

MICROMECHANICS MODELING OF UNIT CELLS USING CUF BEAM MODELS AND THE MECHANICS OF STRUCTURE GENOME

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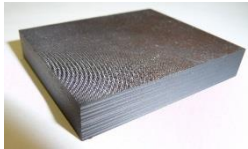
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September 20th, 2017, TU Eindhoven, Netherlands

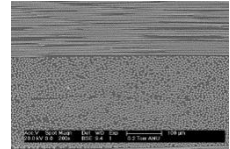
Goals



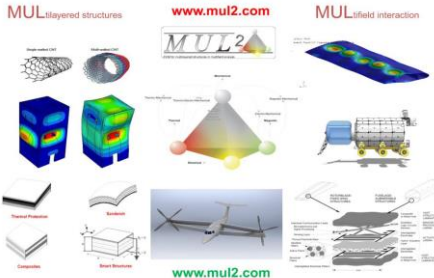
Macro-scale



Meso-scale



Micro-scale



Mechanics of Structure Genome
+
Carrera Unified Formulation

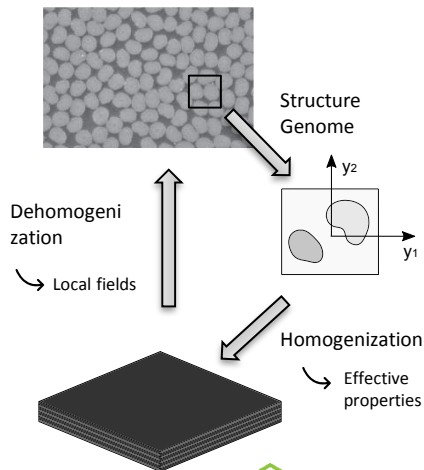


- ☐ 1D micromechanics
- ☐ Increase efficiency
- ☐ Reduce modeling time

Overview

- ☐ Mechanics of Structure Genome (MSG) for micromechanical analysis
- ☐ Carrera Unified Formulation (CUF) : higher-order beam models for unit cells
- ☐ Hierarchical Legendre Expansions (HLE) as theory of structure
- ☐ Numerical results: fiber reinforced and particle reinforced composites
- ☐ Conclusions and perspectives

Micromechanics modeling: MSG



Principle of Minimum Information Loss

- Express the kinematics as a sum of the global displacements and the local fluctuations

$$u_i = \bar{u}_i + \delta\chi_i$$

$$\varepsilon_{ij} = \bar{\varepsilon}_{ij} + \chi_{(i,j)}$$

- Express the energy of the original model as

$$U(\varepsilon_{ij}) = U(\bar{\varepsilon}_{ij}, \chi_{(i,j)})$$

- Using the Variational Asymptotic Method, minimize the energy to solve the fluctuations

$$\min_{\chi} U(\bar{\varepsilon}_{ij}, \chi_{(i,j)}) - U(\bar{\varepsilon}_{ij})$$

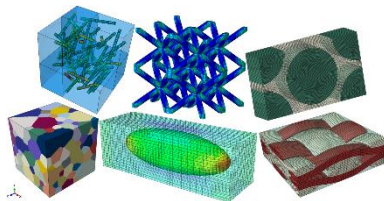
No ad-hoc assumptions

One load step

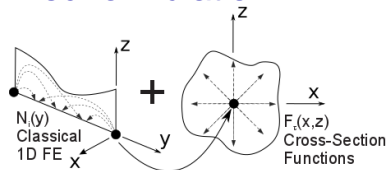
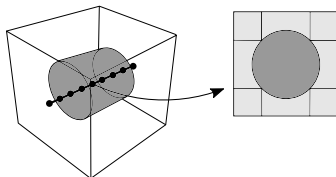
Different local solutions for a single run of the code

[1] Yu W. A unified theory for constitutive modeling of composites. J Mech Mater Struct (2016);11(4): pp 379–411.

Beam kinematics through the unified formulation



*Abaqus Plugin



Displacement field:

$$\mathbf{u}(x, y, z) = N_i(y) F_t(x, z) \mathbf{u}_{ti}$$

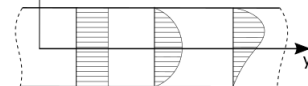
Fundamental Nucleus:

$$K_{xx}^{ijrs} = \tilde{C}_{22} \int_{\Omega} F_{t,x} F_{s,x} d\Omega \int_I N_i N_j dy + \dots$$

□ transverse stresses

$$F_t =$$

$$\begin{matrix} z, x \\ o(x, z) & o(x^3, z^3) & o(x^5, z^5) \end{matrix}$$



Hierarchical Legendre Expansions, HLE

Vertex polynomials

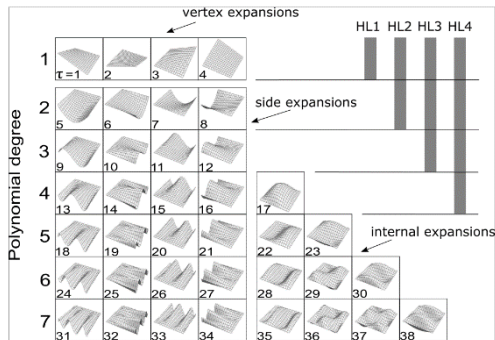
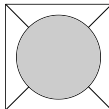
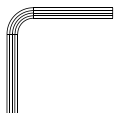
$$F_{\tau} = \frac{1}{4}(1 - r_{\tau}r)(1 - s_{\tau}s)$$

Side polynomials

$$F_{\tau} = \frac{1}{2}(1 - s)\varphi_p(r)$$

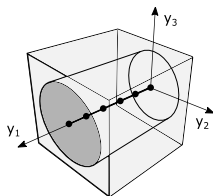
Internal polynomials

$$F_{\tau} = \varphi_{pr}(r)\varphi_{ps}(s) \quad p_r + p_s = p$$



- ☐ Hierarchical kinematics
- ☐ Non-local distribution of unknowns
- ☐ Geometrically exact curved sections:
blending function method

Numerical results: square pack



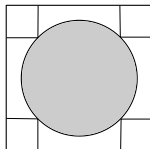
Graphite-epoxy composite
- Volume fraction = 0.6

Component	E_1	E_2	G_{12}	G_{23}	ν_{12}	ν_{23}
Fiber	235	15	28	5.6	0.2	0.25
Matrix	4.8		1.8		0.34	

*E,G in GPa

Model:

- ☐ 1 beam element
- ☐ 9 HLE expansions
- ☐ 957 DOFs, 0.234 s
- ☐ Exact geometry
- ☐ P-refinement



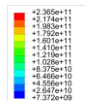
Homogenization

Model	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	G_{23} [GPa]	ν_{12}	ν_{23}
References						
FEM [5]	142.6	9.60	6.00	3.10	0.25	0.35
MOC [2]	143	9.6	5.47	3.08	0.25	0.35
GMC [21]	143.0	9.47	5.68	3.03	0.253	0.358
HFGMC [22]	142.9	9.61	6.09	3.10	0.252	0.350
ECM [23]	143	9.6	5.85	3.07	0.25	0.35
SwiftComp	142.9	9.61	6.10	3.12	0.252	0.350
CUF-MSG						
HL2	143.17	9.70	6.29	3.19	0.252	0.346
HL4	143.16	9.64	6.09	3.12	0.252	0.349
HL6	143.16	9.62	6.09	3.12	0.252	0.350
HL8	143.16	9.62	6.08	3.12	0.252	0.350

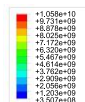
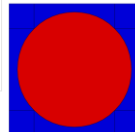
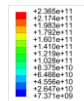
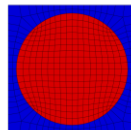
Dehomogenization

SwiftComp 2D

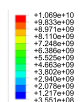
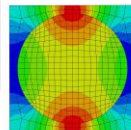
CUF



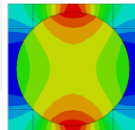
σ_{11}



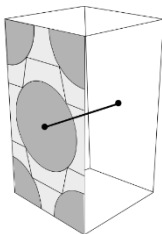
σ_{12}



σ_{12}



Numerical results: hexagonal pack



Carbon-epoxy composite
- Volume fraction = 0.6

Component	E_1	E_2	G_{12}	G_{23}	ν_{12}	ν_{23}
Fiber	276	19.5	70	5.74	0.28	0.7
Matrix	4.76		1.74		0.37	

* E, G in GPa

Model:

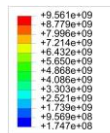
- ☐ 1 beam element
- ☐ 15 HLE expansions
- ☐ 1206 DOFs

Homogenization

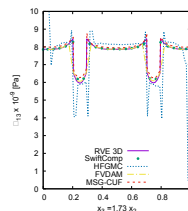
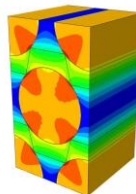
Model	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
Literature									
FVDAM	167.30	10.67	10.67	6.38	6.39	3.33	0.310	0.310	0.600
HFGMC	167.40	10.71	10.69	6.58	6.54	3.36	0.312	0.312	0.603
SwiftComp	167.33	10.67	10.67	6.38	6.39	3.33	0.312	0.312	0.600
FEA RVE	167.33	10.67	10.67	6.38	6.39	3.33	0.312	0.312	0.600
MSG-HLE									
HL7	167.65	10.68	10.68	6.40	6.61	3.34	0.312	0.312	0.600

*Micromechanics Simulation Challenge

Dehomogenization



σ_{13}

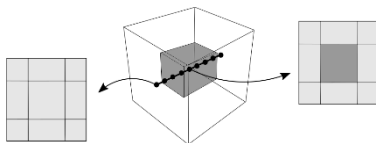


Computational time

Model	FVDAM	HFGMC	SwiftComp	MSG-HLE	FEA RVE
Homogenization	4	-	0.26	0.19	-
Localization	0.88	-	0.93	0.73	-
Total	4.88	1.151	1.19	0.92	42.00

[4] A.G. de Miguel A. Pagani, W.Yu and E. Carrera. A. Pagani, *Micromechanics of periodically heterogeneous materials using higher-order beam theories and the mechanics of structure genome*. Composite Structures (2017) 180: pp 484-496.

Numerical results: particle inclusion



Al₂O₃/Al composite
- Volume fraction = 0.125

Component	E	ν
Particle	350	0.3
Matrix	70	0.3

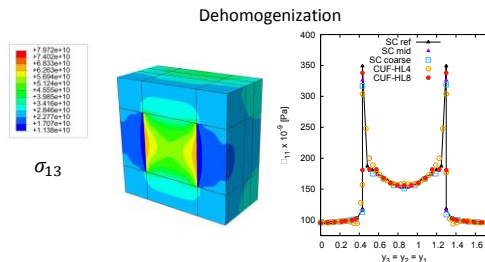
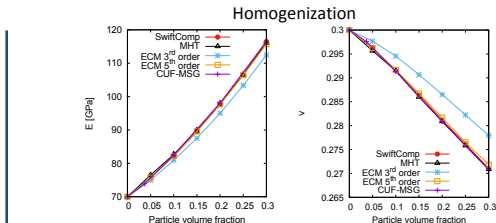
* E, G in GPa

Model:

- ☐ 6 cubic beam elements
- ☐ 9 HLE expansions
- ☐ 2160 - 17226 DOFs

Challenges:

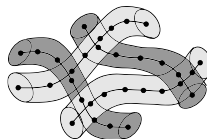
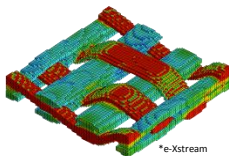
- Variation of phases in all directions
- Stress concentration in corners



Model	SC refined	SC mid	SC coarse	HL4	HL8
Homogenization	138.65	18.23	3.21	1.17	10.78
Dehomogenization	120.02	41.08	14.62	3.96	6.71
Total	258.67	59.31	17.82	5.13	17.49

Conclusions and perspectives

- ❑ Assessment of the model: MSG/CUF coupling can be a highly efficient tool for the micromechanics analysis of periodically heterogeneous materials
- ❑ The accuracy of the micromechanics analysis is controlled by the polynomial order of the expansions: no need of iterative refinements of the mesh
- ❑ Mapping of the exact geometry of the components through the bending function method
- ❑ Fibers and inclusions can be modelled by only a single expansion over the cross-section of the beam: great reduction of the complexity of the model with no loss of accuracy
- ❑ Multiscale analysis: high-order beams for macro, meso and micro scales
- ❑ Future developments: more complex SG, woven fabrics, multifield analysis (electric, thermal, magnetic), damage.





FULLCOMP project

- ☐ FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures
- ☐ Funded by the European Commission under a Marie Skłodowska –Curie Innovative Training Networks grant: 12 PhD students

- ☐ Partners:

- i. Politecnico di Torino (Italy)
- ii. University of Bristol (UK)
- iii. ENSMA Bordeaux (France)
- iv. Leibniz Universitaet Hannover (Germany)
- v. LIST (Luxemburg)
- vi. ELAN-AUSY GmbH (Germany)
- vii. Universidade do Porto (Portugal)
- viii. University of Washington (USA)
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Thank you for the attention,
any questions?



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