

## REFINED 1D FINITE ELEMENTS FOR THIN-WALLED CURVED STRUCTURES

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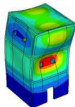
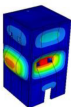
**Keywords:** Beams, Finite Elements, Refined models, Composites

**Abstract.** *This paper presents numerical results on the static and dynamic analysis of thin-walled, composite structures. The results are obtained via 1D finite elements based on refined beam theories. The Carrera Unified Formulation (CUF) is employed to build the refined theories. In the CUF framework, structural models can be obtained using expansions of the unknown variables along the cross-section of the beam. Any expansion type can be employed; for instance, polynomial, exponential, harmonic, etc.. Moreover, the order of the expansion can be set as an input, and chosen via a convergence analysis. Such features stem from the use of a few fundamental nuclei to obtain the governing equations and the finite element matrices. The formal expressions of the nuclei are independent of the order and the type of the expansion. 1D CUF models can provide 3D-like accuracies with low computational cost. Moreover, non-classical effects, such as warping, can be dealt with straightforwardly. This paper shows the latest extension of 1D CUF models. Legendre polynomials are employed as expansion functions of the displacement variables. The use of Legendre polynomials allows the parametrization of the cross-section geometry to tackle complex geometries, such as curved boundaries. The Principle of Virtual Displacements (PVD) is employed to obtain the finite element matrices. Various structural configurations are considered, including composite, thin-walled, curved structures. The results are compared with those from literature and 3D finite element models.*



# MUL2 - Our Research Group

## MUL<sub>ti</sub>layered structures



Thermal Protection



Composites

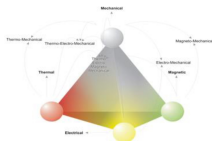


Sandwich



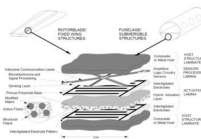
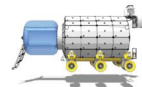
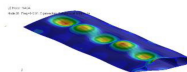
Smart Structures

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## MUL<sub>ti</sub>field interaction



## Marie Curie Project on Composites



[www.fullcomp.net](http://www.fullcomp.net)

# The FULLCOMP project

- FULLY integrated analysis, design, manufacturing and health-monitoring of COMPOSITE structures
- The FULLCOMP project is funded by the European Commission under a Marie Skłodowska-Curie Innovative Training Networks grant for European Training Networks (ETN).
- The FULLCOMP partners are:
  - 1 Politecnico di Torino (Italy) - Coordinator
  - 2 University of Bristol (UK)
  - 3 Ecole Nationale Supérieure d'arts et Métiers (Bordeaux, France)
  - 4 Leibniz Universität Hannover (Germany)
  - 5 University of Porto (Portugal)
  - 6 University of Washington (USA)
  - 7 RMIT (Australia)
  - 8 Luxembourg Institute of Technology
  - 9 Elan-Ausy, Hamburg, (Germany)
- FULLCOMP has recruited 12 PhD students who will work in an international framework to develop integrated analysis tools to improve the design of composite structures.
- The full spectrum of the design of composite structures will be dealt with, such as manufacturing, health-monitoring, failure, modeling, multiscale approaches, testing, prognosis, and prognostic. The FULLCOMP research activity is aimed at many engineering fields, e.g. aeronautics, automotive, mechanical, wind energy, and space.
- [www.fullcomp.net](http://www.fullcomp.net)



# Overview

- 1 Description of the Carrera Unified Formulation for refined models (CUF).
- 2 Main 1D CUF capabilities overview: 1D Taylor- and Lagrange-based models, and the novel Hierarchic Legendre-based model.
- 3 Implementation of mapping techniques to obtain geometrically-exact models.
- 4 Introduction to the Component-Wise approach (CW) on composite structures.
- 5 Numerical examples dealing with different applications.

# Brief Overview of Beam Refinement Methods and Contributors

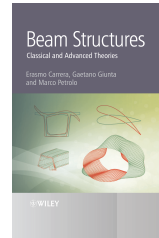
- 1 Shear correction factors (Timoshenko, Sokolnikoff, Cowper, Gruttmann, etc.).
- 2 Warping functions and Saint-Venant solutions (El Fatmi, Ladéveze, etc.).
- 3 Variational asymptotic method (Berdichevsky, Hodges, Yu, etc.).
- 4 Generalized beam theory (Schardt, Camotim, Silvestre, etc.).
- 5 Higher-order models (Washizu, Reddy, Kapania, Carrera, etc.).

# 1D Advanced Structural Models

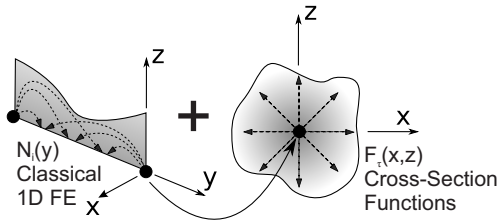
## Actual Wing



## Our Model



## 1D Carrera Unified Formulation, CUF - FEM Version



# Fundamental Nucleus Equations by CUF

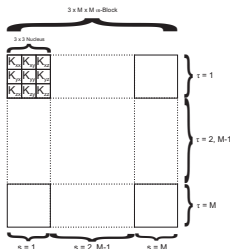
## Finite Element Formulation (FEM)

$$\mathbf{u} = F_{\tau}(x, z) N_i(y) \mathbf{u}_{\tau i} \quad (1D)$$

$$\delta L_{int} = \delta \mathbf{q}_{\tau i}^T \mathbf{K}^{ijrs} \mathbf{q}_{sj}$$

$$\delta L_{ext} = \mathbf{P} \delta \mathbf{u}^T$$

- $i, j$  - Shape function indexes (depend on the FE discretization).
- $\tau, s$  - Expansion function indexes (depend on the model order).



## Assembly Technique

### Fundamental Nucleus

$$K_{xx}^{ijrs} = \tilde{C}_{22} \int_{\Omega} F_{\tau,x} F_{s,x} d\Omega \int_I N_i N_j dy + \dots$$

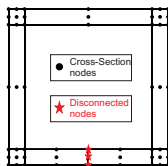
# Taylor Expansions, TE

$$\begin{array}{lll}
 u_x = & u_{x_1} & + x u_{x_2} + z u_{x_3} & + x^2 u_{x_4} + xz u_{x_5} + z^2 u_{x_6} + \dots \\
 u_y = & u_{y_1} & + x u_{y_2} + z u_{y_3} & + x^2 u_{y_4} + xz u_{y_5} + z^2 u_{y_6} + \dots \\
 u_z = & u_{z_1} & + x u_{z_2} + z u_{z_3} & + x^2 u_{z_4} + xz u_{z_5} + z^2 u_{z_6} + \dots
 \end{array}$$

$$\begin{array}{lll}
 \underbrace{N = 0}_{\tau = 1} & \underbrace{N = 1}_{\tau = 2, \tau = 3} & \underbrace{N = 2}_{\tau = 4, \tau = 5, \tau = 6} \\
 3 \text{ DOFs} & 9 \text{ DOFs} & 18 \text{ DOFs}
 \end{array}$$

Classical models, such as [Timoshenko](#), can be obtained as particular cases of the linear models.

# Lagrange Expansions, LE



$$\begin{aligned}
 u_x(x, y, z) &= F_1(x, z) u_{x_1}(y) + \dots + F_\tau(x, z) u_{x_\tau}(y) + \dots + F_9(x, z) u_{x_9}(y) \\
 u_y(x, y, z) &= F_1(x, z) u_{y_1}(y) + \dots + F_\tau(x, z) u_{y_\tau}(y) + \dots + F_9(x, z) u_{y_9}(y) \\
 u_z(x, y, z) &= F_1(x, z) u_{z_1}(y) + \dots + F_\tau(x, z) u_{z_\tau}(y) + \dots + F_9(x, z) u_{z_9}(y)
 \end{aligned}$$

## L9 polynomials - Isoparametric

$$F_1 = \frac{1}{4}(r^2 + r r_\tau)(s^2 + s s_\tau)$$

## Cross-section domains

# Hierarchic Legendre Expansions

## Vertex polynomials

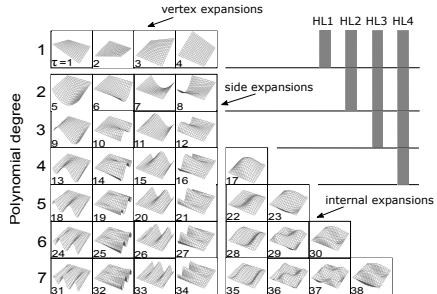
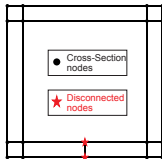
$$F_{\tau} = \frac{1}{4}(1 - r_{\tau}r)(1 - s_{\tau}s)$$

## Side polynomials

$$F_{\tau} = \frac{1}{2}(1 - s)\phi_p(r)$$

## Internal polynomials

$$F_{\tau} = \phi_{p_r}(r)\phi_{p_s}(s) \quad p_r + p_s = p$$

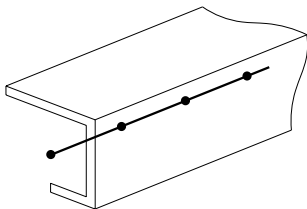
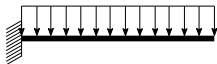


e.g. HL4 = 4<sup>th</sup> order expansion, 17×3 DOFs

- ~ TE - generalized displacements unknowns, **hierarchical kinematics**
- ~ LE - cross-section discretization

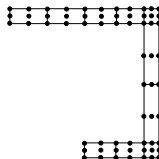
# C-section beam

$$p = 1000 \text{ N/m}^2$$

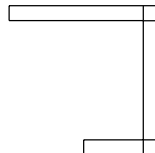


● Al:  $E = 75 \text{ GPa}$ ,  $\nu = 0.33$

LE modeling

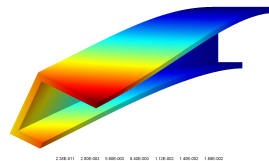


HLE modeling



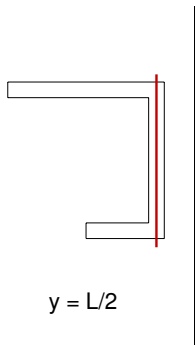
Displacement solutions

model	$-u_z \times 10^2 \text{ m}$	DOF
MSC Nastran	1.560	177000
L9	1.545	5301
HL1	1.479	1116
HL2	1.538	2604
HL3	1.543	4092
HL4	1.548	6045
HL5	1.551	8463
HL6	1.554	11346
HL7	1.555	14694
HL8	1.556	18507

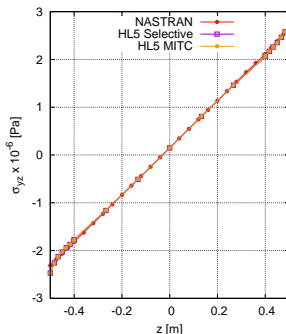


## Stress solutions

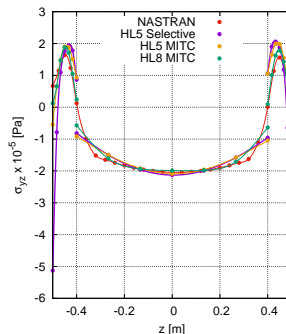
- Mixed interpolation of tensorial components (MITC)
- Accurate description of 3D-like states of stress



### Normal stress



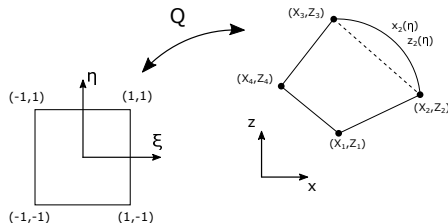
### Shear stress





# Cross-sectional mapping

- HLE models: coarse section discretizations, large domains
- Need of representing the exact geometry of curved domains
- Blending function method → **non isoparametric expansions** (i.e. the mapping functions,  $Q$ , do not correspond with the expansion functions,  $F_\tau$ )



$$x_2(\eta) = a_x + b_x\eta + c_x\eta^2 + d_x\eta^3$$

$$z_2(\eta) = a_z + b_z\eta + c_z\eta^2 + d_z\eta^3$$



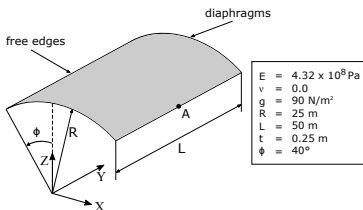
Mapping functions

$$Q_x = F_\tau(\xi, \eta)X_\tau + \underbrace{\left(x_2(\eta) - \left(\frac{1-\eta}{2}X_2 + \frac{1+\eta}{2}X_3\right)\right)}_{\text{blending function term}} \frac{1+\xi}{2}$$

$$Q_z = \underbrace{F_\tau(\xi, \eta)Z_\tau}_{\text{isoparametric term}} + \underbrace{\left(z_2(\eta) - \left(\frac{1-\eta}{2}Z_2 + \frac{1+\eta}{2}Z_3\right)\right)}_{\text{blending function term}} \frac{1+\xi}{2}$$

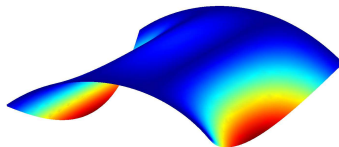
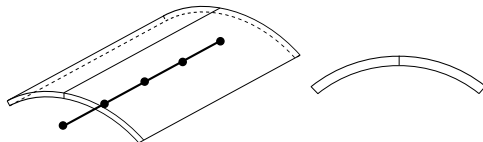
# Shell-like structures

## ● Benchmark test: Scordelis-Lo roof



[1] A.C. Scordelis and K.S. Lo,  
Computer Analysis of Cylindrical Shells  
Journal of the American Concrete Institute,  
61(5), pp. 561-593, 1964.

### HLE modelling



Scaled deformation

● Scordelis-Lo reference solution:  $u_{ref} = -0.3086$  m [1]

$$u^* = u / u_{ref}$$

2D and 3D elements						1D HL5 model			
Mesh	4x4	6x6	8x8	10x10	13x13	Mesh	2	4	10
QUAD4 [2]	1.029	0.998	0.988	0.984	-	B2	0.879	0.987	0.996
HEXA8 [2]	1.007	0.992	0.985	-	-	B3	1.010	0.998	0.997
Koiter [3]	0.957	-	-	0.977	0.980	B4	0.995	0.997	0.997
Naghdi [3]	0.957	-	-	0.978	0.982	B4(MITC)	0.995	0.997	0.997



[2] R.H. Macneal and R.L.

Harder,

A proposed standard set of problems to test finite element accuracy

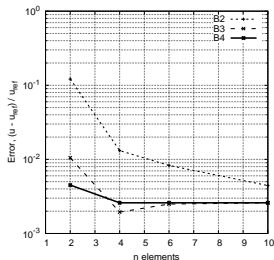
Finite Elements in Analysis and Design, 1(1), pp. 3-20, 1985.



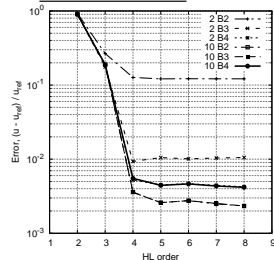
[3] C. Chinosi, L. Della

Croce, and T. Scapolla, Hierarchic finite elements for thin Naghdi shell model International Journal of Solids and Structures, 35(16), pp. 1863-180, 1998.

# of elements

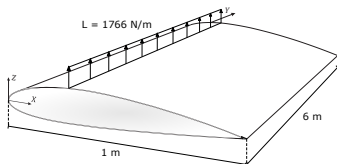


Expansion order



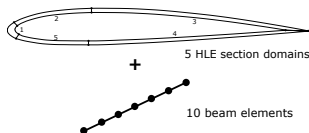
# Wing structure

## Airfoil NACA 2415

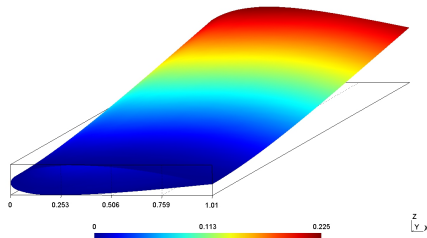


● Aluminum:  $E = 75 \text{ GPa}$ ,  $\nu = 0.33$

## HLE mapped model

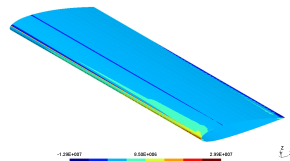
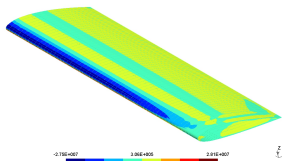
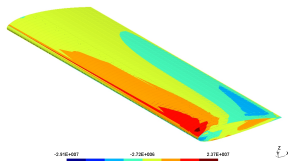
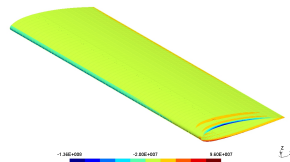
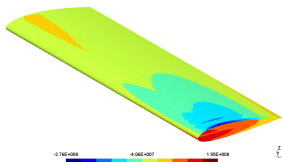
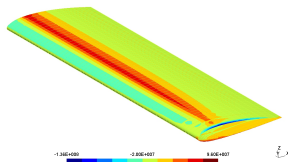


## Shell-like displacement solutions

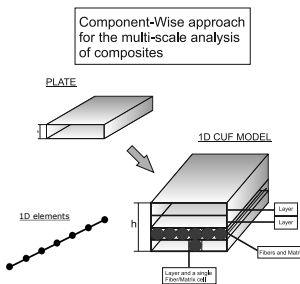
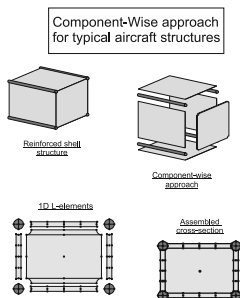


Model	$u_z$ m	$\sigma_{yy}$ MPa	$\sigma_{yz}$ MPa	DOFs
	Load point, $y = L$	Load point, $y = L/2$	Leading edge, $y = L/2$ m	
MSC Nastran	0.225	32.301	8.761	395280
HL8	0.223	32.000	8.839	17670

## 3D-like stress solutions

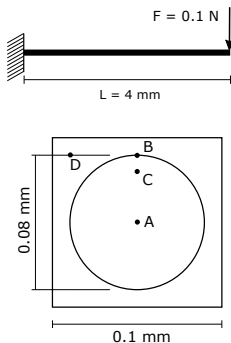


# The Component-Wise Approach



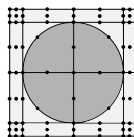
- Only 1D beam elements are used to model all components.
- No need of reference surfaces. This might be useful in a CAD-FEM interface scenario.
- No need of homogenization techniques.

# Fiber-matrix cell

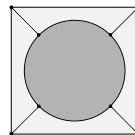


	E [GPa]	$\nu$
Fiber	202.038	0.2128
Matrix	3.252	0.355

## Component-Wise models



LE model



HLE mapped model

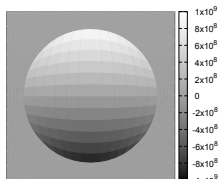
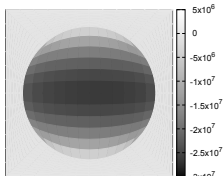
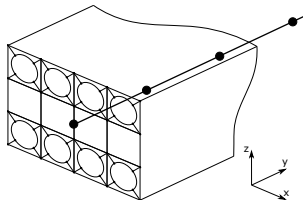
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1D elements

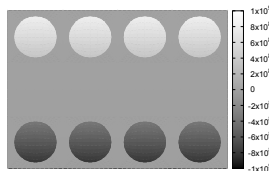
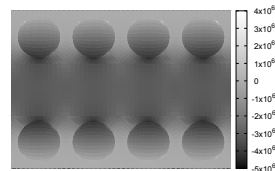
model	$u_x \times 10^2$ [mm] Point A, $y = L$	$\sigma_{yy} \times 10^{-8}$ [Pa] Point B, $y = L/2$	$\sigma_{yy} \times 10^{-8}$ [Pa] Point C, $y = L/2$	$\sigma_{yz} \times 10^{-8}$ [Pa] Point D, $y = L/2$	DOFs
MSC Nastran	-7.818	9.492	7.094	-2.383	268215
TE (N=5)	-7.795	9.327	7.090	-2.375	7623
LE (12L9+8L6)	-7.933	9.450	7.046	-2.500	7533
HLE (5HL5)	-7.773	9.383	7.070	-2.746	6603

## Stress field


 $\sigma_{yy}$ 

 $\sigma_{yz}$ 


- Cross-ply  $[0^\circ, 90^\circ, 0^\circ]$
- Loadcase: pure bending
- DOFs  $\sim 40000$

Component	$E_L$	$E_T$	$E_z$	$G_{LT}$	$G_{Lz}$	$G_{Tz}$	$\nu_{LT}$	$\nu_{Lz}$	$\nu_{Tz}$
Fiber (ortho)	202.038	12.134	12.134	8.358	8.358	47.756	0.2128	0.2128	0.2704
Layer (ortho)	103.173	5.145	5.145	2.107	2.107	2.353	0.2835	0.2835	0.3124
Matrix (iso)		3.252			1.200			0.355	


 $\sigma_{yy}$ 

 $\sigma_{yz}$



# Main Conclusions and Perspectives

- 1 1D CUF structural models are powerful tools to analyze structures for different applications, including aerospace and civil structures and composites.
- 2 Geometrically-exact models are obtained with HLE mapped models in an efficient manner.
- 3 The accuracy is controlled by the polynomial order of the expansions, no need of iterative refinements of the cross-section domain.
- 4 The Component-Wise approach is able to analyze complex structures by means of a unique structural formulation.
- 5 CW 1D Models require at least 10 times less DOFs than Solid models.

## Future extensions

- Modal analysis, dynamic analysis, buckling, non-linear problems.
- Composite structures: efficient multiscale analysis of fiber-reinforced components.
- Curved beams: study of any-shape structure through 1D models.

Thank you for the kind attention,  
any questions?



