Node-Dependent Kinematic One-dimensional FEM Models for the Analysis of Beams with Piezo-patches

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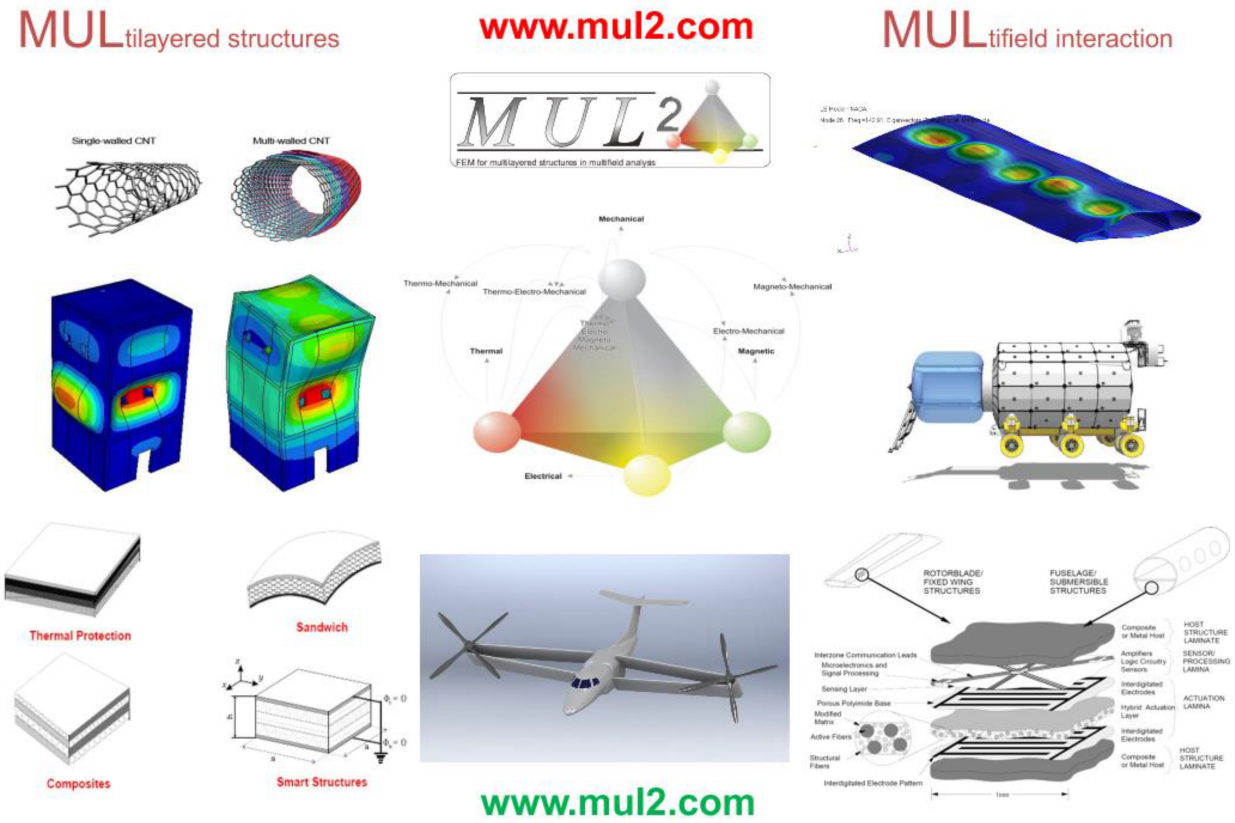

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Overview

1. Carrera Unified Formulation (CUF) for refined 1D models.
2. Node-dependent kinematics (NDK).
3. Modeling of piezo-patches with NDK beam elements.
4. Numerical examples.
5. Conclusions.

An Example: A Higher-order Deformation Beam Theory Written in CUF

– Displacement description

$$\begin{cases} u = F_1 u_1 & +F_2 u_2 & +F_3 u_3 & +F_4 u_4 & +F_5 u_5 & +F_6 u_6 + \cdots \\ v = F_1 v_1 & +F_2 v_2 & +F_3 v_3 & +F_4 v_4 & +F_5 v_5 & +F_6 v_6 + \cdots \\ w = F_1 w_1 & +F_2 w_2 & +F_3 w_3 & +F_4 w_4 & +F_5 w_5 & +F_6 w_6 + \cdots \end{cases}$$

– F_τ

$$F_1 = 1,$$
$$F_2 = x, \quad F_3 = z,$$
$$F_4 = x^2, \quad F_5 = xz, \quad F_6 = z^2,$$
$$\ldots$$

– FEM discretization

$$\boldsymbol{u}(x,y,z) = N_i(y) \cdot \boldsymbol{u}_i(x,z) = N_i(y) \cdot F_\tau(x,z) \cdot \boldsymbol{u}_{i\tau}$$

– PVD

$$\boldsymbol{u}(x,y,z) = F_\tau(x,z) N_i(y)\, \boldsymbol{u}_{i\tau} \qquad \delta\boldsymbol{u}(x,y,z) = F_s(x,z) N_j(y)\, \delta\boldsymbol{u}_{js}$$

$$\delta L_{int} = \int_V \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV = \int_V \delta\boldsymbol{u}^T \boldsymbol{b}^T \boldsymbol{C}\boldsymbol{b}\boldsymbol{u} dV$$

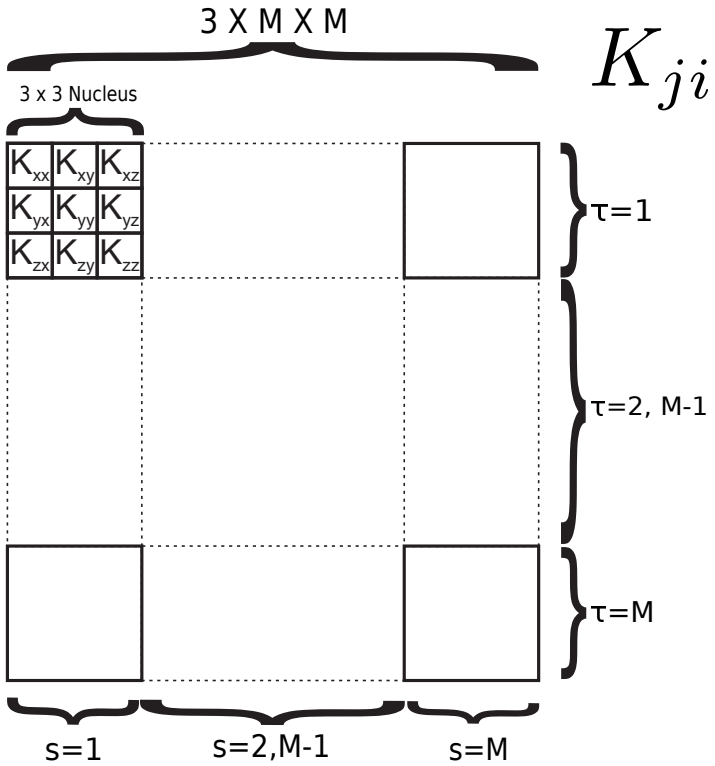
$$= \int_V \delta\boldsymbol{u}_{js}^T N_j \boldsymbol{I} F_s \boldsymbol{b}^T \boldsymbol{C}\boldsymbol{b} F_\tau \boldsymbol{I} N_i \boldsymbol{u}_{i\tau} dV$$

$$= \delta\boldsymbol{u}_{js}^T \cdot \int_V \delta N_j \boldsymbol{I} F_s \boldsymbol{b}^T \boldsymbol{C}\boldsymbol{b} F_\tau \boldsymbol{I} N_i dV \cdot \boldsymbol{u}_{i\tau} = \delta\boldsymbol{u}_{js}^T \cdot \boldsymbol{K}_{ij\tau s} \cdot \boldsymbol{u}_{i\tau}$$

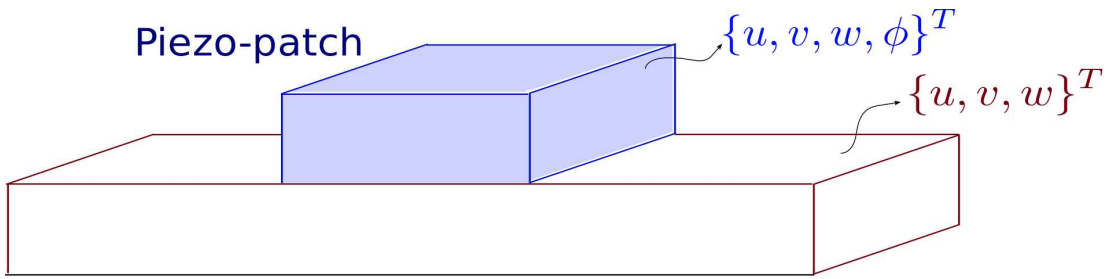
$$\delta L_{ext} = \int_V \delta\boldsymbol{u}^T \boldsymbol{P} dV = \delta\boldsymbol{u}_{js}^T \int_V N_j \boldsymbol{I} F_s \boldsymbol{P} dV = \delta\boldsymbol{u}_{js}^T \boldsymbol{P}_{js}$$

$$\boldsymbol{K}_{ij\tau s} = \int_V N_j \boldsymbol{I} F_s \boldsymbol{b}^T \boldsymbol{C}\boldsymbol{b} F_\tau \boldsymbol{I} N_i dV$$

Fundamental Nucleus (FN)



Refined 1D Models for Electro-mechanical Problems



$$\boldsymbol{u}(x, y, z) = \{u, v, w\}^T = F_\tau(x, z)\boldsymbol{u}_\tau(y)$$

$$\boldsymbol{q}(x, y, z) = \{u, v, w, \phi\}^T = F_\tau(x, z)\boldsymbol{q}_\tau(y)$$

ESL model adopting Taylor Expansions (TE)

$$\boldsymbol{q}(x, y, z) = \sum_{\tau=1}^{N} x^{i_\tau} z^{j_\tau} \boldsymbol{q}_\tau(y)$$

Note:
– F_τ are defined on the whole cross-section domain;
– Higher-order DOFs: mathematical weighting factors.

LW model employing Lagrange Expansions (LE)

$$\boldsymbol{q}^k(x, y, z) = \sum_{\tau=1}^{N} L_\tau(x, z)\boldsymbol{q}_\tau^k(y)$$

Note:
– F_τ^k defined on each layer section domain;
– All DOFs are physically meaningful.

Beam Elements with Node-Dependent Kinematics (NDK)

– 1D Models with NDK

$$\boldsymbol{u} = F_\tau N_i \cdot \boldsymbol{I} \cdot \boldsymbol{u}_{i\tau}$$

$$\boldsymbol{u} = F_\tau^i N_i \cdot \boldsymbol{I} \cdot \boldsymbol{u}_{i\tau}$$

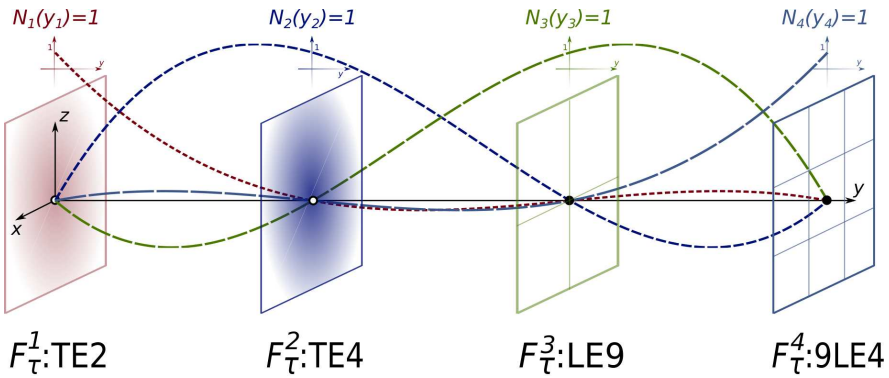
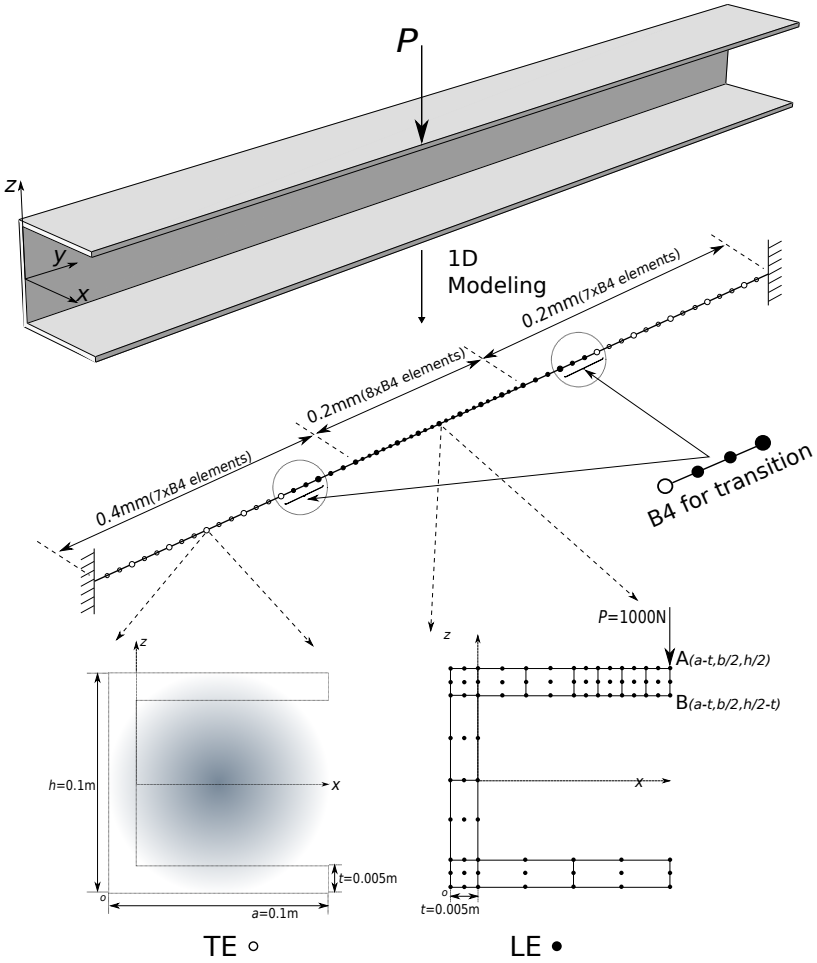


Figure: A B4 element with NDK.

- Variable LW/ESL nodal capabilities.
- Global-local analysis
- Modeling of patches

– Application: Local kinematic refinement



Electro-mechanical Constitutive Relations

– Electro-mechanical Constitutive Equations

$$\boldsymbol{E} = \{E_x, E_y, E_z\}^T = \{\partial_x, \partial_y, \partial_z\}^T \phi$$

$$\bar{\boldsymbol{\epsilon}} = \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xz}, \varepsilon_{yz}, \varepsilon_{xy}, E_x, E_y, E_z\}^T = \boldsymbol{D}\boldsymbol{q}$$

$$\bar{\boldsymbol{\sigma}} = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xz}, \sigma_{yz}, \sigma_{xy}, D_x, D_y, D_z\}^T = \tilde{\boldsymbol{H}}\bar{\boldsymbol{\epsilon}}$$

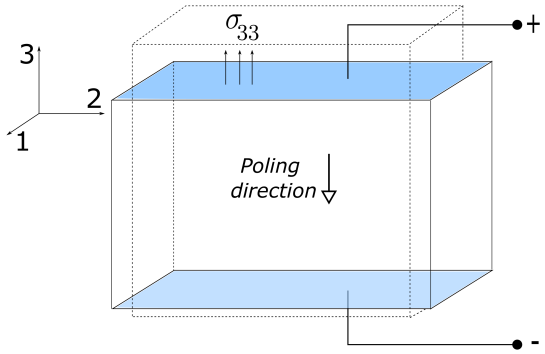
$$\boldsymbol{D} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & 0 & \frac{\partial}{\partial y} \\ 0 & 0 & 0 & \frac{\partial}{\partial z} \end{bmatrix}, \quad \tilde{\boldsymbol{H}} = \begin{bmatrix} \tilde{\boldsymbol{C}}_{6\times6} & -\tilde{\boldsymbol{e}}^T_{6\times3} \\ \tilde{\boldsymbol{e}}_{3\times6} & \tilde{\boldsymbol{\chi}}_{3\times3} \end{bmatrix}.$$

$$\tilde{\boldsymbol{H}} = \boldsymbol{\Lambda}\boldsymbol{H}_m\boldsymbol{\Lambda}^T$$

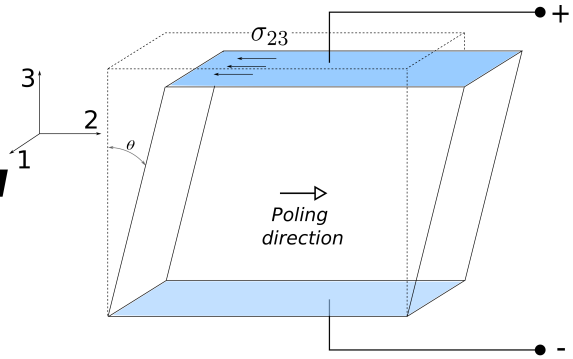
$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{T}_{6\times6} & \\ & \boldsymbol{R}_{3\times3} \end{bmatrix}$$

– Extension/Shear Actuation Mechanism

a)***EAM***



b)***SAM***



– Governing Equation

$$\delta L_{int} = \delta \boldsymbol{q}_{js} \cdot \int_V N_j \boldsymbol{I} F^j_s \boldsymbol{D}^T \tilde{\boldsymbol{H}} \boldsymbol{D} F^i_\tau \boldsymbol{I} N_i dV \cdot \boldsymbol{q}_{i\tau} = \delta \boldsymbol{q}_{sj} \cdot \boldsymbol{K}_{ij\tau s} \cdot \boldsymbol{q}_{i\tau}$$

Modeling of Piezo-patches

– FEM Discretization

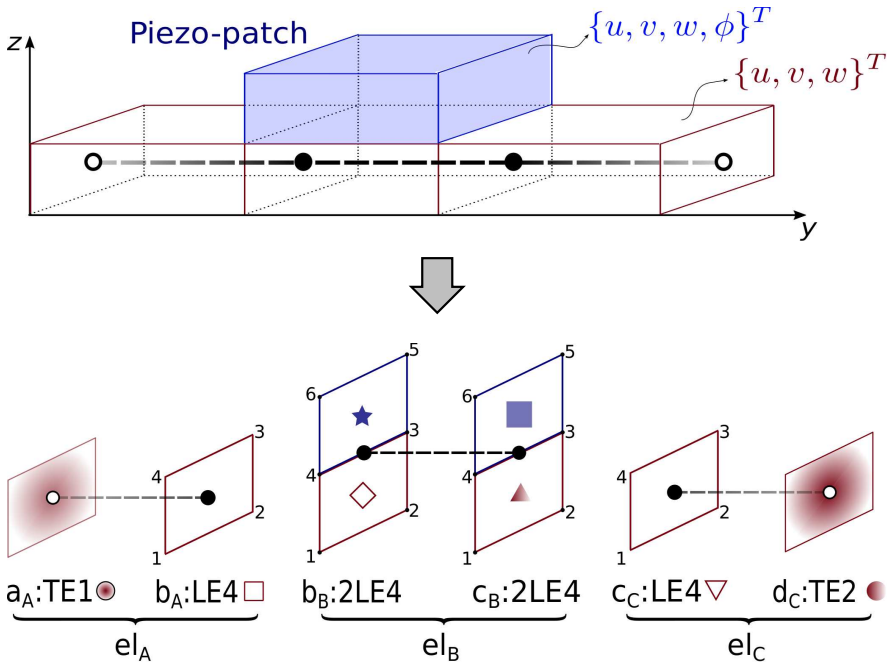
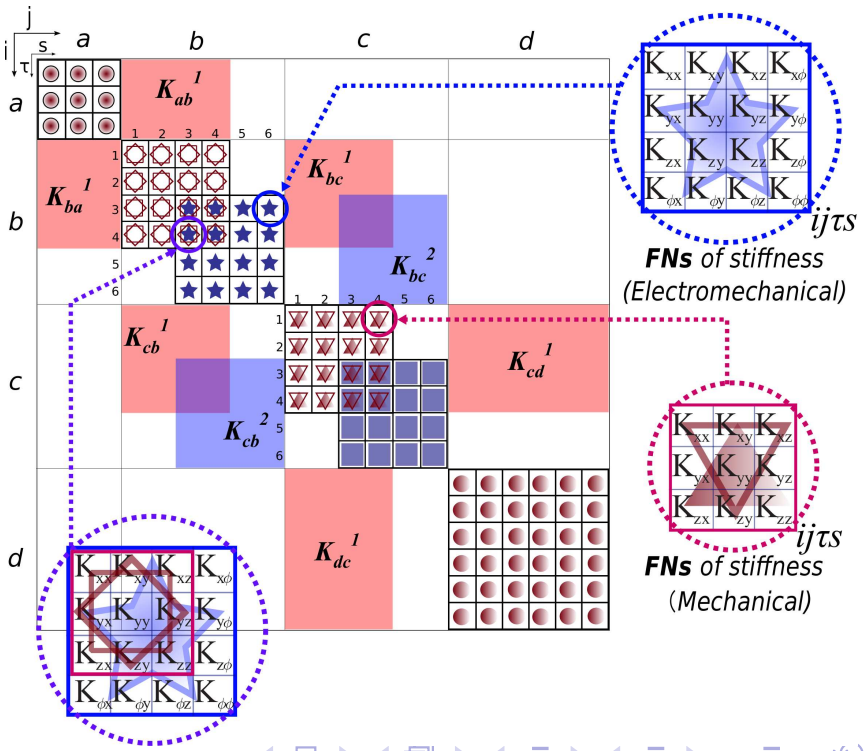


Figure: 1D modeling of piezo-patch.

– Electro-mechanical *FN*: $\boldsymbol{K}_{ij\tau s}$

$$\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} & K_{x\phi} \\ K_{yx} & K_{yy} & K_{yz} & K_{y\phi} \\ K_{zx} & K_{zy} & K_{zz} & K_{z\phi} \\ K_{\phi x} & K_{\phi y} & K_{\phi z} & K_{\phi\phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{MM}_{3\times3} & \boldsymbol{ME}_{3\times1} \\ \boldsymbol{EM}_{1\times3} & \boldsymbol{EE} \end{bmatrix}$$

–Assembly of Stiffness Matrix



Beams with piezo-pathces with EAM and SAM

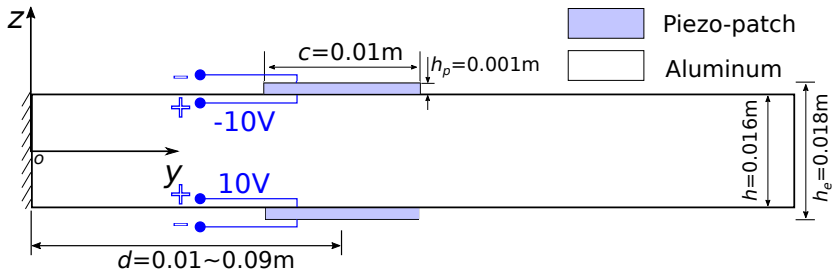


Figure: Extension mechanism (EAM).

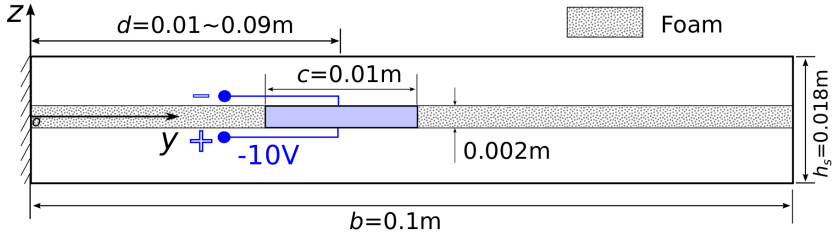


Figure: Shear mechanism (SAM).

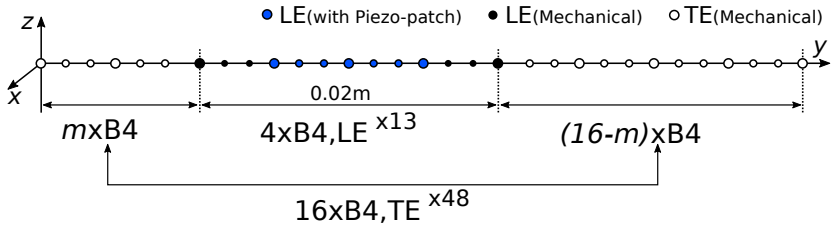
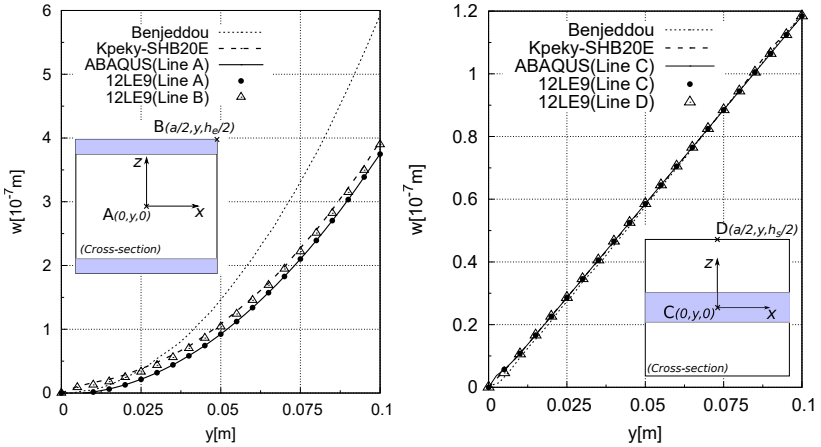


Figure: FEM discretization.

– Case A: Piezo-patches cover the whole longitudinal range



(a) EAM (b) SAM

Table: Results with mono-kinematics model

	$w[10^{-7}\,\mathrm{m}]$			
	EAM		SAM	
	$(0,b,0)$	$(\frac{a}{2},b,\frac{h_e}{2})$	$(0,b,0)$	$(\frac{a}{2},b,\frac{h_s}{2})$
ABAQUS	3.749	3.913	1.184	1.184
12LE9	3.748	3.897	1.184	1.184

Beams with piezo-pathces with EAM and SAM

– Case B: Piezo-patches with variable position

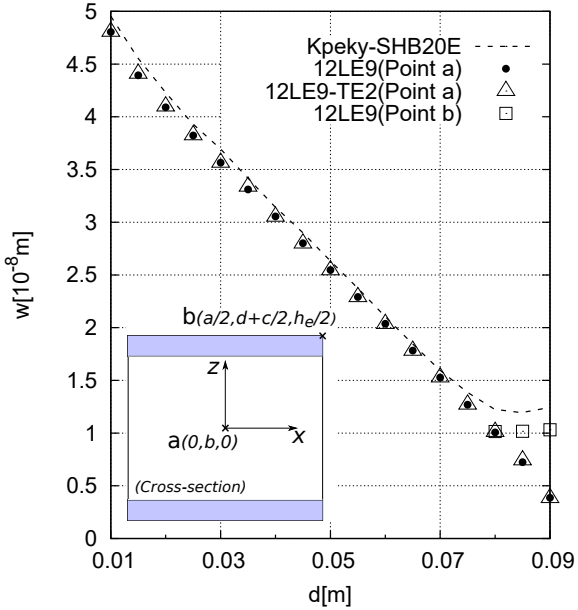
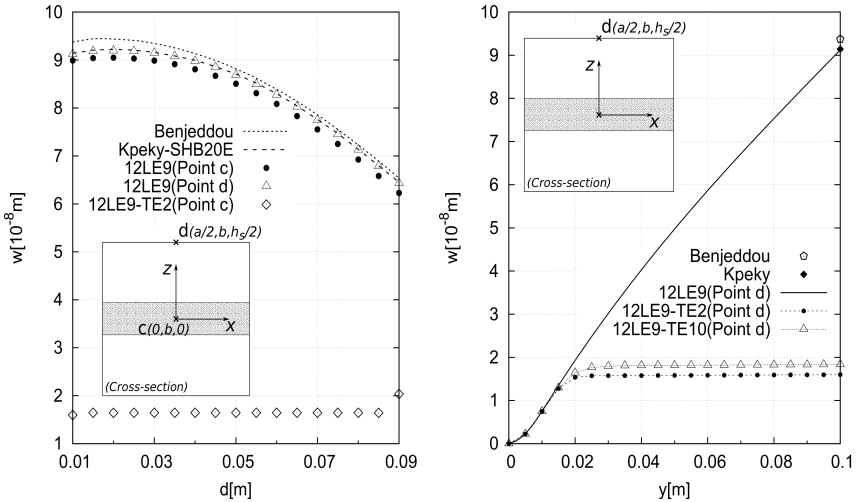


Figure: Extension mechanism (EAM).

d[m]	$w^*[10^{-8}$m] at Point a$(0,b,0)$ (EAM)	
	12LE9	12LE9-TE2
0.01	4.805	4.805
0.03	3.565	3.563
0.05	2.546	2.543
0.07	1.527	1.527
0.09	0.3863	0.3826
DOFs	5765	3317



(a) d=0.01-0.09m (b) d=0.01m

Figure: Shear mechanism (SAM).

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A unified beam finite element model for extension and shear piezoelectric actuation mechanisms
Journal of Intelligent Material Systems and Structures, 8(12), pp.1012-1025.

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Linear and quadratic solid–shell finite elements SHB8PSE and SHB20E for the modeling of piezoelectric sandwich structures
Mechanics of Advanced Materials and Structures. 2017 Mar 30:1-20.

Cantilever beam with a surface-mounted piezo-patch

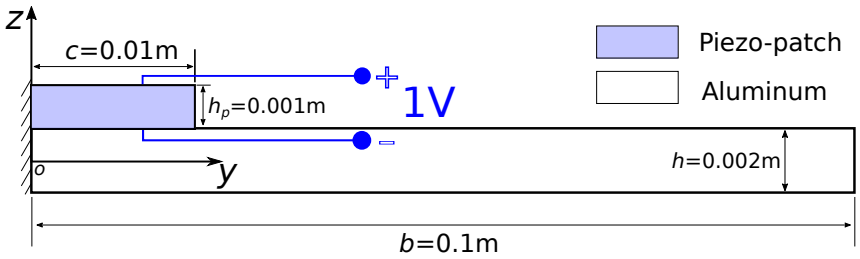


Figure: Side view.

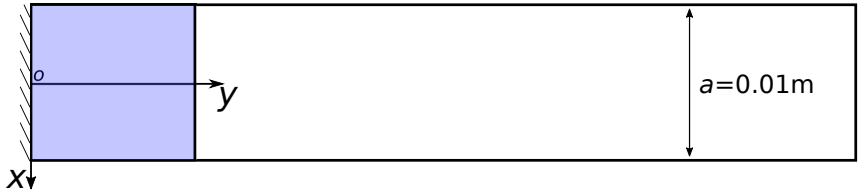


Figure: Top view.

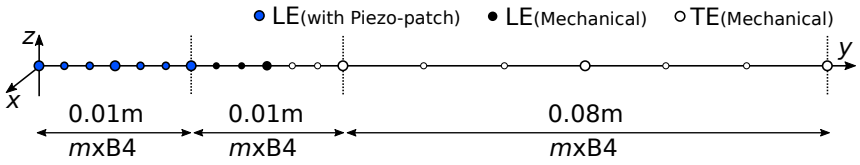
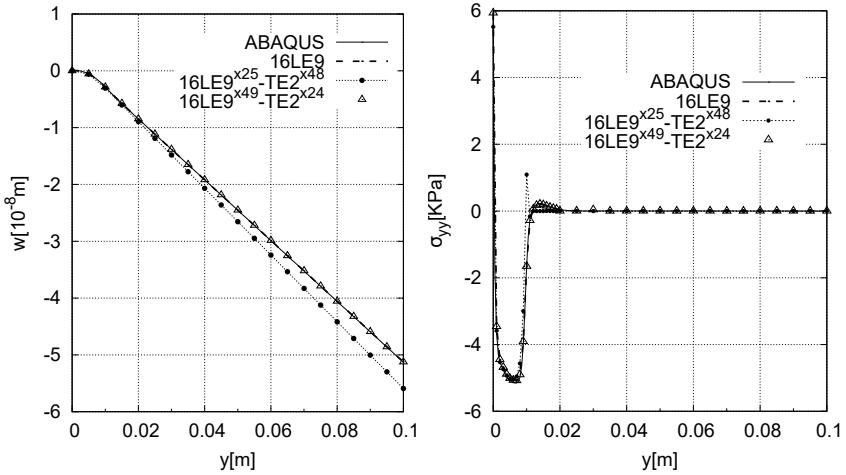


Figure: FEM discretization.

E_1,E_2 [GPa]	E_3 [GPa]	G_{12} [GPa]	G_{13},G_{23} [GPa]	ν_{12}	ν_{13}, ν_{23}
81.3	64.5	30.6	25.6	0.329	0.432

e_{31},e_{32} C/m^2	e_{33} C/m^2	e_{15},e_{24} C/m^2	χ_{11},χ_{22}	χ_{33}
-5.2	15.8	12.72	1475χ_0	1300χ_0

Vacuum permittivity: $\chi_0 = 8.85 \times 10^{-12}$ F/m



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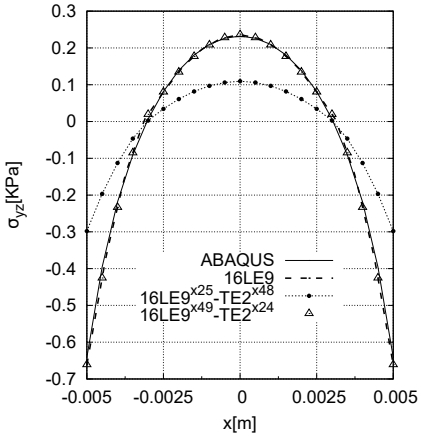
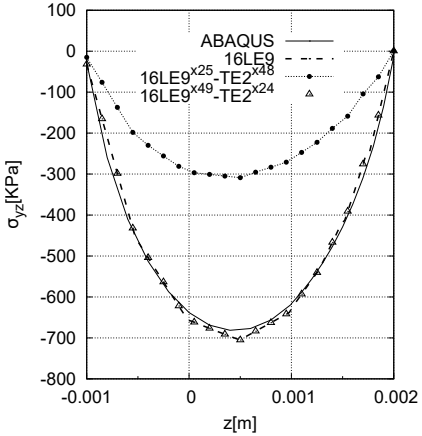
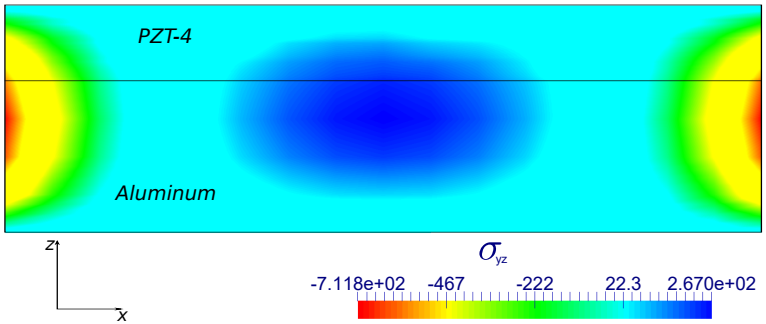
Coupling of hierarchical piezoelectric plate finite elements via Arlequin method.
Journal of Intelligent Material Systems and Structures, 23(7), pp.749-764.

Cantilever beam with a surface-mounted piezo-patch

Mesh	Kinematics	$-u_z[10^{-8}\text{m}]$ $(0, \frac{b}{2}, 0)$	$-u_z[10^{-8}\text{m}]$ $(0, b, 0)$	$-\sigma_{yy}$[KPa] $(0, \frac{c}{2}, -\frac{h}{2})$	$-\sigma_{yz}$[KPa] $(\frac{a}{2}, \frac{c}{2}, 0)$	DOFs
12×B4	4LE9	2.482	5.192	5.878	0.5149	2250
12×B4	16LE9	2.444	5.109	5.131	0.6692	12852
24×B4	16LE9	2.452	5.125	5.009	0.6612	25164
24×B4	$16LE9^{\times 25}$-$TE2^{\times 48}$	2.656	5.592	5.028	0.2979	14346
24×B4	$16LE9^{\times 49}$-$TE2^{\times 24}$	2.452	5.125	5.009	0.6612	19908
ABAQUS		2.451	5.125	5.087	0.6381	196281
Biscani-2D(LD3)		2.309	4.871	–	–	–







Main Conclusions

With node-dependent kinematic beam elements:

1. Models with variable LW/ESL nodal capabilities can be conveniently formulated;
2. The abrupt change of the cross-section introduced by the patches can be considered;
3. Slender structures with surface mounted or embedded piezo-patches can be efficiently modeled;
4. Mechanical and electro-mechanical constitutive relations can be separately applied to the base structure and the piezoelectric actuators, with the help of LW models;
5. The structural responses under piezoelectric actuation can be properly captured with reduced computational costs;
6. When applied in the modeling of shear mechanism, the adopted kinematics should be able to capture the shearing effects appropriately.

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