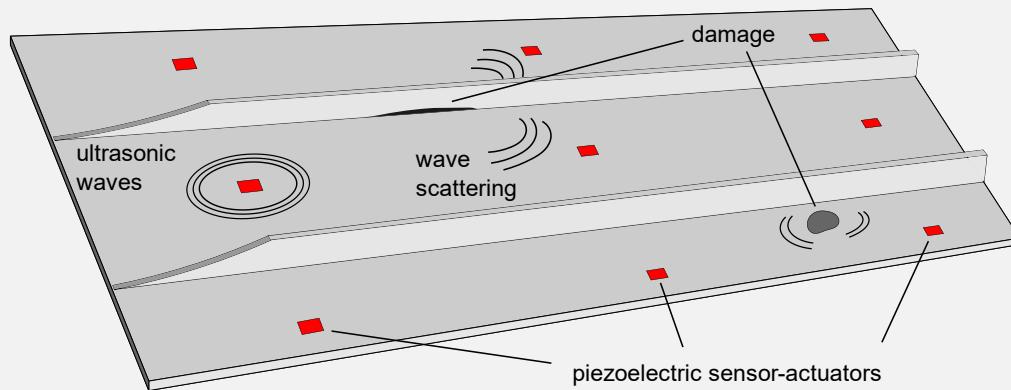
- 
- 01 Motivation and scope**
  - 02 Hierarchical one-dimensional finite elements**
  - 03 Analysis of composite structures via hierarchical beams**
  - 04 Simulation of Lamb waves for structural health monitoring**
  - 05 Conclusions**

# Lamb wave analysis: challenges

Structural health monitoring systems: on-line detection and evaluation of structural damage in reinforced structures

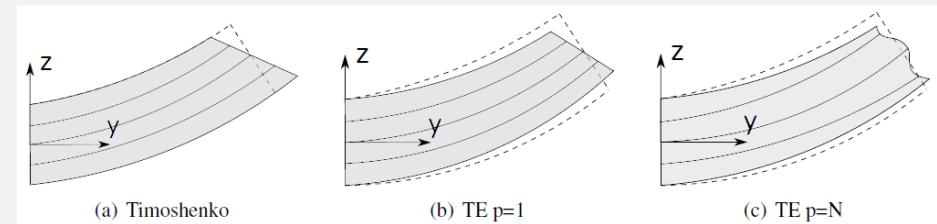
Guided plate waves, a.k.a. **Lamb waves**, are convenient for SHM due to:

- High dispersion: long distances
- Short wavelengths: high sensitivity to small defects



## Simulation challenges:

- Highly refined discretizations both in space and time.
- Large domains
- Transverse approximation

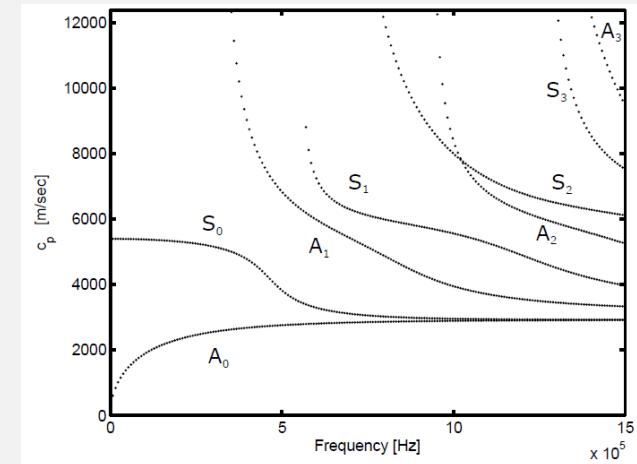


# Lamb wave analysis: state of the art

## Analytical and semi-analytical solutions: obtention of dispersion curves

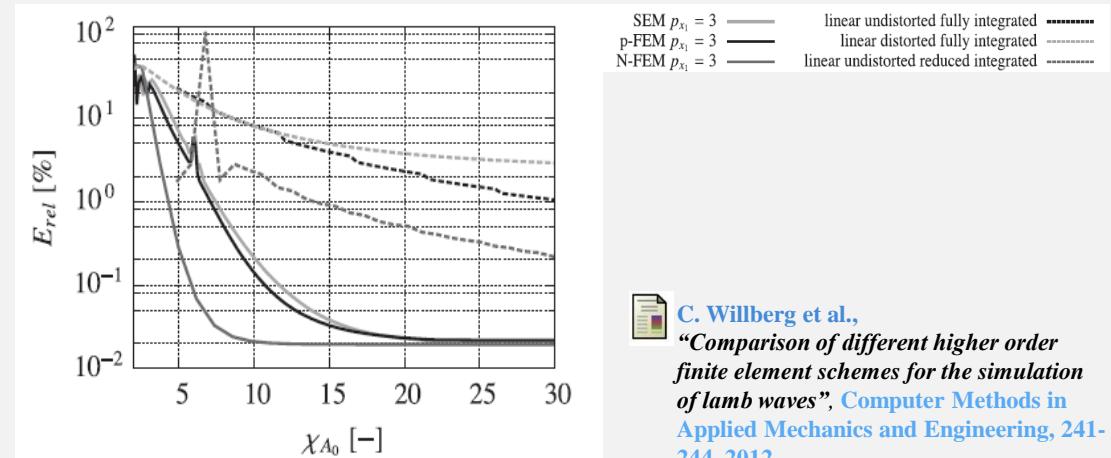
-  Transfer matrix method: [Lowe, M.J.S. \(1995\)](#)
-  Global matrix method: [Obenchain, M. B. and Cesnik, C. E. S. \(2013\)](#)
-  Mass-spring lattice method: [Yim, H. and Choi, Y. \(2000\)](#)
-  Semi-analytical finite element: [Datta, S. K. et al. \(1988\)](#)
-  Global-local methods: [Delsanto, P. \(1992\)](#), [Lammering, R. et al. \(2018\)](#)

 [Bocchini, P., Marzani, A., and Viola, GUIGUW code](#)



## Numerical solutions: higher-order shape functions for better convergence

-  p-version of FEM: [Willberg, C. at al. \(2012\)](#)
-  Isogeometric analysis: [Dedè, L. et al. \(2015\)](#)
-  Spectral element method: [Komatitsch, D. and Tromp, J. \(2002\)](#)
-  Wave FEM: [Ham, S. and Bathe, K-J. \(2012\)](#)



 [C. Willberg et al., "Comparison of different higher order finite element schemes for the simulation of lamb waves", Computer Methods in Applied Mechanics and Engineering, 241-244, 2012.](#)

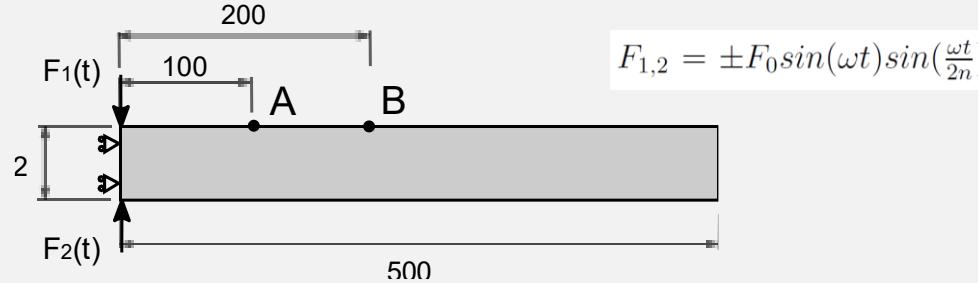
# Lamb wave analysis: benchmark case

## Example: aluminum strip



C. Willberg et al.

*“Comparison of different higher order finite element schemes for the simulation of lamb waves”, Computer Methods in Applied Mechanics and Engineering, 241-244, 2012.*



### Model:

- Hanning window:  $f=477.5$  kHz,  $n=32$  cycles
- Time step:  $3.0E-8$
- Implicit time scheme: Newmark

## Results

Analytical solution: Rayleigh-Lamb equation

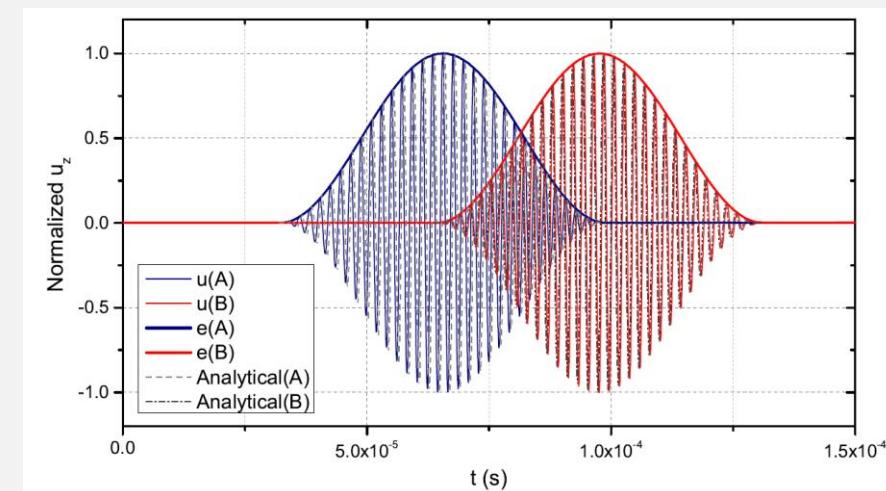
$$\frac{\tan(\beta \frac{d}{2})}{\tan(\alpha \frac{d}{2})} = - \left[ \frac{4\alpha\beta k^2}{(k^2 - \beta^2)^2} \right]^a$$

$$\alpha^2 = \omega^2/c_1^2 - k^2$$

$$\beta^2 = \omega^2/c_2^2 - k^2$$

$\omega$  = angular frequency

$k$  = wave number



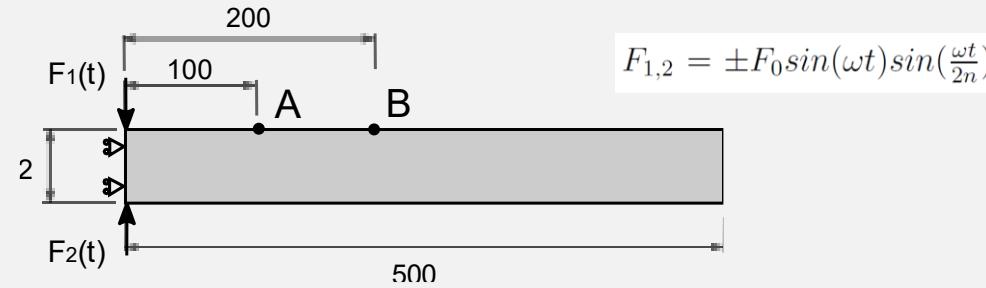
# Lamb wave analysis: benchmark case

## Example: aluminum strip



C. Willberg et al.

*“Comparison of different higher order finite element schemes for the simulation of lamb waves”, Computer Methods in Applied Mechanics and Engineering, 241-244, 2012.*



Model:

- Hanning window:  $f=477.5$  kHz,  $n=32$  cycles
- Time step:  $3.0E-8$
- Implicit time scheme: Newmark

## Time solver

- Newmark method

1. Assembly of the mass matrix  $\mathbf{M}$  and stiffness  $\mathbf{K}$  (the damping is neglected)
2. Assign the time step  $\Delta t$  and compute the dynamic stiffness matrix

$$\bar{\mathbf{K}} = \mathbf{K} + \frac{1}{\Delta t^2 \beta} \mathbf{M}$$

3. Initialize  $\mathbf{U}_0$ ,  $\dot{\mathbf{U}}_0$  and  $\ddot{\mathbf{U}}_0$  for  $t = t_0$ .
4. Factorize the dynamic stiffness matrix  $\bar{\mathbf{K}} = \mathbf{L} \mathbf{D} \mathbf{L}^T$
5. Start the loop on the time steps
6. Compute the dynamic force vector at  $t + \Delta t$ :

$$\bar{\mathbf{P}}_{t+\Delta t} = \mathbf{P}_{t+\Delta t} + \mathbf{M} \left( \frac{1}{\Delta t^2 \beta} \mathbf{U}_t + \frac{1}{\Delta t \beta} \dot{\mathbf{U}}_t + \left( 1 - \frac{1}{2\beta} \right) \ddot{\mathbf{U}}_t \right)$$

7. Solve at current time step  $\mathbf{U}_{t+\Delta t} = \bar{\mathbf{K}}^{-1} \bar{\mathbf{P}}_{t+\Delta t}$
8. Compute  $\dot{\mathbf{U}}$  and  $\ddot{\mathbf{U}}$  at current step:

$$\begin{aligned} \dot{\mathbf{U}}_{t+\Delta t} &= \frac{\gamma}{\Delta t \beta} (\mathbf{U}_{t+\Delta t} - \mathbf{U}_t) + \left( 1 - \frac{\gamma}{\beta} \right) \dot{\mathbf{U}}_t + \left( 1 - \frac{\gamma}{2\beta} \right) \Delta t \ddot{\mathbf{U}}_t \\ \ddot{\mathbf{U}}_{t+\Delta t} &= \frac{1}{\Delta t^2 \beta} (\mathbf{U}_{t+\Delta t} - \mathbf{U}_t) - \frac{1}{\Delta t \beta} \dot{\mathbf{U}}_t + \left( \frac{1}{2\beta} - 1 \right) \ddot{\mathbf{U}}_t \end{aligned}$$

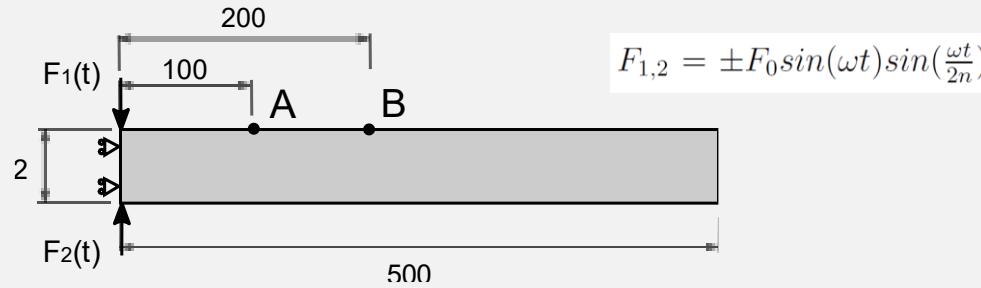
# Lamb wave analysis: benchmark case

## Example: aluminum strip



C. Willberg et al.

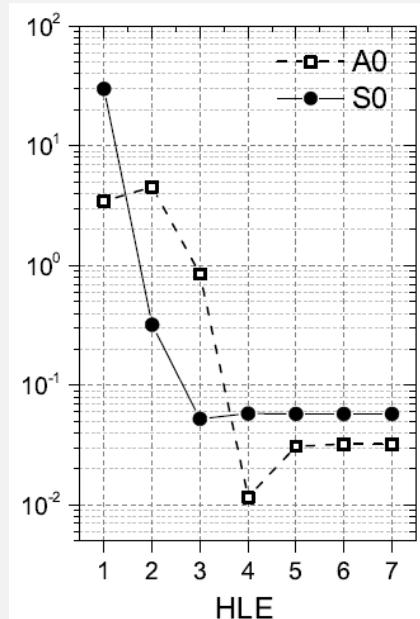
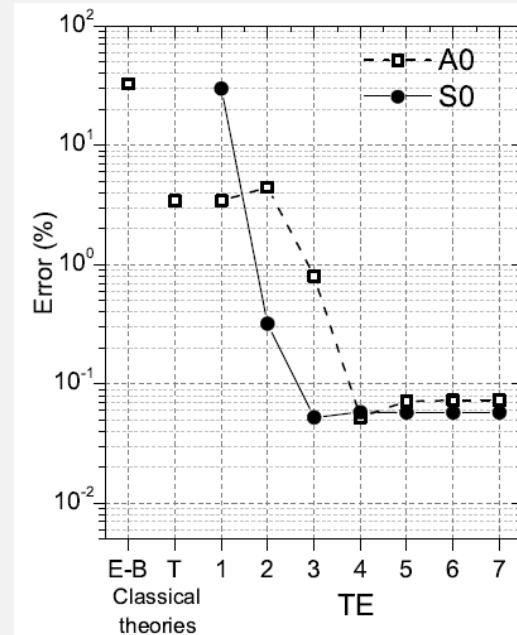
*“Comparison of different higher order finite element schemes for the simulation of lamb waves”, Computer Methods in Applied Mechanics and Engineering, 241-244, 2012.*



### Model:

- Hanning window:  $f=477.5$  kHz,  $n=32$  cycles
- Time step:  $3.0E-8$
- Implicit time scheme: Newmark

## Numerical convergence



\*Error based on the time of flight

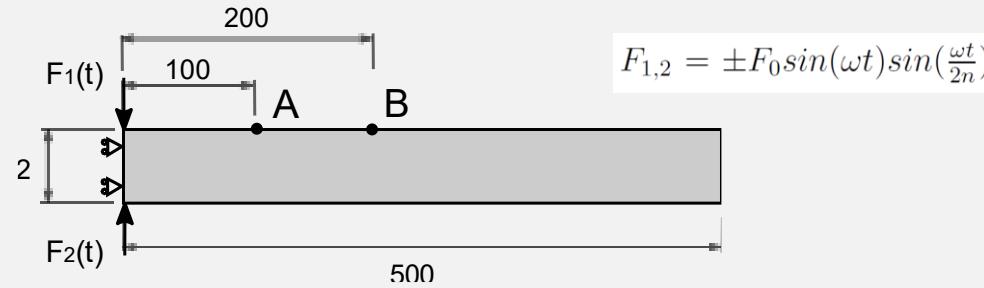
# Lamb wave analysis: benchmark case

## Example: aluminum strip



C. Willberg et al.

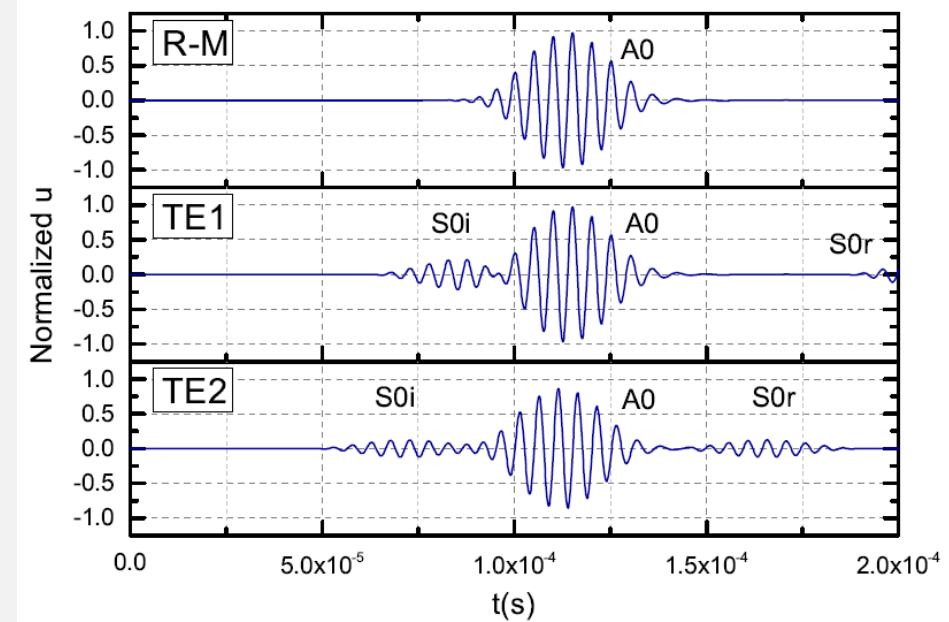
*“Comparison of different higher order finite element schemes for the simulation of lamb waves”, Computer Methods in Applied Mechanics and Engineering, 241-244, 2012.*



Model:

- Hanning window:  $f=200$  kHz,  $n=10$  cycles
- Time step:  $3.0E-8$
- Implicit time scheme: Newmark

## Computed signals

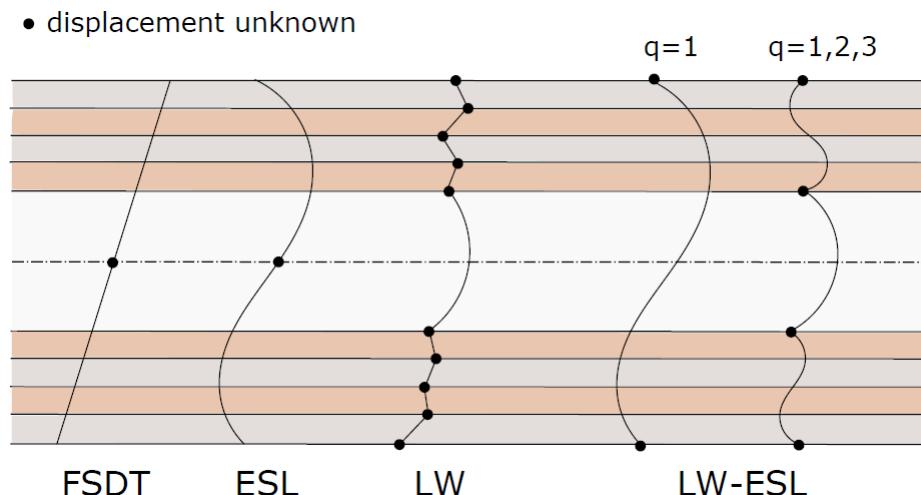


# Ultrasonic wave analysis in laminates

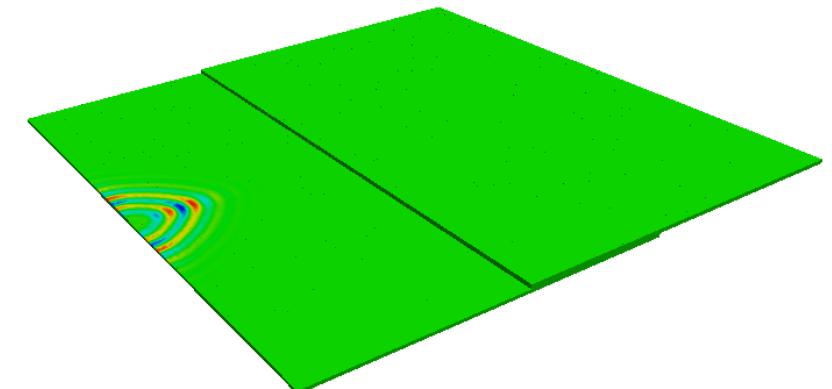
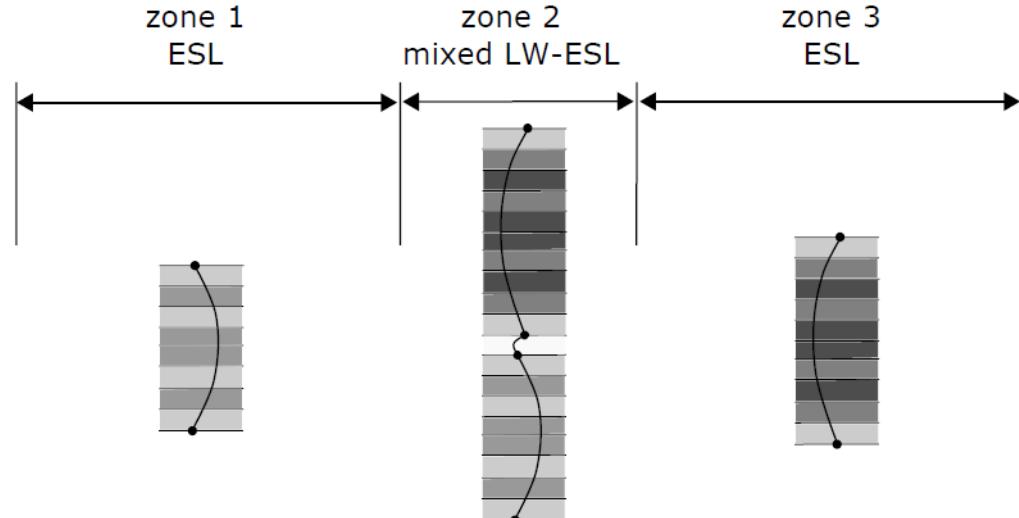
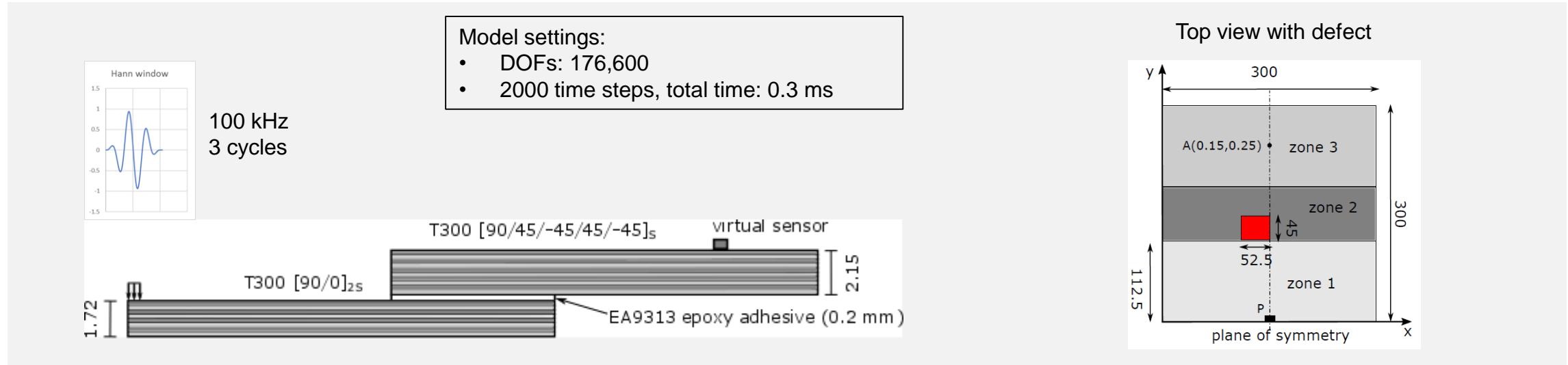
## Multilayered theories for plates

- FSDT
- Equivalent single layer
- Layer-wise
- Mixed ESL-LW

$$E_{\tau_{(z)}^q s_{(z)}}^q = \sum_{k=1}^{n_l^q} \tilde{C}_{\alpha\beta}^k \int_{z_b^k}^{z_t^k} F_{\tau_{(z)}} F_{s_{(z)}} dz = \sum_{k=1}^{n_l^q} \tilde{C}_{\alpha\beta}^k \int_{r_b^k}^{r_t^k} F_{\tau_{(r)}} F_{s_{(r)}} |\mathbf{J}_r^k| dr$$



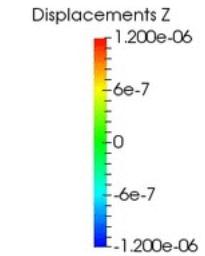
# Composite adhesive joint



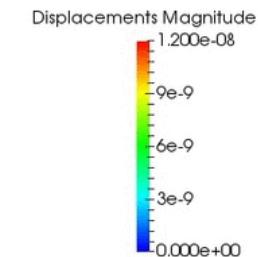
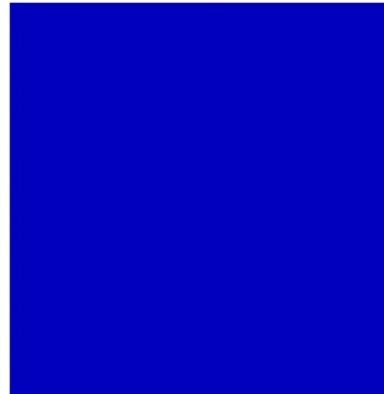
# Composite adhesive joint

Pristine structure

A0

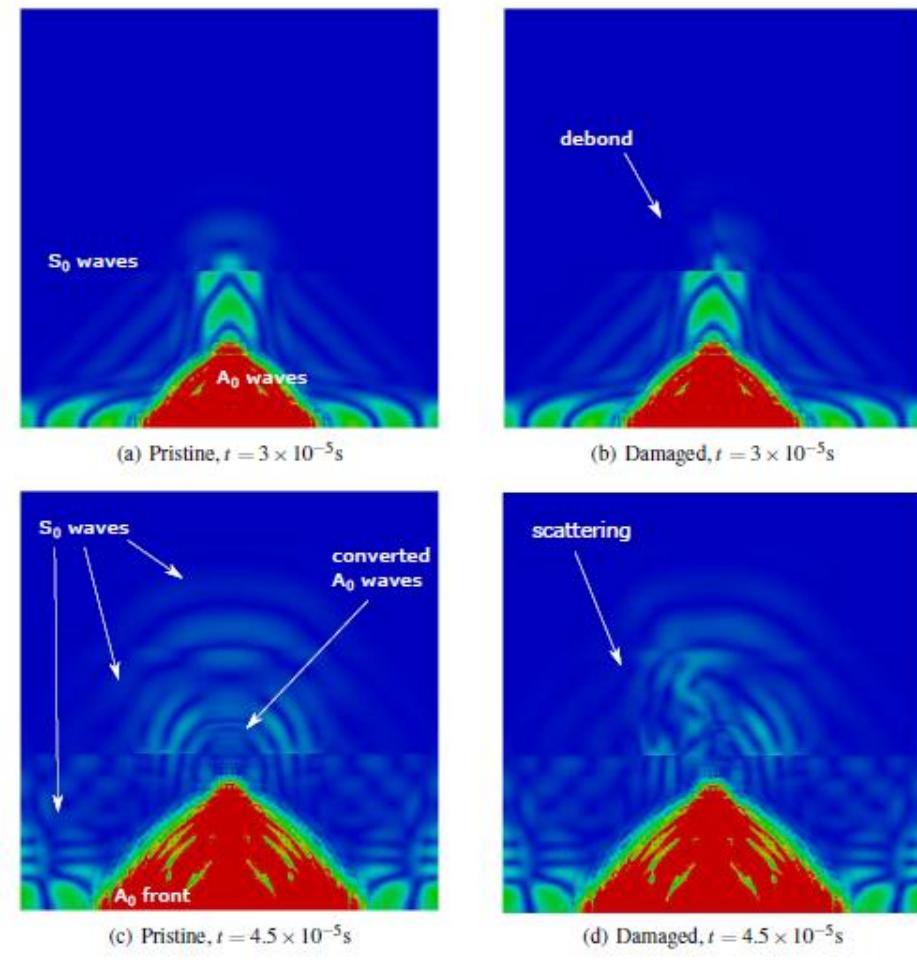


S0

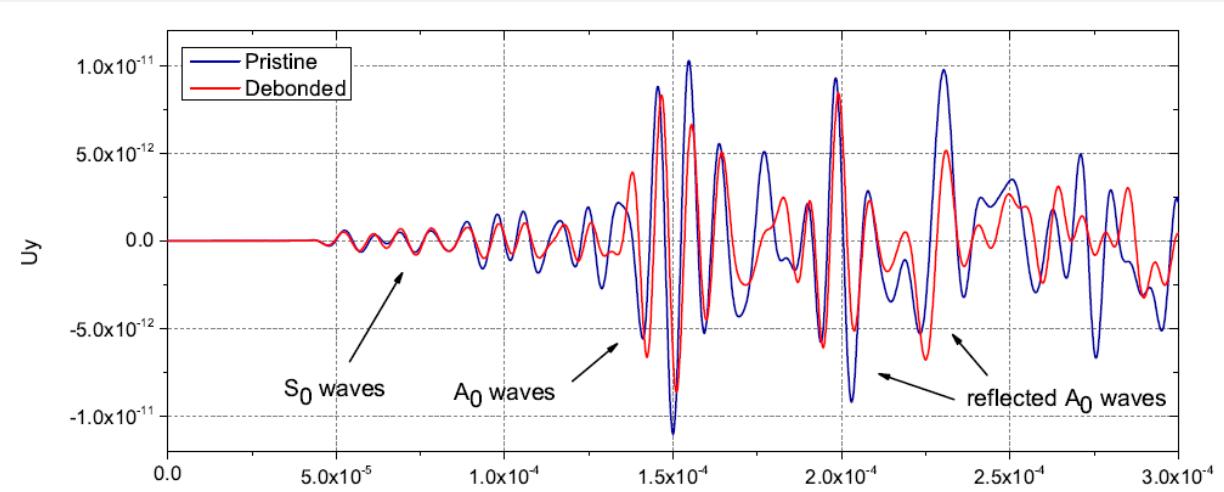


# Composite adhesive joint

Damaged structure

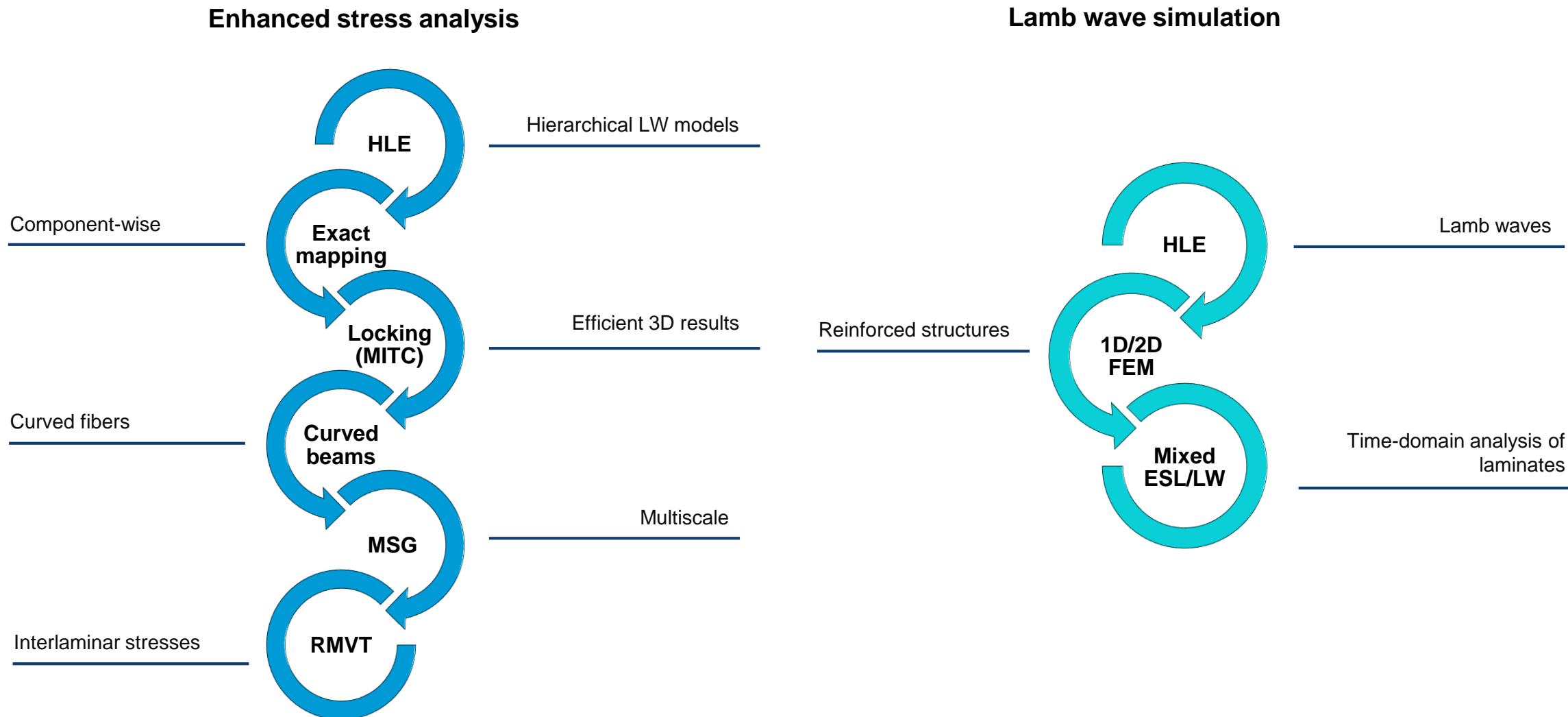


Scattering of the signals at virtual sensor



- 
- 01 Motivation and scope**
  - 02 Hierarchical one-dimensional finite elements**
  - 03 Analysis of composite structures via hierarchical beams**
  - 04 Simulation of Lamb waves for structural health monitoring**
  - 05 Conclusions**

# Summary and conclusions



# Summary and conclusions

1

## Transverse formulation

Development of a hierarchical modeling framework that can be applied in all the scales of the composite simulation

- ✓ link between scales

2

## High fidelity solutions

Implementation of advanced finite elements with HLE

- ✓ 3D stresses, interlaminar continuity
- ✓ Accurate simulation of waves propagating in laminates

3

## Computational efficiency

The use of dimensionally reduced models enables to reduce the computational size of the analysis:

- ✓ Up to various orders of magnitude
- ✓ No remeshing

# Outcome



## Research outcome

- 15 journal publications
- 16 conferences proceedings
- 1 book chapter



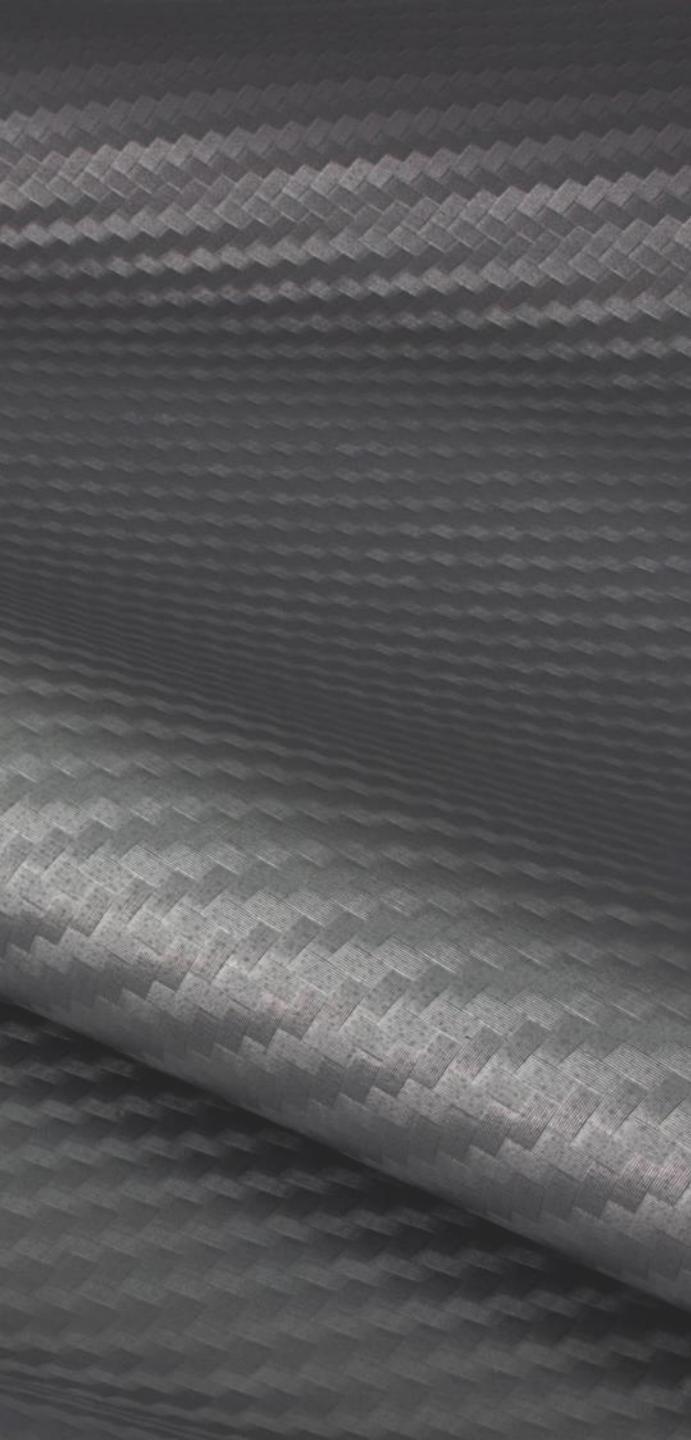
## Secondments

- Visiting scholar at Purdue University (Aug – Dec 2016)
- Visiting PhD student at Thales Alenia Space (July 2018)
- Visiting scholar at CALTECH (Jan – Feb 2019)



## Projects

- Joint Project PoliTO – Embraer ‘Global-Local Analysis of Composite Wing Structures’
- Joint Project PoliTO – Embraer ‘Validation Project’

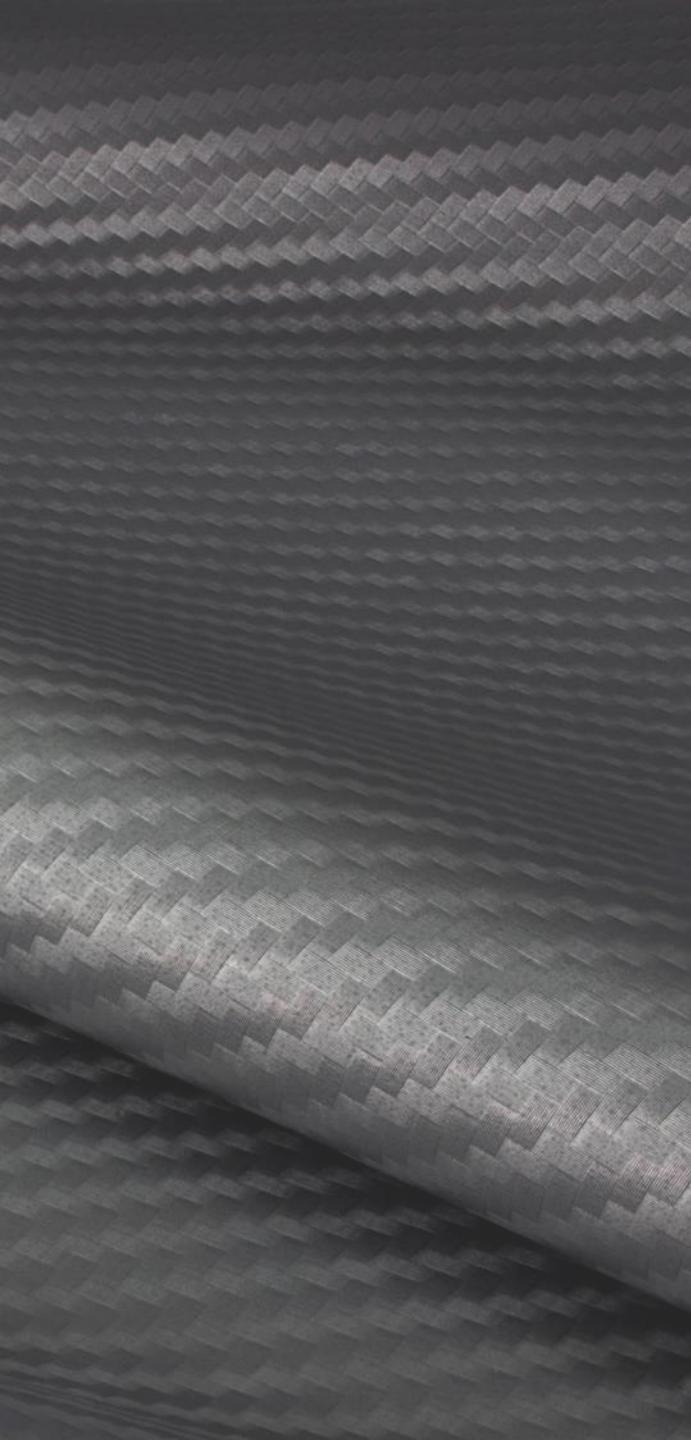


**Thank you for the attention**



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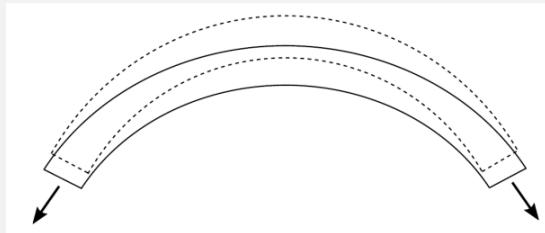
# Back up slides



# Curved beams

## Membrane locking

Numerical stiffening due to the coupling of membrane and bending effects in curved structures



- Arch finite elements



Fried, I. "Shape functions and the accuracy of arch finite elements", *AIAA Journal* 1973, 11(3):287 – 291

- Reduced integration



Noor, A. K. and Peters, J. M. "Mixed models and reduced/selective integration displacement models for nonlinear analysis of curved beams", *International Journal for Numerical Methods in Engineering* 1981, 17(4):615–631

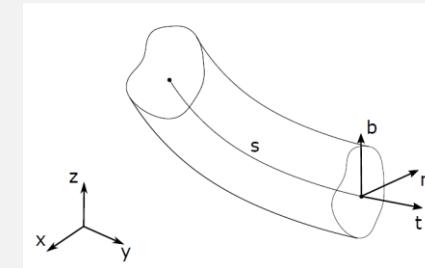
- MITC



Huang, H. C. and Hinton, E. "A new nine node degenerated shell element with enhanced membrane and shear interpolation", *International Journal for Numerical Methods in Engineering* 1986, 22(1):73–92.

## MITC for membrane and shear locking

- Equations written in the Frenet-Serret system:



- Geometrical relations:

$$\varepsilon_{ss} = \frac{1}{H} \left( \frac{\partial u_s}{\partial s} - \kappa u_\xi \right),$$

$$\varepsilon_{\xi\xi} = \frac{\partial u_\xi}{\partial \xi},$$

$$\varepsilon_{\eta\eta} = \frac{\partial u_\eta}{\partial \eta},$$

$$\varepsilon_{\xi\eta} = \frac{\partial u_\xi}{\partial \eta} + \frac{\partial u_\eta}{\partial \xi},$$

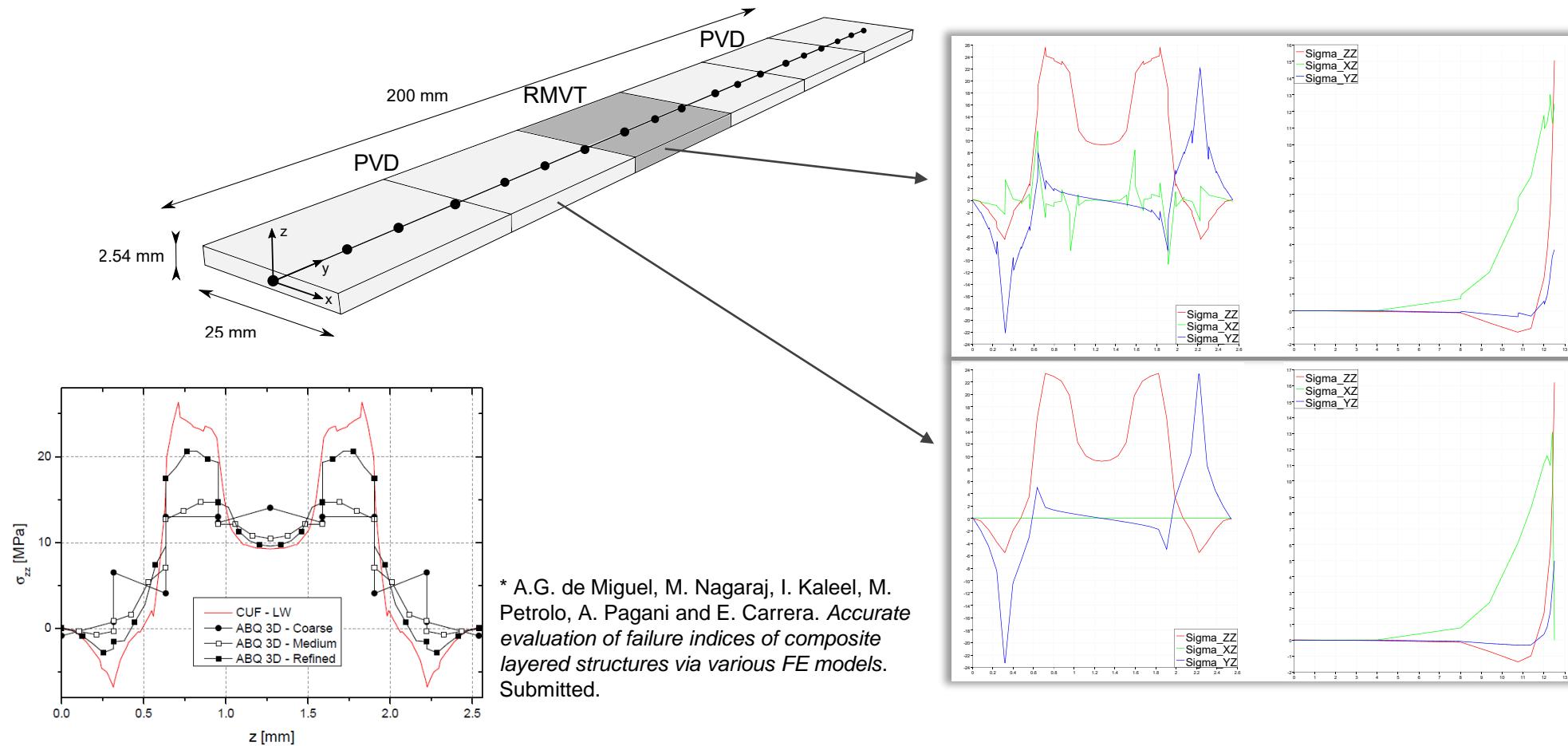
$$\varepsilon_{s\eta} = \frac{1}{H} \left( \frac{\partial u_\eta}{\partial s} \right) + \frac{\partial u_s}{\partial \eta},$$

$$\varepsilon_{s\xi} = \frac{1}{H} \left( \frac{\partial u_\xi}{\partial s} + \kappa u_s \right) + \frac{\partial u_s}{\partial \xi},$$

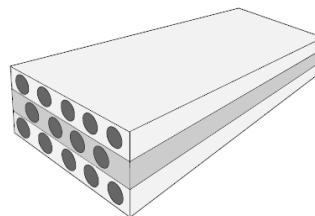
- Use of assumed membrane and shear strains

$$\bar{\boldsymbol{\varepsilon}}_C = \bar{N}_m F_\tau (\mathbf{D}_M N_i \mathbf{I}_3)_m \mathbf{u}_{\tau i} + \bar{N}_m F_\tau (\mathbf{D}_{S_\parallel} N_i \mathbf{I}_3)_m \mathbf{u}_{\tau i} + \bar{N}_m (\mathbf{D}_{S_\perp} F_\tau \mathbf{I}_3) N_{i_m} \mathbf{u}_{\tau i} .$$

# Stress concentrations: RMVT



# HLE for composite analysis



## Micro-scale

- 3D (RVE)
  -  [Sun, C. and Vaidya, R. \(1996\)](#)
- Mathematical Homogenization (MHT)
  -  [Mori, T. and Tanaka, K. \(1973\)](#)
- Method of cells (GMC, HFGMC)
  -  [Aboudi, J. \(1982\)](#)

## Meso-scale

- 3D FEM, 2D plane strain
  -  [Allix, O. et al. \(2013\)](#)
- Layer-wise theories
  -  [Reddy, J.N. \(1989\)](#)
- Stress recovery methods
  -  [Whitney, J. \(1972\)](#)

## Macro-scale

- First-order shear deformation theories (FSDT)
  -  [Mindlin, R.D. \(1951\)](#)