

Overview

- 1 Description of the Carrera Unified Formulation (CUF) for refined beam models.
- 2 Main 1D CUF models: Taylor Expansions and the novel Hierarchic Legendre Expansions.
- 3 Numerical examples and recent developments.
- 4 Application of CUF models for the analysis of composite structures.
- 5 Conclusions and future developments.

Brief Overview of Beam Refinement Methods and Contributors

- 1 Shear correction factors (Timoshenko, Sokolnikoff, Cowper, Gruttmann, etc.).
- 2 Warping functions and Saint-Venant solutions (El Fatmi, Ladéveze, etc.).
- 3 Variational asymptotic method (Berdichevsky, Hodges, Yu, etc.).
- 4 Generalized beam theory (Schardt, Camotim, Silvestre, etc.).
- 5 Higher-order models (Washizu, Reddy, Kapania, Carrera, etc.).

Higher-order methods: prof. J.N. Reddy

Third-order shear deformation theory

$$u(x, y, z) = u_0(x, y) + z\phi_x(x, y) + z^3\left(-\frac{4}{3h^2}\right)\left(\phi_x + \frac{\partial w_0}{\partial x}\right)$$

$$v(x, y, z) = v_0(x, y) + z\phi_y(x, y) + z^3\left(-\frac{4}{3h^2}\right)\left(\phi_y + \frac{\partial w_0}{\partial y}\right)$$

$$w(x, y, z) = w_0(x, y)$$

- Zero transverse shear stress at faces
- No need of shear correction factors

Layerwise theory

- A compromise between 3D formulations and classical plate theories
- No aspect ratio limitations
- Accurate description of interlaminar stresses

Laminated structures



J.N. Reddy,

Mechanics of Laminated Composite Plate and Shells: Theory and Analysis
CRC Press, 2004.

Carrera Unified Formulation (CUF)

CUF displacement field

$$\mathbf{u} = F_\tau(x, z)N_i(y) \mathbf{u}_{\tau i} \quad (1D)$$

PVD - Static

$$\delta L_{int} = \delta \mathbf{u}_{\tau i}^T \mathbf{K}^{ijrs} \mathbf{u}_{sj}$$

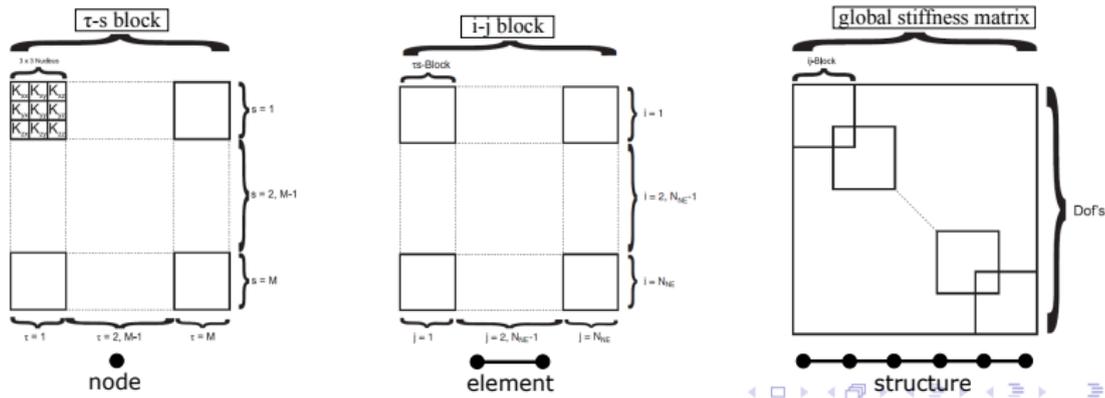
$$\delta L_{ext} = \mathbf{P} \delta \mathbf{u}^T$$

fundamental nucleus

$$\mathbf{K}_{xx}^{ijrs} = \tilde{\mathbf{C}}_{22} \int_{\Omega} F_{\tau,x} F_{s,x} d\Omega \int_l N_i N_j dy + \dots$$

- i, j - Shape function indexes.
- τ, s - Expansion function indexes

Assembly Technique

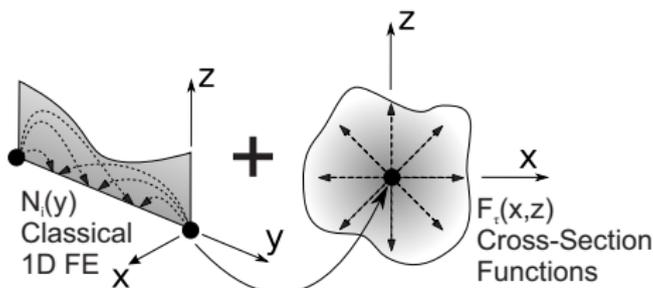


Taylor-based CUF 1D Models, TE

$$\begin{array}{l}
 u_x(x, y, z) = \underbrace{u_{x_1}(y)}_{N=0} + x u_{x_2}(y) + z u_{x_3}(y) + \underbrace{x^2 u_{x_4}(y) + xz u_{x_5}(y) + z^2 u_{x_6}(y) + \dots}_{N=2} \\
 u_y(x, y, z) = \underbrace{u_{y_1}(y)}_{N=0} + x u_{y_2}(y) + z u_{y_3}(y) + \underbrace{x^2 u_{y_4}(y) + xz u_{y_5}(y) + z^2 u_{y_6}(y) + \dots}_{N=2} \\
 u_z(x, y, z) = \underbrace{u_{z_1}(y)}_{N=0} + x u_{z_2}(y) + z u_{z_3}(y) + \underbrace{x^2 u_{z_4}(y) + xz u_{z_5}(y) + z^2 u_{z_6}(y) + \dots}_{N=2}
 \end{array}$$

$$\begin{array}{l}
 N = 0 \\
 \tau = 1 \\
 3 \text{ DOFs}
 \end{array}
 \qquad
 \begin{array}{l}
 N = 1 \\
 \tau = 2, \tau = 3 \\
 9 \text{ DOFs}
 \end{array}
 \qquad
 \begin{array}{l}
 N = 2 \\
 \tau = 4, \tau = 5, \tau = 6 \\
 18 \text{ DOFs}
 \end{array}$$

Classical models, such as [Timoshenko](#), can be obtained as particular cases of the linear models.



$$\begin{aligned}
 F_1(x, z) &= 1 \\
 F_2(x, z) &= x \quad F_3(x, z) = z \\
 F_4(x, z) &= x^2 \quad F_5(x, z) = xz \quad F_6(x, z) = z^2
 \end{aligned}$$

Hierarchical Legendre Expansions, HLE

Vertex polynomials

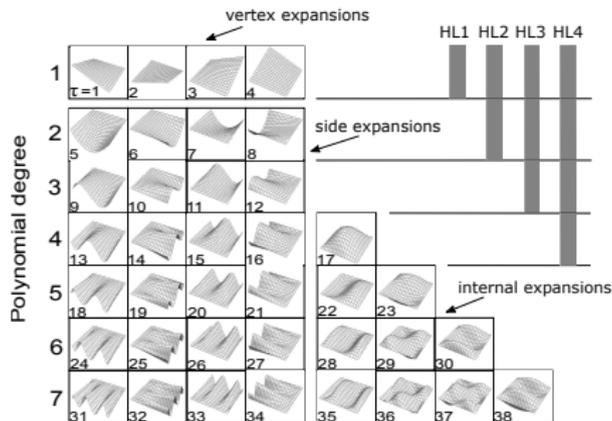
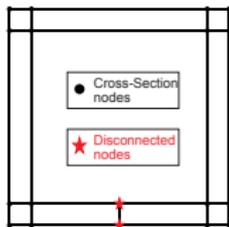
$$F_r = \frac{1}{4}(1 - r_r r)(1 - s_r s)$$

Side polynomials

$$F_r = \frac{1}{2}(1 - s)\phi_p(r)$$

Internal polynomials

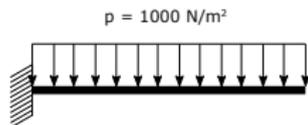
$$F_r = \phi_{p_r}(r)\phi_{p_s}(s) \quad p_r + p_s = p$$



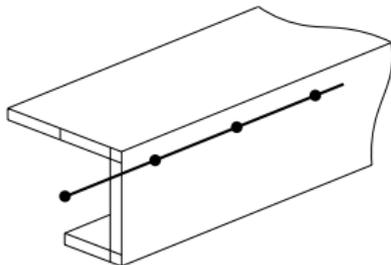
e.g. HL4 = 4th order expansion, 17×3 DOFs

- ~ generalized displacements unknowns, hierarchical kinematics
- ~ cross-section discretization

C-section beam



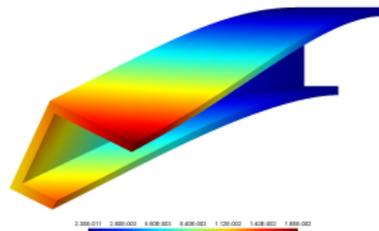
$$L = 20 \text{ m}, \quad b_t = h = 1 \text{ m}, \\ b_b = 0.5 \text{ m}, \quad t = 0.1 \text{ m}$$



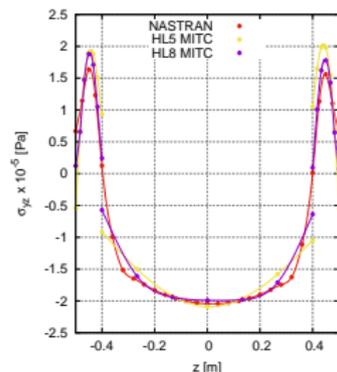
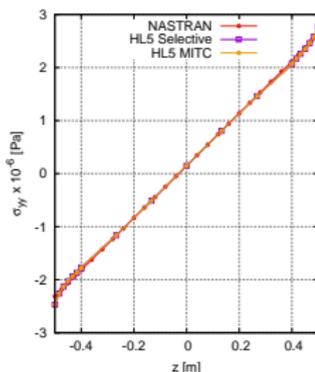
● Al: $E = 75 \text{ Gpa}$, $\nu = 0.33$

Displacement solutions

model	$-u_z \times 10^{-2} \text{ m}$	DOF
MSC Nastran	1.560	177000
HL1	1.479	1116
HL2	1.538	2604
HL3	1.543	4092
HL4	1.548	6045
HL5	1.551	8463
HL6	1.554	11346
HL7	1.555	14694
HL8	1.556	18507



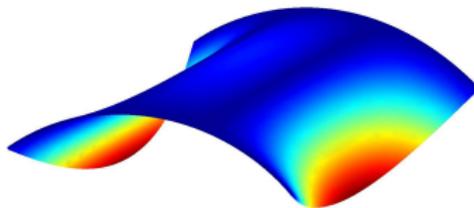
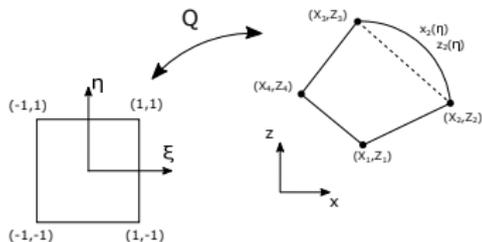
Stress solutions



Advanced capabilities

Cross-sectional mapping

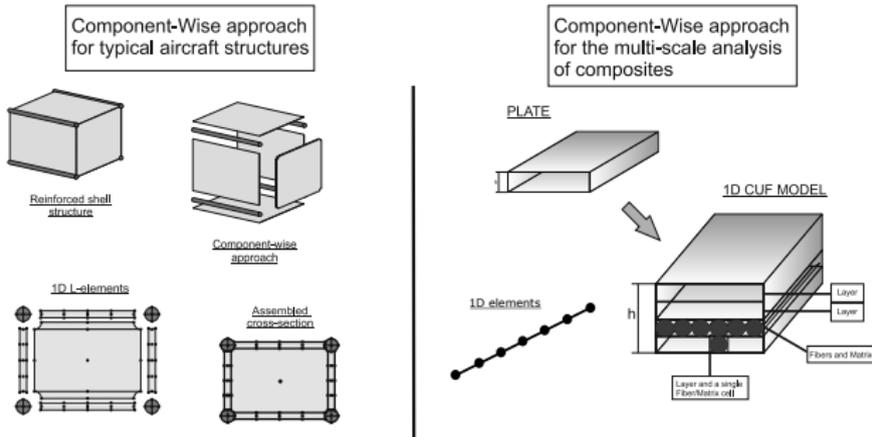
- HLE models: coarse section discretizations, large domains
- Blending function method → **non isoparametric expansions** (i.e. the mapping functions, Q , do not correspond with the expansion functions, F_τ)



Mixed Interpolation of Tensorial Components (MITC)

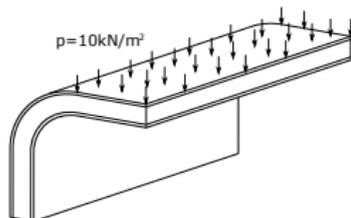
- High-order mixed interpolated beam elements
- Overcome shear locking phenomena
- Accurate description of transverse shear stresses

The Component-Wise Approach

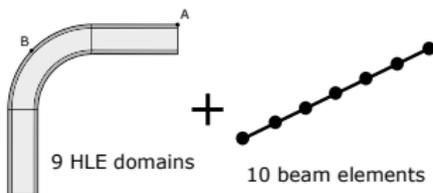


- Only 1D beam elements are used to model all components.
- Layerwise approach for laminates.
- No need of reference surfaces. This might be useful in a CAD-FEM interface scenario.
- No need of homogenization techniques.

Curved sandwich

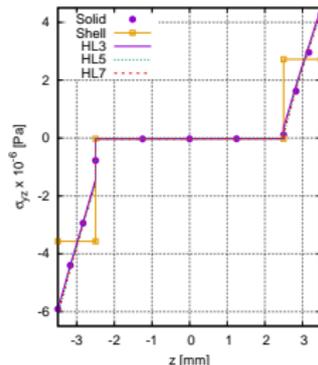
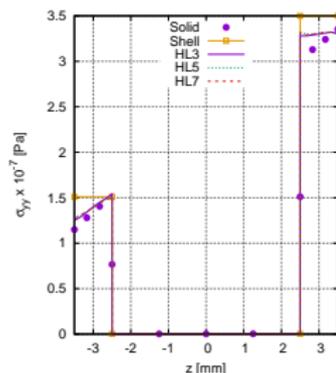


$L = 1 \text{ m}$, $h = w = 40 \text{ mm}$,
 $t_f = 1 \text{ mm}$, $t_c = 5 \text{ mm}$

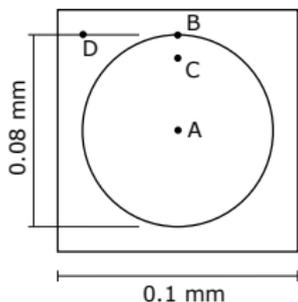
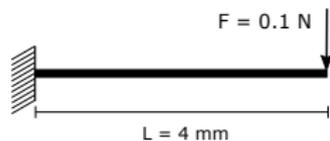


- Faces: $E = 75 \text{ GPa}$, $\nu = 0.33$
- Core: $E = 0.1063 \text{ GPa}$, $\nu = 0.32$

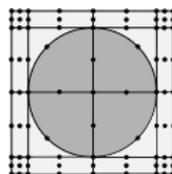
model	$u_z \times 10^2 \text{ m}$	$\sigma_{yy} \times 10^{-7} \text{ Pa}$	$\sigma_{yz} \times 10^{-6} \text{ Pa}$	DOFs
	Point A	point B	Point B	
MSC Nastran solutions				
Solid	-3.715	3.353	4.324	166617
Shell	-3.900	3.496	2.721	71000
HLE model solutions				
HL2	-3.032	3.421	-6.551	3720
HL3	-3.711	3.307	4.484	952
HL4	-3.712	3.293	4.466	9021
HL5	-3.712	3.288	4.276	12927
HL6	-3.712	3.290	4.256	17670
HL7	-3.712	3.298	4.308	23250
HL8	-3.712	3.297	4.321	29667



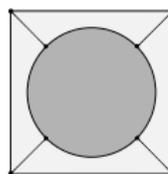
Fiber-matrix cell



	E [GPa]	ν
Fiber	202.038	0.2128
Matrix	3.252	0.355



LE model



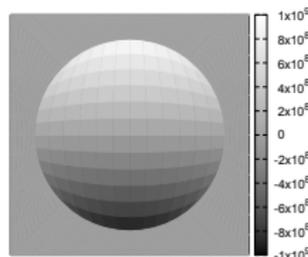
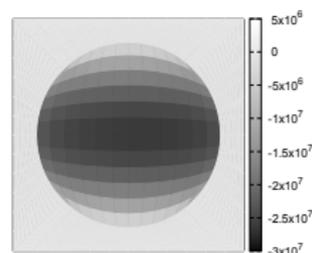
HLE mapped model

+

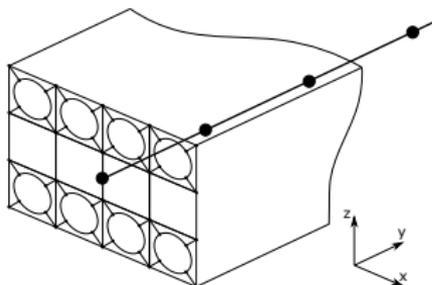


1D elements

model	$u_z \times 10^2$ [mm] Point A, $y = L$	$\sigma_{yy} \times 10^{-8}$ [Pa] Point B, $y = L/2$	$\sigma_{yy} \times 10^{-8}$ [Pa] Point C, $y = L/2$	$\sigma_{yz} \times 10^{-8}$ [Pa] Point D, $y = L/2$	DOFs
MSC Nastran	-7.818	9.492	7.094	-2.383	268215
TE (N=5)	-7.795	9.327	7.090	-2.375	7623
LE (12L9+8L6)	-7.933	9.450	7.046	-2.500	7533
HLE (5HL5)	-7.773	9.383	7.070	-2.746	6603

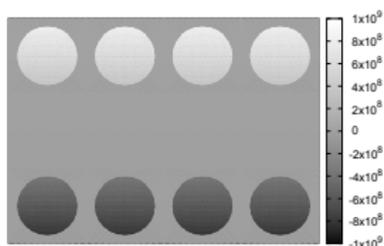
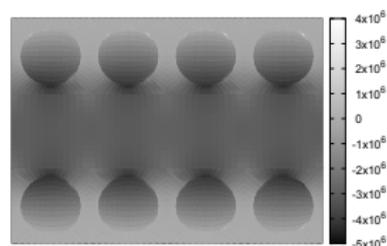
 σ_{yy}  σ_{yz}

Cross-ply



- Cross-ply $[0^\circ, 90^\circ, 0^\circ]$
- Loadcase: pure bending
- DOFs ~ 40000

Component	E_L	E_T	E_z	G_{LT}	G_{Lz}	G_{Tz}	ν_{LT}	ν_{Lz}	ν_{Tz}
Fiber (ortho)	202.038	12.134	12.134	8.358	8.358	47.756	0.2128	0.2128	0.2704
Layer (ortho)	103.173	5.145	5.145	2.107	2.107	2.353	0.2835	0.2835	0.3124
Matrix (iso)		3.252			1.200			0.355	


 σ_{yy}

 σ_{yz}


Main Conclusions and Perspectives

- 1 Any theory of structure can be devised in the framework of CUF, making it ideal for assessing other formulations.
- 2 The accuracy is mainly controlled by the polynomial order and refinement of the expansions.
- 3 The Component-Wise approach is able to analyze multi-component structures by means of a unique structural formulation.
- 4 Reduction of DOFs by one order of magnitude, or more, in comparison with commercial solid models for the same level of accuracy.

Ongoing developments and future extensions

- Composite structures: efficient multiscale analysis of fiber-reinforced components (FULLCOMP).
- Extension of CUF to non-linear problems: progressive damage, plasticity, etc.
- Introduction of curved beams: extension of CUF models to a new range of structural geometries.

Thank you for the kind attention,
any questions?



