Node-Dependent Kinematic One-dimensional FEM Models for the Analysis of Beams with Piezo-patches

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8th Conference on Smart Structures and Materials
6th International Conference on Smart Materials and Nanotechnology in Engineering
SMART2017
6 June 2017, Madrid
MUL2 - Our Research Group

Marie Curie Project on Composites

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Overview

1. Carrera Unified Formulation (CUF) for refined 1D models.
5. Conclusions.
An Example: A Higher-order Deformation Beam Theory Written in CUF

- Displacement description

\[
\begin{align*}
    u &= F_1 u_1 + F_2 u_2 + F_3 u_3 + F_4 u_4 + F_5 u_5 + F_6 u_6 + \cdots \\
    v &= F_1 v_1 + F_2 v_2 + F_3 v_3 + F_4 v_4 + F_5 v_5 + F_6 v_6 + \cdots \\
    w &= F_1 w_1 + F_2 w_2 + F_3 w_3 + F_4 w_4 + F_5 w_5 + F_6 w_6 + \cdots \\
    \end{align*}
\]

- FEM discretization

\[
\begin{align*}
    u(x, y, z) &= N_i(y) \cdot u_i(x, z) = N_i(y) \cdot F_\tau(x, z) \cdot u_{i\tau} \\
    \end{align*}
\]

- PVD

\[
\begin{align*}
    u(x, y, z) &= F_\tau(x, z) N_i(y) u_{i\tau} \\
    \delta u(x, y, z) &= F_\tau(x, z) N_j(y) \delta u_{j\tau}
\end{align*}
\]

\[
\delta L_{int} = \int_V \delta \varepsilon^T \sigma dV = \int_V \delta u^T b^T C b dV
\]

\[
\begin{align*}
    &= \int_V \delta u_{j\tau}^T N_j I F_s b^T C b F_\tau I N_i u_{i\tau} dV \\
    &= \delta u_{j\tau}^T \cdot \int_V \delta N_j I F_s b^T C b F_\tau I N_i dV \cdot u_{i\tau} = \delta u_{j\tau}^T \cdot \mathbf{K}_{i\tau} \cdot u_{i\tau}
\end{align*}
\]

\[
\delta L_{ext} = \int_V \delta u^T P dV = \delta u_{j\tau}^T \int_V N_j I F_s P dV = \delta u_{j\tau}^T P_{j\tau}
\]

**Fundamental Nucleus (FN)**

\[
K_{i\tau} = \int_V N_j I F_s b^T C b F_\tau I N_i dV
\]
Refined 1D Models for Electro-mechanical Problems

\[ u(x, y, z) = \{u, v, w\}^T = F_\tau(x, z)u_\tau(y) \]

\[ q(x, y, z) = \{u, v, w, \phi\}^T = F_\tau(x, z)q_\tau(y) \]

ESL model adopting Taylor Expansions (TE)

\[ q(x, y, z) = \sum_{\tau=1}^{N} x^i z^j q_\tau(y) \]

Note:
- \( F_\tau \) are defined on the whole cross-section domain;
- Higher-order DOFs: mathematical weighting factors.

LW model employing Lagrange Expansions (LE)

\[ q^k(x, y, z) = \sum_{\tau=1}^{N} L_\tau(x, z)q^k_\tau(y) \]

Note:
- \( F^k_\tau \) defined on each layer section domain;
- All DOFs are physically meaningful.
Beam Elements with Node-Dependent Kinematics (NDK)

– 1D Models with NDK

\[ u = F^i T N_i \cdot I \cdot u^i T \]

– Application: Local kinematic refinement

Variable LW/ESL nodal capabilities.
Global-local analysis
Modeling of patches

Figure: A B4 element with NDK.
Electro-mechanical Constitutive Relations

- Electro-mechanical Constitutive Equations

\[ E = \{E_x, E_y, E_z\}^T = \{\partial_x, \partial_y, \partial_z\}^T \phi \]
\[ \bar{\epsilon} = \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xz}, \varepsilon_{yz}, \varepsilon_{xy}, E_x, E_y, E_z\}^T = Dq \]
\[ \bar{\sigma} = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xz}, \sigma_{yz}, \sigma_{xy}, D_x, D_y, D_z\}^T = \tilde{H}\bar{\epsilon} \]

\[ D = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 & 0 \\
0 & \frac{\partial}{\partial y} & 0 & 0 \\
0 & 0 & \frac{\partial}{\partial z} & 0 \\
\frac{\partial}{\partial z} & \frac{\partial}{\partial y} & 0 & 0 \\
\frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 \\
0 & 0 & 0 & \frac{\partial}{\partial z} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \]
\[ \tilde{H} = \begin{bmatrix}
\tilde{C}_{6\times6} & -\tilde{e}^T_{6\times3} \\
\tilde{e}_{3\times6} & \tilde{X}_{3\times3} \\
\end{bmatrix}. \]

\[ \tilde{H} = \Lambda H_m \Lambda^T \]
\[ \Lambda = \begin{bmatrix}
T_{6\times6} & \mathbf{R}_{3\times3} \\
\end{bmatrix} \]

- Extension/Shear Actuation Mechanism

a) EAM

b) SAM

- Governing Equation

\[ \delta L_{\text{int}} = \delta q_{js} \cdot \int_V N_j I F_s^j \tilde{H} D_i^T \tilde{H} D_i N_i dV \cdot q_{ir} = \delta q_{sj} \cdot K_{ijrs} \cdot q_{ir} \]
Modeling of Piezo-patches

– FEM Discretization

\[
\begin{bmatrix}
K_{xx} & K_{xy} & K_{xz} & K_{x\phi} \\
K_{yx} & K_{yy} & K_{yz} & K_{y\phi} \\
K_{zx} & K_{zy} & K_{zz} & K_{z\phi} \\
K_{\phi x} & K_{\phi y} & K_{\phi z} & K_{\phi \phi}
\end{bmatrix}
= 
\begin{bmatrix}
MM_{3\times3} & ME_{3\times1}
\end{bmatrix}
\]
Beams with piezo-patches with EAM and SAM

Figure: Extension mechanism (EAM).

Figure: Shear mechanism (SAM).

Table: Results with mono-kinematics model
Beams with piezo-patches with EAM and SAM

– Case B: Piezo-patches with variable position

**Figure: Extension mechanism (EAM).**

<table>
<thead>
<tr>
<th>$d$ [m]</th>
<th>$w^*[10^{-8} \text{m}]$ at Point $(0,b,0)$ (EAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>4.805</td>
</tr>
<tr>
<td>0.03</td>
<td>3.565</td>
</tr>
<tr>
<td>0.05</td>
<td>2.546</td>
</tr>
<tr>
<td>0.07</td>
<td>1.527</td>
</tr>
<tr>
<td>0.09</td>
<td>0.3863</td>
</tr>
<tr>
<td>DOFs</td>
<td>5765</td>
</tr>
</tbody>
</table>

**Figure: Shear mechanism (SAM).**

Benjeddou, A and Trindade, MA and Ohayon, R,
A unified beam finite element model for extension and shear piezoelectric actuation mechanisms

Kpeky F, Abed-Meraim F, Boudaoud H, Daya EM,
Linear and quadratic solid–shell finite elements SHB8PSE and SHB20E for the modeling of piezoelectric sandwich structures
Cantilever beam with a surface-mounted piezo-patch

**Figure: Side view.**

- Beam dimensions: $c=0.01m$, $b=0.1m$
- Piezo-patch thickness: $h_p=0.001m$
- Aluminum thickness: $h=0.002m$

**Figure: Top view.**

- Beam dimensions: $a=0.01m$

**Figure: FEM discretization.**

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<table>
<thead>
<tr>
<th>$E_1$ [GPa]</th>
<th>$E_2$ [GPa]</th>
<th>$E_3$ [GPa]</th>
<th>$G_{12}$ [GPa]</th>
<th>$G_{13},G_{23}$ [GPa]</th>
<th>$\nu_{12}$</th>
<th>$\nu_{13},\nu_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>81.3</td>
<td>64.5</td>
<td>30.6</td>
<td>25.6</td>
<td>0.329</td>
<td>0.432</td>
<td></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>$e_{31},e_{32}$ [C/m²]</th>
<th>$e_{33}$ [C/m²]</th>
<th>$e_{15},e_{24}$ [C/m²]</th>
<th>$\chi_{11},\chi_{22}$</th>
<th>$\chi_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.2</td>
<td>15.8</td>
<td>12.72</td>
<td>1475 $\chi_0$</td>
<td>1300 $\chi_0$</td>
</tr>
</tbody>
</table>

Vacuum permittivity: $\chi_0 = 8.85 \times 10^{-12}$ F/m
# Cantilever beam with a surface-mounted piezo-patch

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Kinematics</th>
<th>$-u_z [10^{-8} \text{m}] (0, b, 0)$</th>
<th>$-u_z [10^{-8} \text{m}] (0, \frac{c}{2}, -\frac{h}{2})$</th>
<th>$-\sigma_{yy} [\text{KPa}] (\frac{a}{2}, \frac{c}{2}, 0)$</th>
<th>$-\sigma_{yz} [\text{KPa}] (\frac{a}{2}, \frac{c}{2}, 0)$</th>
<th>DOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>12×B4</td>
<td>4LE9</td>
<td>2.482</td>
<td>5.192</td>
<td>5.878</td>
<td>0.5149</td>
<td>2250</td>
</tr>
<tr>
<td>12×B4</td>
<td>16LE9</td>
<td>2.444</td>
<td>5.109</td>
<td>5.131</td>
<td>0.6692</td>
<td>12852</td>
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<td>24×B4</td>
<td>16LE9</td>
<td>2.452</td>
<td>5.125</td>
<td>5.009</td>
<td>0.6612</td>
<td>25164</td>
</tr>
<tr>
<td>24×B4</td>
<td>16LE9×25-TE2×48</td>
<td>2.656</td>
<td>5.592</td>
<td>5.028</td>
<td>0.2979</td>
<td>14346</td>
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<tr>
<td>24×B4</td>
<td>16LE9×49-TE2×24</td>
<td>2.452</td>
<td>5.125</td>
<td>5.009</td>
<td>0.6612</td>
<td>19908</td>
</tr>
<tr>
<td>ABAQUS</td>
<td></td>
<td>2.451</td>
<td>5.125</td>
<td>5.087</td>
<td>0.6381</td>
<td>196281</td>
</tr>
<tr>
<td>Biscani-2D(LD3)</td>
<td></td>
<td>2.309</td>
<td>4.871</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

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### Mesh Kinematics

- 4LE9
- 16LE9
- 16LE9×25-TE2×48
- 16LE9×49-TE2×24

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### Material Properties

- **PZT-4**
- **Aluminum**

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### Details

**ABAQUS**, **16LE9**, **16LE9×25-TE2×48**, **16LE9×49-TE2×24**

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### Diagrams

- **σ_{yz} [KPa]**
- **σ_{zz} [KPa]**

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Main Conclusions

With node-dependent kinematic beam elements:

1. Models with variable LW/ESL nodal capabilities can be conveniently formulated;

2. The abrupt change of the cross-section introduced by the patches can be considered;

3. Slender structures with surface mounted or embedded piezo-patches can be efficiently modeled;

4. Mechanical and electro-mechanical constitutive relations can be separately applied to the base structure and the piezoelectric actuators, with the help of LW models;

5. The structural responses under piezoelectric actuation can be properly captured with reduced computational costs;

6. When applied in the modeling of shear mechanism, the adopted kinematics should be able to capture the shearing effects appropriately.
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