Modal Interaction and Mode Switching in Post-buckling Analysis of Plates Using a Multi-mode Finite Element Based Reduced Order Model

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Outline of talk

- Motivation
- Methodology
- Tested scenario
- Effect of imperfections on post buckling behavior
- Concluding remarks

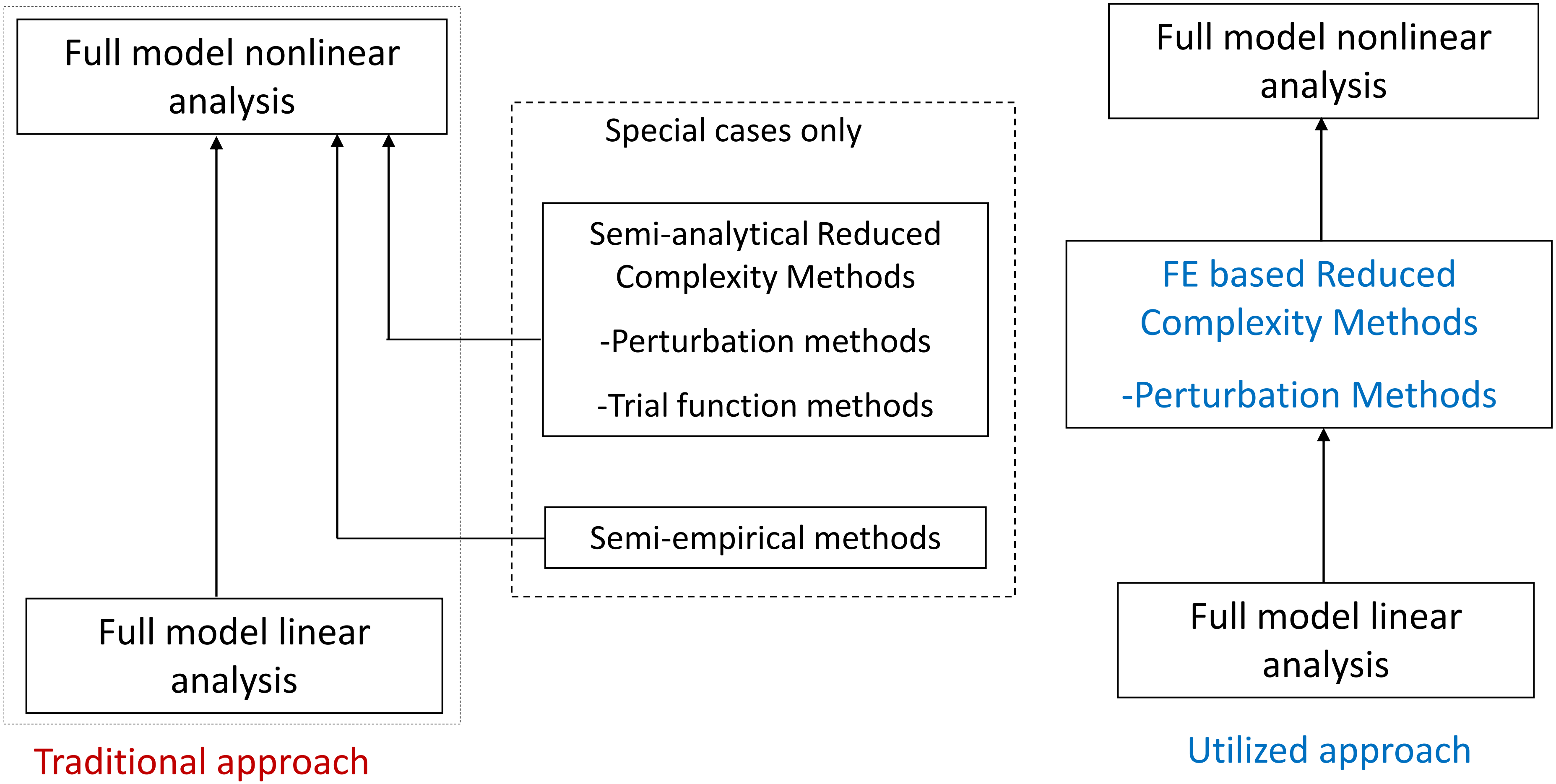
Motivation

- Trend towards optimization
- Imperfections dominate post buckling behavior
- Probabilistic analyses by traditional approaches can be costly
- Design tools accounting for probabilistic imperfection patterns require fast computation of multiple scenarios: use of reduced order models

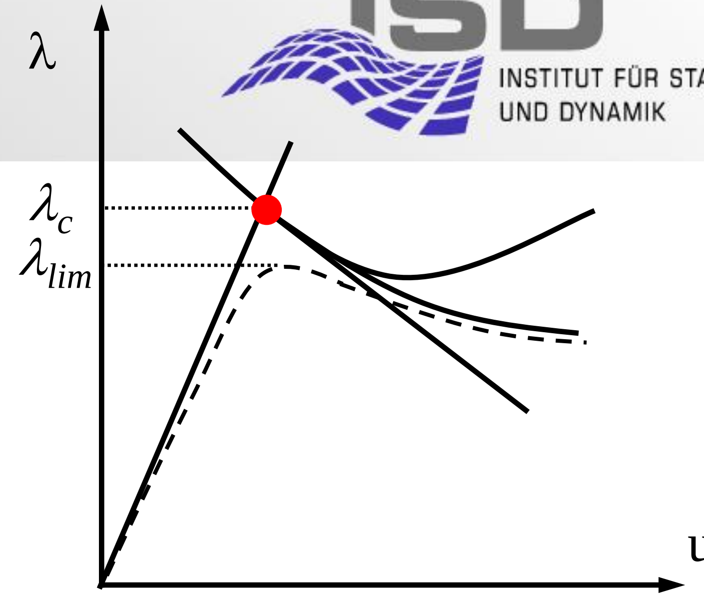
- Reduced order modeling approach based on perturbation analysis introduced earlier shows good agreement with other more intensive analyses for static analysis

- Objective: Further definition of interesting test cases for nonlinear analysis to be used in the current extensions of the reduced order approach
    - Towards dynamic analysis
    - Towards probabilistic analysis

Reduced order modelling: Motivation



ROM within FE framework basics: Perturbation procedure Stability



- Given a proportional load distribution
- we find the linear(ized) prebuckling path $\mathbf{u} = \lambda \mathbf{u_0}$
- the buckling load and mode $\mathbf{u}_1, \lambda_{cr}$
- The postcritical path is expanded about the critical point

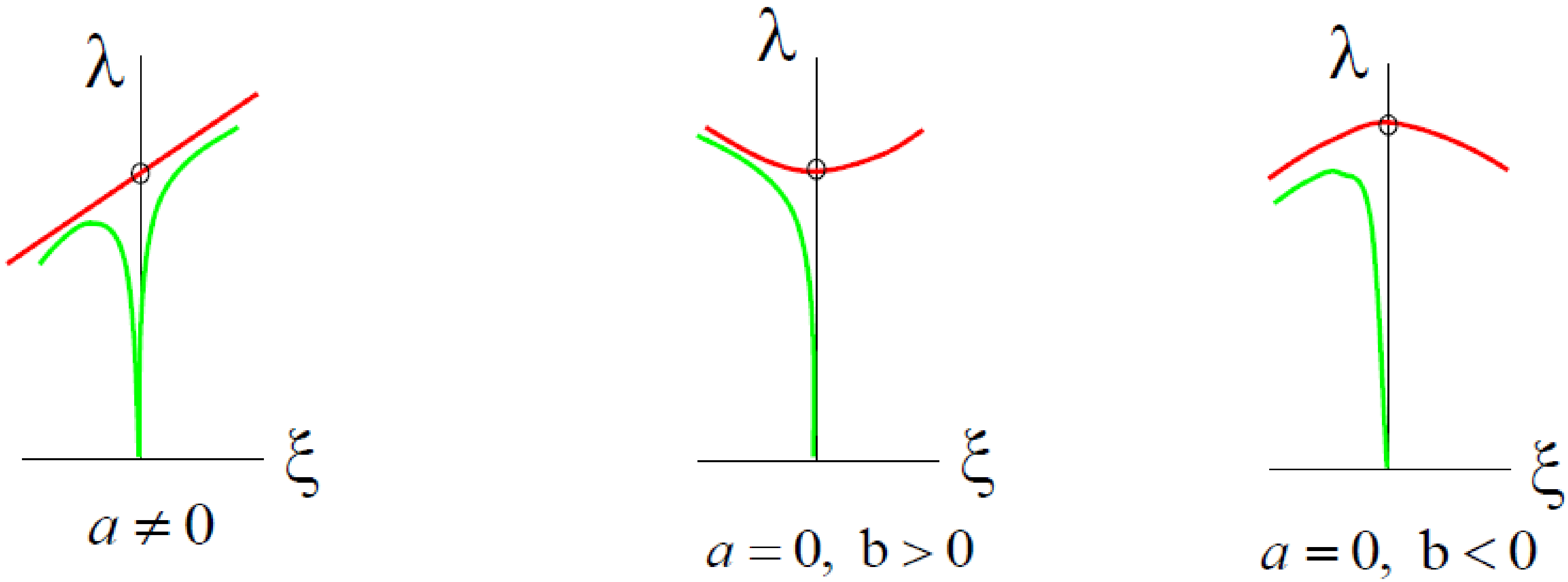
$$\mathbf{u} = \lambda \mathbf{u_0} + \xi \mathbf{u}_1 + \xi^2 \mathbf{u}_2 + \cdots$$

- We find the postcritical slope $a = \dfrac{3\boldsymbol{\sigma}_1 \cdot L_2(\mathbf{u}_1)}{2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\varepsilon}_1}$
- and the postcritical curvature $b = \dfrac{2\boldsymbol{\sigma}_1 \cdot L_{11}(\mathbf{u}_1, \mathbf{u}_2) + \boldsymbol{\sigma}_2 \cdot L_2(\mathbf{u}_1)}{\boldsymbol{\sigma}_1 \boldsymbol{\varepsilon}_1}$
- Given an imperfection pattern the load-deflection path is found: $\dfrac{\lambda}{\lambda_C} = \dfrac{1 + a\xi + b\xi^2}{1 + \dfrac{\xi}{\xi}}$

Methodology

- Perturbation based reduced order model

$$\lambda = \lambda_c + a\lambda_c\xi + b\lambda_c\xi^2 + \cdots$$



FE Implementation: Reduction method for nonlinear static stability analysis

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Fundamental path: $\mathbf{K}_0\mathbf{q}_0 = \mathbf{F}$

↓

Buckling modes: $[\mathbf{K}_0 - \lambda_i \mathbf{K}_G]\,\mathbf{q}_i = \mathbf{0}$

↓

Second order modes: $[\mathbf{K}_0 - \lambda_i \mathbf{K}_G]\,\mathbf{q}_{ij} = \mathbf{g}(\mathbf{q}_i, \mathbf{q}_j)$

↓

Post-buckling coefficients: a, b

↓

The reduced equation: $(\lambda_c - \lambda)\xi + a_s \lambda_c \xi^2 + b_s \lambda_c \xi^3 = \dots$

↓

Displacement field: $\boldsymbol{q} = \boldsymbol{q}_0(\lambda) + \boldsymbol{q}_1 \xi + \boldsymbol{q}_2 \xi^2$

Tested scenario

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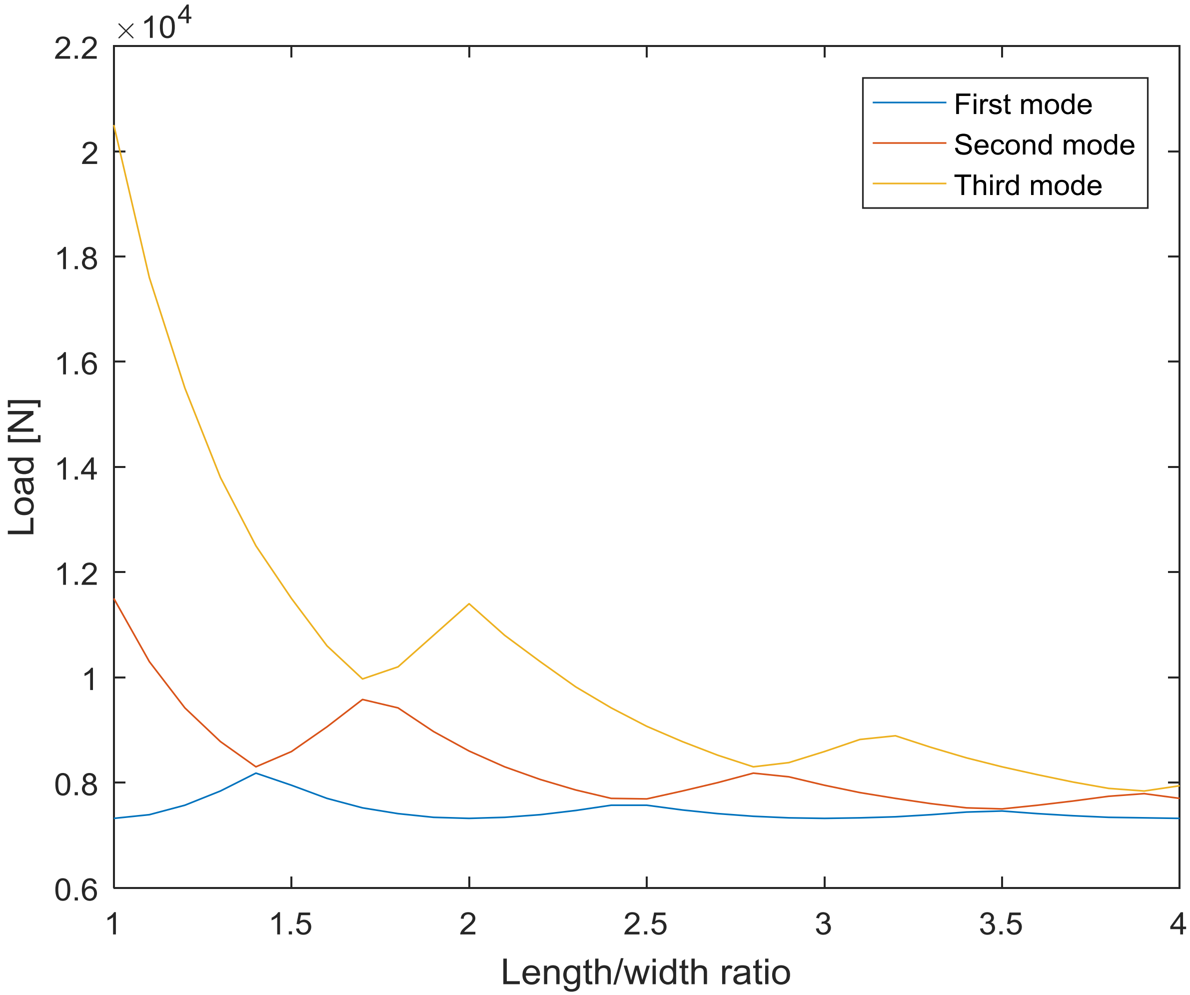
- Scenario to analyze modal interactions of simple plate configurations, making use of current Matlab implementation for isotropic structures:
  - Isotropic plates
  - Quasi-isotropic plates

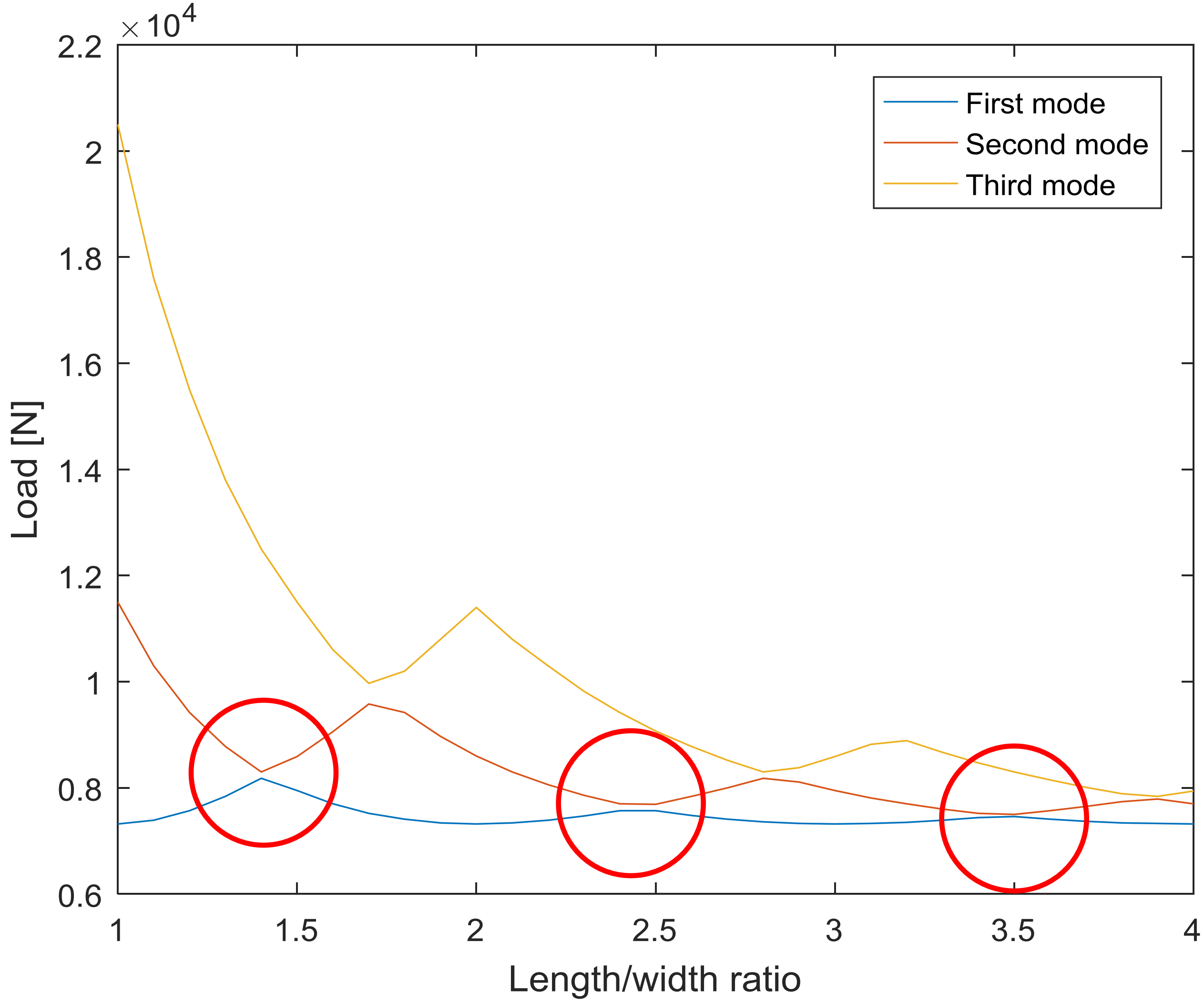
- Plate configuration:
  - 1.5 mm, 12 layer “fully isotropic” laminate (Grédiac, 1998)

- Investigation of nonlinear behavior for
  - Varying height/width ratios
  - Varying imperfection amplitudes

Mode interaction case

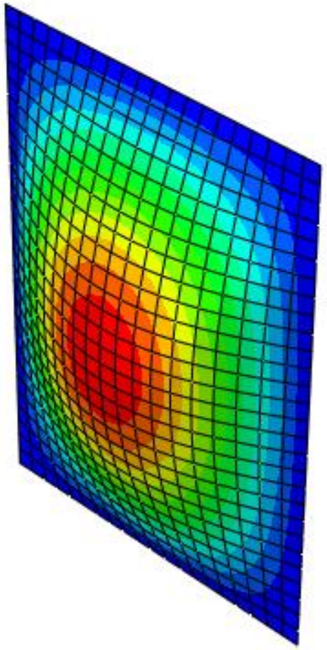


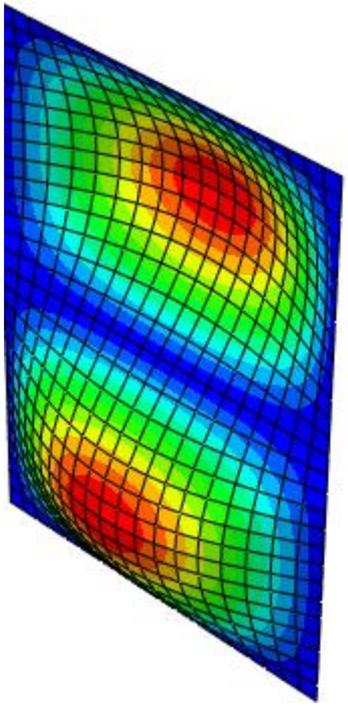
Mode interaction case



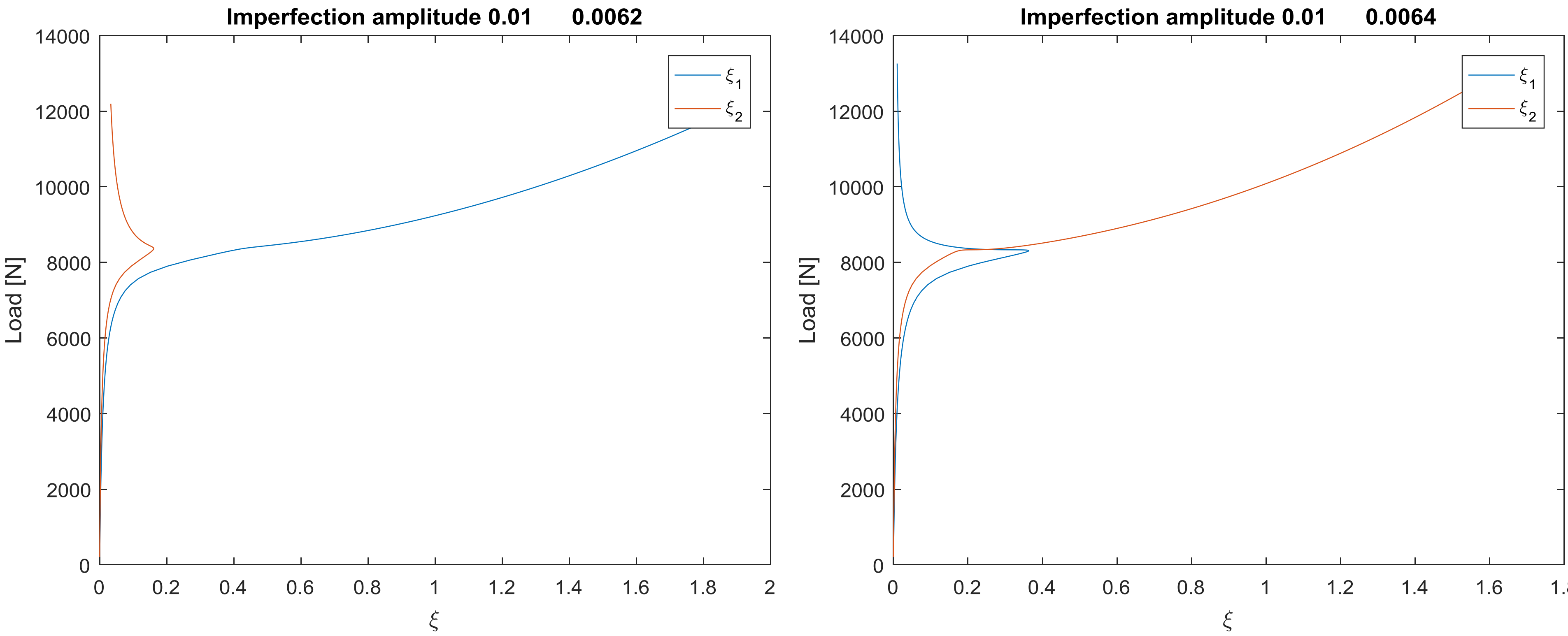
Ratio of 1.4

- 1st and second mode are very close to each other
- Which mode gets triggered depends on the imperfections found in the structure





Effect of imperfections on post buckling
Ratio 1.4



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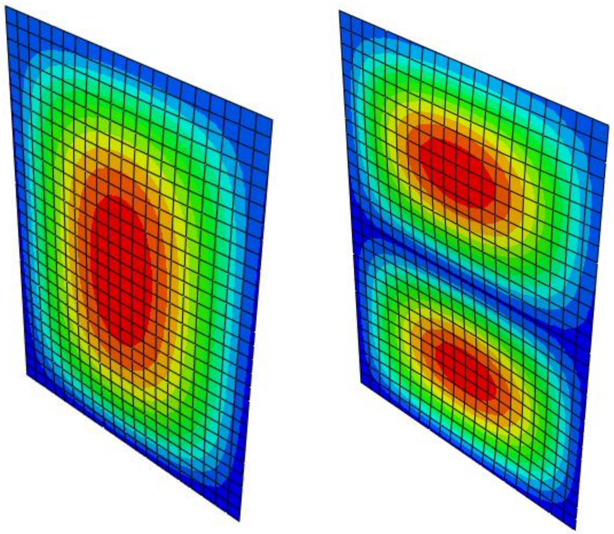
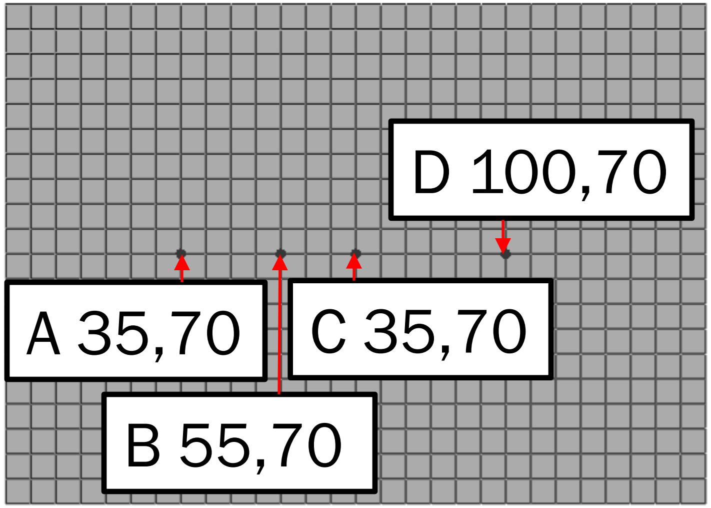
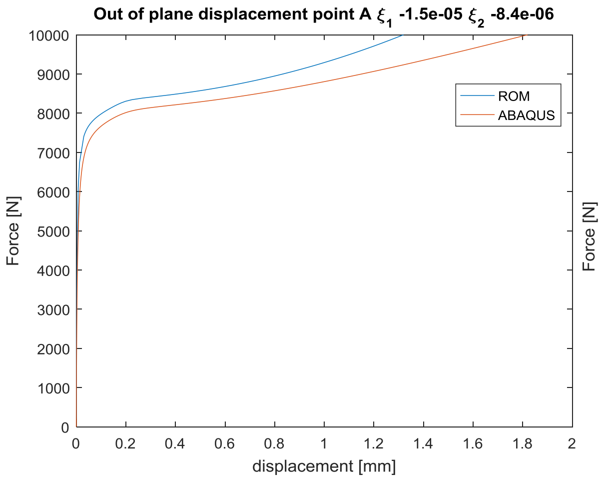
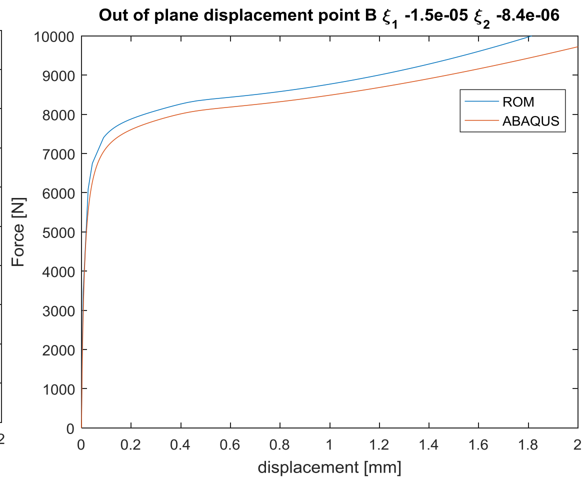
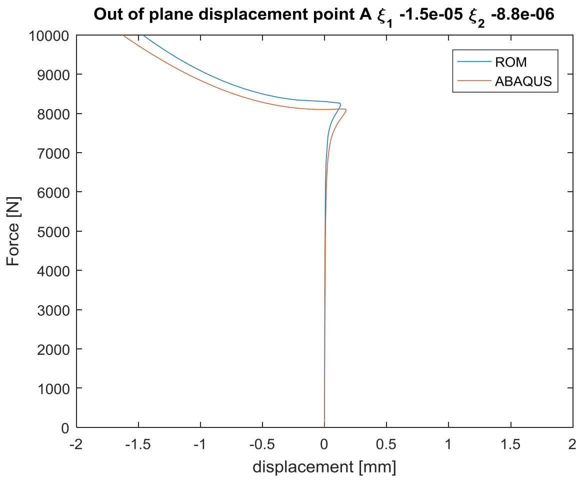
Comparison with ABAQUS, points A & B

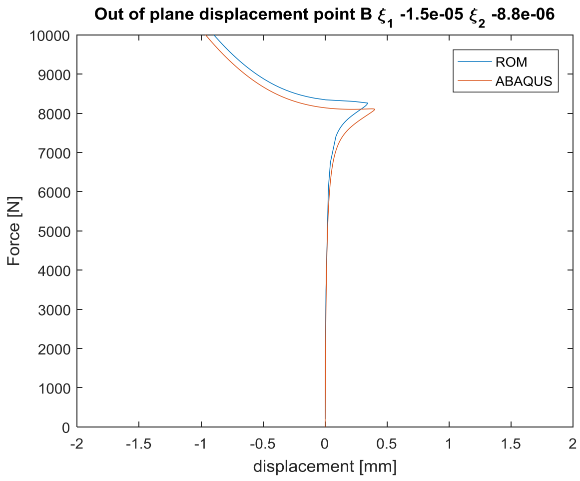




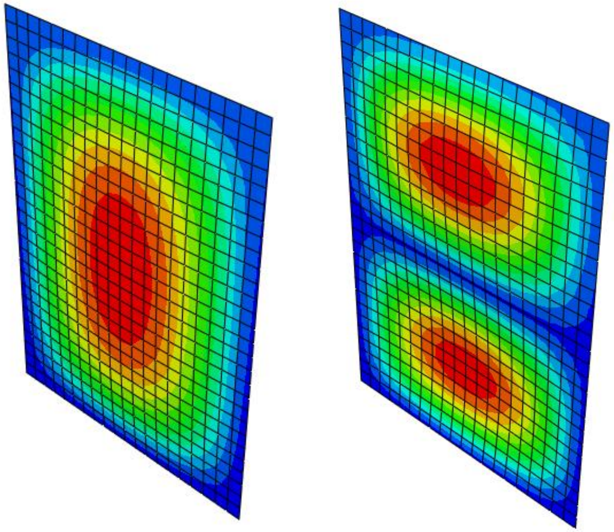


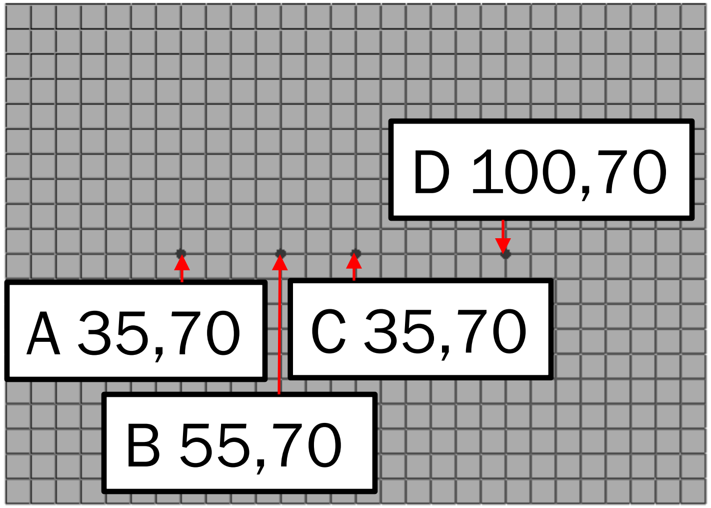


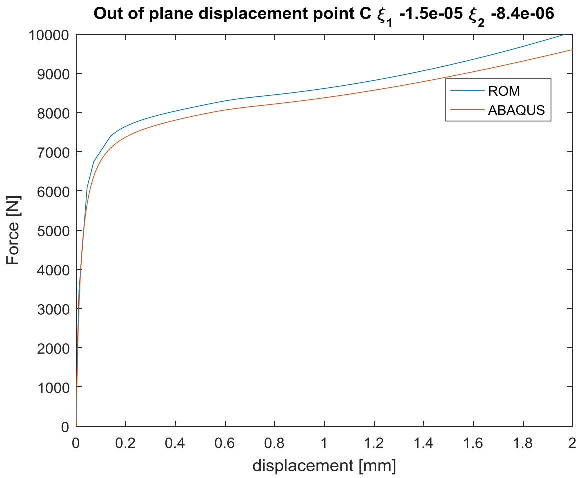


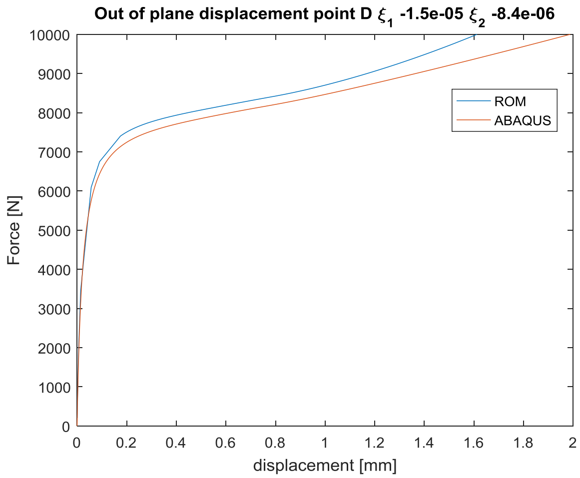


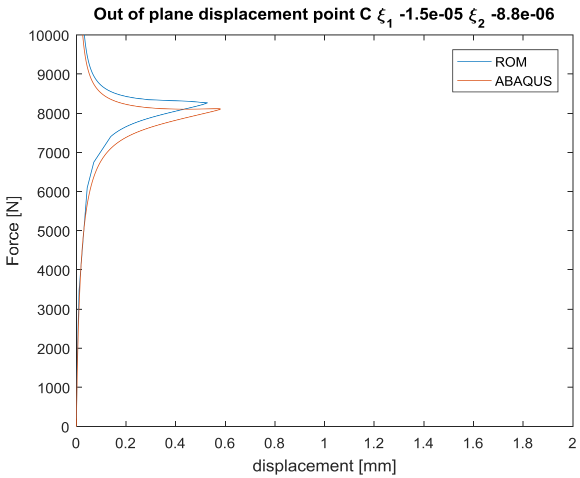
Comparison with ABAQUS, points C & D

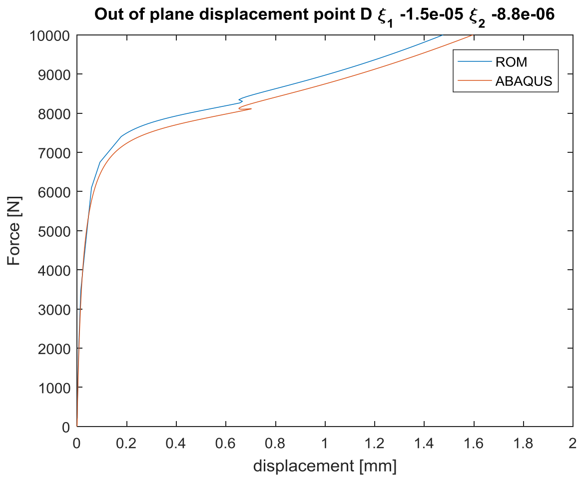




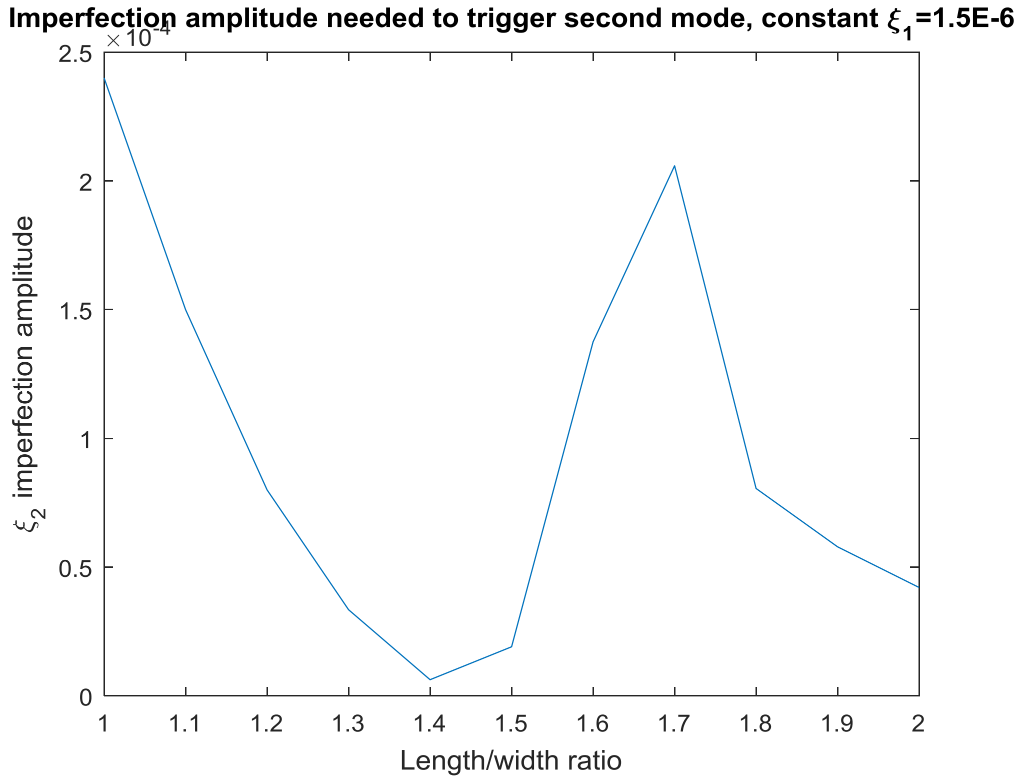


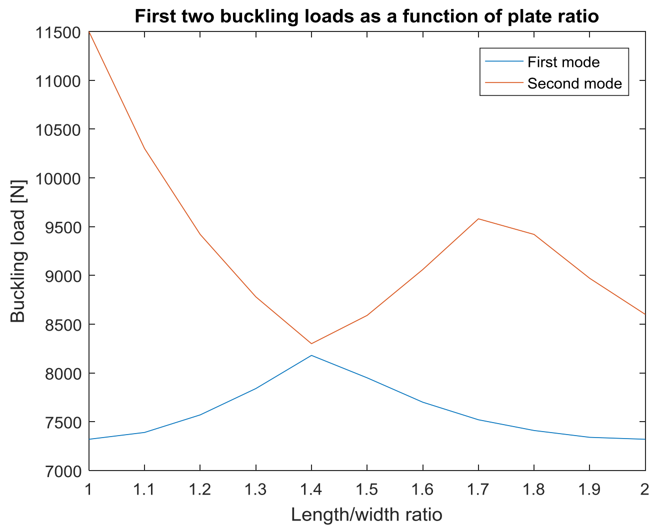


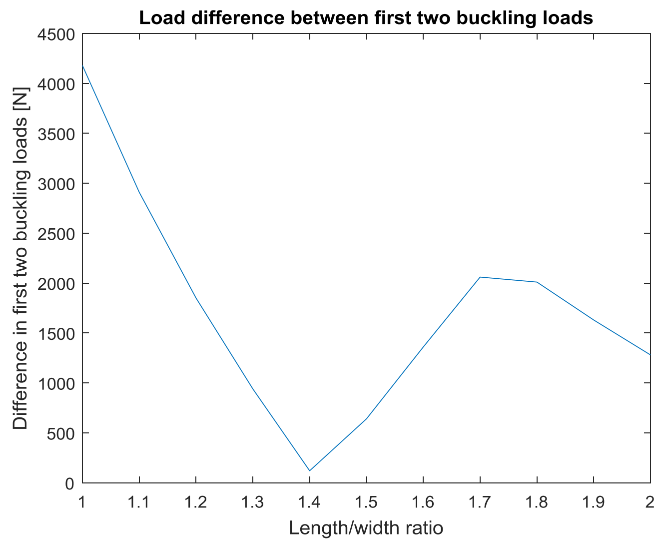




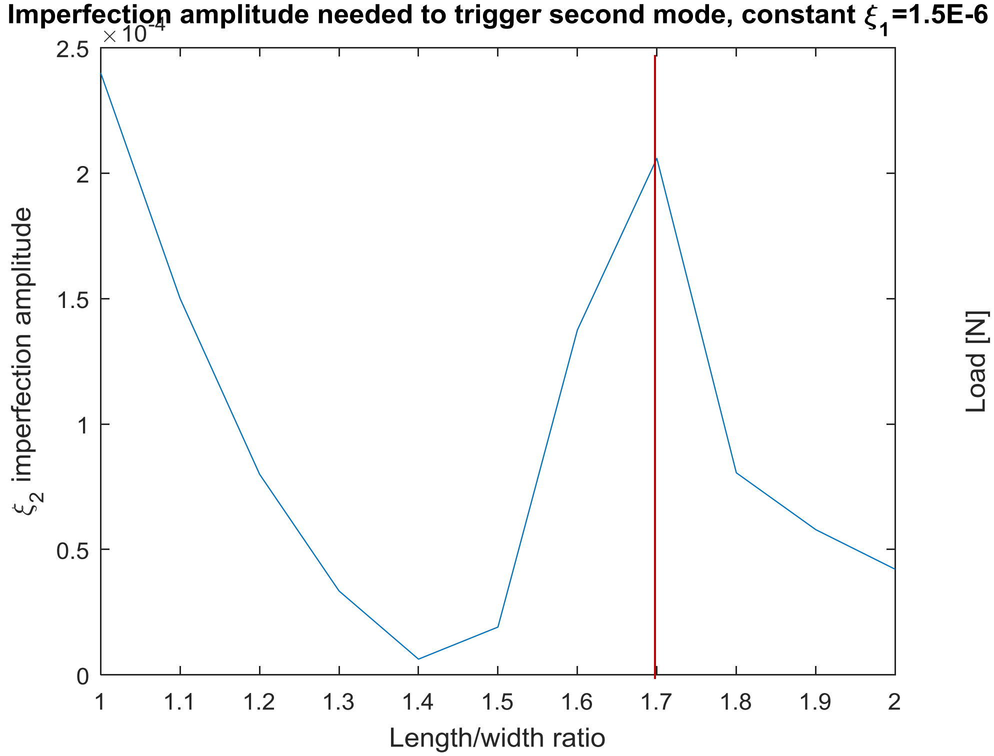
Imperfection amplitude needed for mode switching

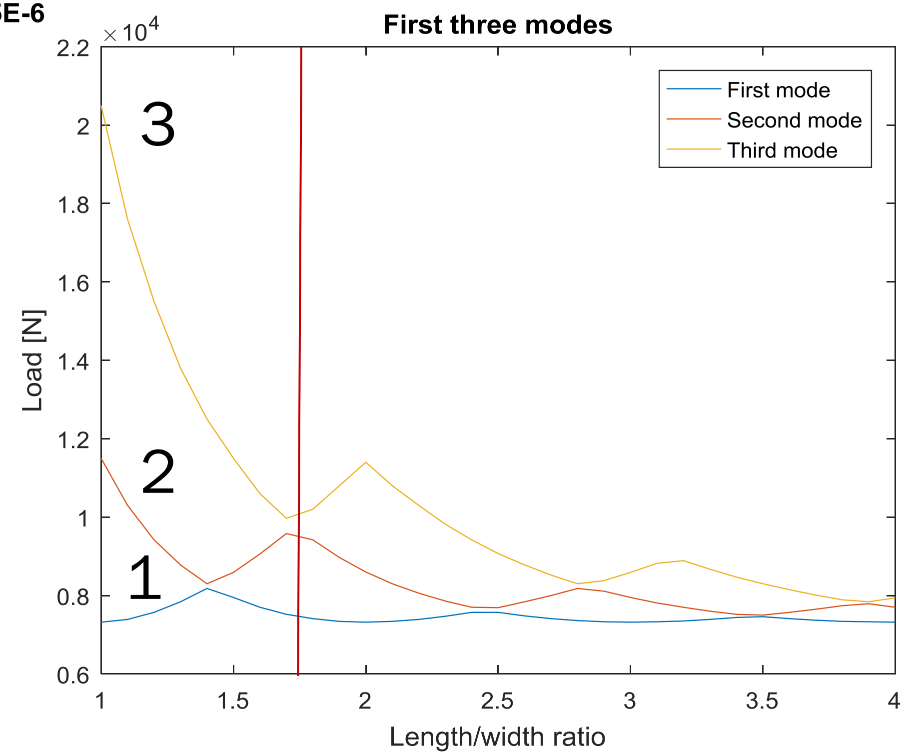






Switching of modes and related imperfection amplitude





Concluding remarks

- Test cases for nonlinear buckling analysis of plates have been investigated
  - Modal interactions have been investigated for specific imperfection shapes: Influence of imperfection amplitude on mode switching depends strongly on how close the modes are to each other
  - Failure mode of the optimized component might be sensitive to the manufacturing tolerances

- Specific test cases can be used as reference cases in the current extensions of the reduced order approach
  - Analyze manufacturing tolerances through probabilistic inputs
  - Determine sensitivity to imperfections in individual manufacturing processes (parameters)

Thank you for your attention!

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