Micromechanics modeling of unit cells using CUF beam models and the mechanics of structure genome

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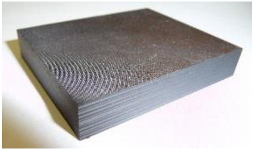
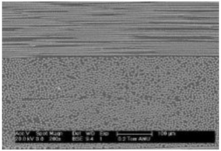
Goals

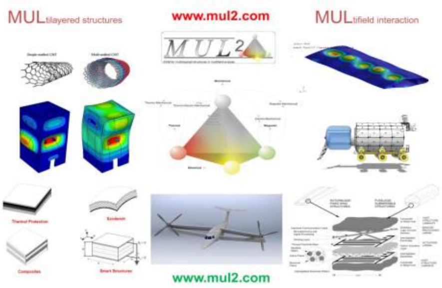
Macro-scale



Meso-scale



Micro-scale



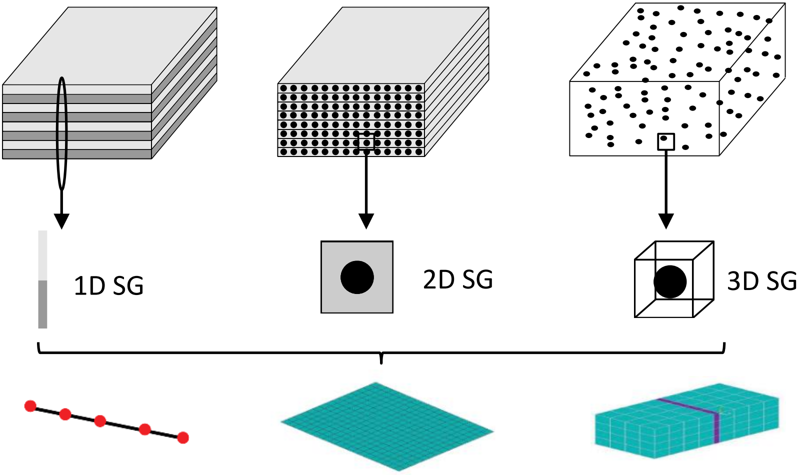
Mechanics of Structure Genome
+
Carrera Unified Formulation

- ❑ 1D micromechanics
- ❑ Increase efficiency
- ❑ Reduce modeling time

Overview

- Governing equations: Mechanics of Structure Genome (MSG) for micromechanical analysis
- Modelling procedure: Carrera Unified Formulation (CUF) and higher-order beam model
- Beam modeling of microstructures
- Numerical results: fiber reinforced and particle reinforced composites
- Conclusions and perspectives

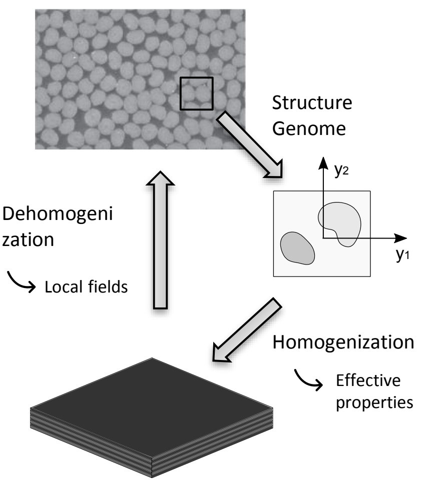
Mechanics of Structure Genome



[1] Yu W. A unified theory for constitutive modeling of composites. J Mech Mater Struct (2016);11(4): pp 379–411.

SwiftComp™ MICROMECHANICS

MSG for micromechanical analysis



Principle of Minimum Information Loss

- ❑ Express the kinematics as a sum of the global displacements and the local fluctuations

$$u_i = \bar{u}_i + \delta\chi_i$$

$$\varepsilon_{ij} = \bar{\varepsilon}_{ij} + \chi_{(i,j)}$$

- ❑ Express the energy of the original model as

$$U(\varepsilon_{ij}) = U(\bar{\varepsilon}_{ij}, \chi_{(i,j)})$$

- ❑ Using the Variational Asymptotic Method, minimize the energy to solve the fluctuations

$$\min_{\chi} \; U\left(\bar{\varepsilon}_{ij},\, \chi_{(i,j)}\right) - U(\bar{\varepsilon}_{ij})$$

- No ad-hoc assumptions
- Straightforward numerical implementation
- Complete set of properties with a single run
- Different sets of local solutions

Beam kinematics through the unified formulation

Euler-Bernoulli

$$u_x(x,y,z) = u_{x_1}(y)$$
$$u_y(x,y,z) = u_{y_1}(y) - x\, u_{x_{1,y}}(y) - \mathrm{z}\, u_{z_{1,y}}(y)$$
$$u_z(x,y,z) = u_{z_1}(y)$$

Timoshenko

$$u_x(x,y,z) = u_{x_1}(y)$$
$$u_y(x,y,z) = u_{y_1}(y) - x\, u_{y_2}(y) - \mathrm{z} u_{y_3}(y)$$
$$u_z(x,y,z) = u_{z_1}(y)$$

Saint Venant

$$u_x(x,y,z) = u_{x_1}(y) - z u_{x_2}(y)$$
$$u_y(x,y,z) = u_{y_1}(y) - x\, u_{y_2}(y) - z u_{y_3}(y) + \psi(x,z) u_{y_3}(y)$$
$$u_z(x,y,z) = u_{z_1}(y) + x\, u_{z_2}(y)$$

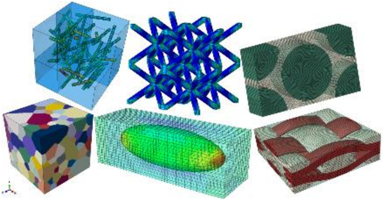
K. Washizu: " *For a complete removal of the inconsistency and an improvement of the accuracy of the beam theory*" -> enrich beam kinematics with higher-order terms

Carrera Unified Formulation

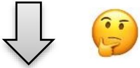
$$u_x(x,y,z) = F_1(x,z)\, u_{x_1}(y) + F_2(x,z)\, u_{x_2}(y) + F_3(x,z)\, u_{x_3}(y) + \ldots + F_M(x,z)\, u_{x_M}(y)$$
$$u_y(x,y,z) = F_1(x,z)\, u_{y_1}(y) + F_2(x,z)\, u_{y_2}(y) + F_3(x,z)\, u_{y_3}(y) + \ldots + F_M(x,z)\, u_{y_M}(y)$$
$$u_z(x,y,z) = F_1(x,z)\, u_{z_1}(y) + F_2(x,z)\, u_{z_2}(y) + F_3(x,z)\, u_{z_3}(y) + \ldots + F_M(x,z)\, u_{z_M}(y)$$

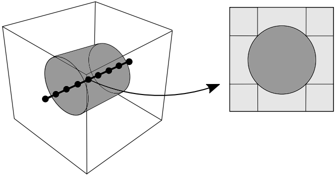
$$\boxed{\mathbf{u}(x,y,x) = F_\tau(x,z)\, \mathbf{u}_\tau(y) \qquad \tau = 1, \ldots, M}$$

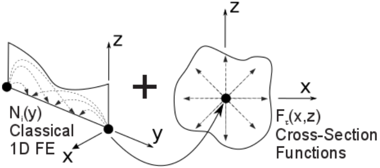
Refined beam elements for unit cells



*Abaqus Plugin







Displacement field:

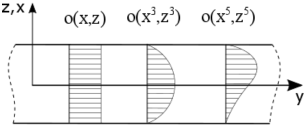
$$\boldsymbol{\chi}(x,y,z) = N_i(y)\, F_\tau(x,z)\, \boldsymbol{\chi}_{\tau i}$$

Fundamental Nucleus:

$$K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_\Omega F_{\tau,x} F_{s,x}\, d\Omega \int_l N_i N_j dy + \ldots$$

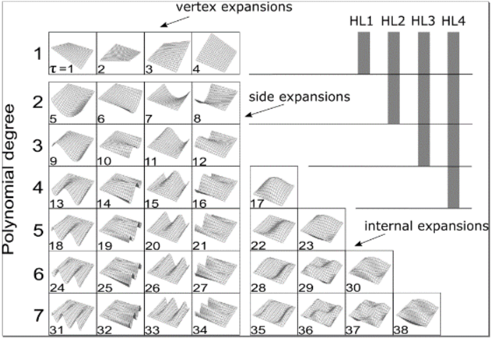
- [ ] transverse stresses

$F_\tau =$



[2] Carrera E., Cinefra M., Petrolo M. and Zappino E. Finite element analysis of structures through unified formulation. John Wiley & Sons; 2014.

Hierarchical Legendre Expansions, HLE



<u>Vertex polynomials</u>

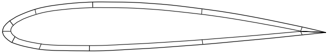
$$F_\tau = \frac{1}{4}(1 - r_\tau r)(1 - s_\tau s)$$

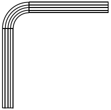
<u>Side polynomials</u>

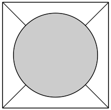
$$F_\tau = \frac{1}{2}(1 - s)\varphi_p(r)$$

<u>Internal polynomials</u>

$$F_\tau = \varphi_{p_r}(r)\varphi_{p_s}(s) \qquad p_r + p_s = p$$



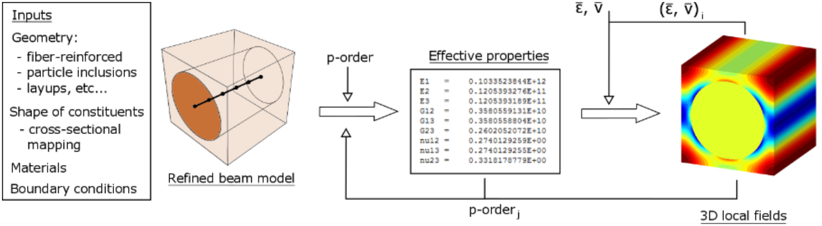




- ❑ Hierarchical kinematics
- ❑ Non-local distribution of unknowns
- ❑ Geometrically exact curved sections using the blending function method

[3] A. Pagani, A.G. de Miguel and E. Carrera. *Cross-sectional mapping for refined beam elements with applications to shell-like structures*. Computational Mechanics (2017) pp 1-18.

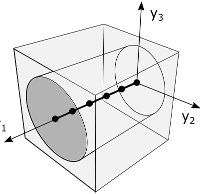
Code* description



1. Modeling of the section of the constituents - coarse domains
2. Discretization of the reference axis
3. Asignment of the properties for each constituent
4. Input the order of the expansion of the domains, *p*
5. Homogenization -> effective properties
6. Input the global solutions
7. Dehomogenization -> local fields

*Travel Scholarship and Code Competition at ASC2017

Numerical results: square pack



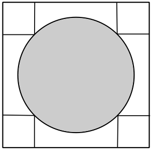
Graphite-epoxy composite
- Volume fraction = 0.6

Component	E_1	E_2	G_{12}	G_{23}	ν_{12}	ν_{23}
Fiber	235	15	28	5.6	0.2	0.25
Matrix	4.8		1.8		0.34	

*E,G in GPa

Model:

- ❑ 1 beam element
- ❑ 9 HLE expansions
- ❑ 957 DOFs, 0.234 s

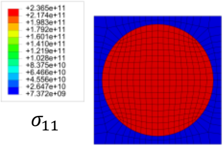


Homogenization

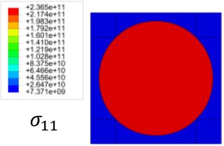
Model	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	G_{23} [GPa]	ν_{12}	ν_{23}
References						
FEM [5]	142.6	9.60	6.00	3.10	0.25	0.35
MOC [2]	143	9.6	5.47	3.08	0.25	0.35
GMC [21]	143.0	9.47	5.68	3.03	0.253	0.358
HFGMC [22]	142.9	9.61	6.09	3.10	0.252	0.350
ECM [23]	143	9.6	5.85	3.07	0.25	0.35
SwiftComp	142.9	9.61	6.10	3.12	0.252	0.350
CUF-MSG						
HL2	143.17	9.70	6.29	3.19	0.252	0.346
HL4	143.16	9.64	6.09	3.12	0.252	0.349
HL6	143.16	9.62	6.09	3.12	0.252	0.350
HL8	143.16	9.62	6.08	3.12	0.252	0.350

Dehomogenization

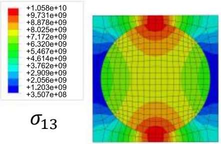
SwiftComp 2D *CUF*



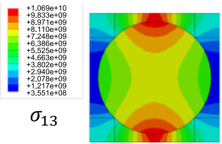
σ_{11}



σ_{11}

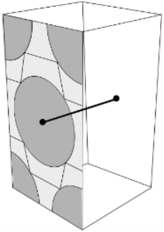


σ_{13}



σ_{13}

Numerical results: hexagonal pack



Carbon-epoxy composite

- Volume fraction = 0.6

Component	E_1	E_2	G_{12}	G_{23}	ν_{12}	ν_{23}
Fiber	276	19.5	70	5.74	0.28	0.7
Matrix	4.76		1.74		0.37	

*E,G in GPa

Model:

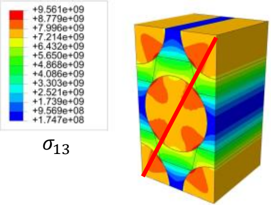
- ❑ 1 beam element
- ❑ 15 HLE expansions
- ❑ 1206 DOFs

Homogenization

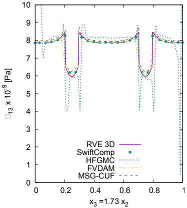
Model	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
Literature									
FVDAM	167.30	10.67	10.67	6.38	6.39	3.33	0.310	0.310	0.600
HFGMC	167.40	10.71	10.69	6.58	6.54	3.36	0.312	0.312	0.603
SwiftComp	167.33	10.67	10.67	6.38	6.39	3.33	0.312	0.312	0.600
FEA RVE	167.33	10.67	10.67	6.38	6.39	3.33	0.312	0.312	0.600
MSG-HLE									
HL7	167.65	10.68	10.68	6.40	6.61	3.34	0.312	0.312	0.600

*Micromechanics Simulation Challenge

Dehomogenization



σ_{13}

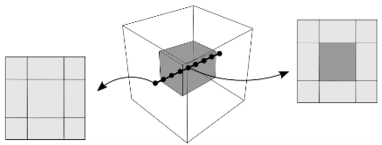


Computational time

Model	FVDAM	HFGMC	SwiftComp	MSG-HLE	FEA RVE
Homogenization	4	-	0.26	0.19	-
Localization	0.88	-	0.93	0.73	-
Total	4.88	1.151	1.19	0.92	42.00

[4] A.G. de Miguel A. Pagani, W.Yu and E. Carrera. A. Pagani, *Micromechanics of periodically heterogeneous materials using higher-order beam theories and the mechanics of structure genome*. Composite Structures (2017) 180: pp 484-496.

Numerical results: particle inclusion



Al2O3/Al composite

Component	E	ν
Particle	350	0.3
Matrix	70	0.3

*E,G in GPa

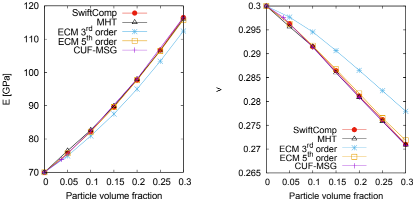
Model:

- ❑ 6 cubic beam elements
- ❑ 9 HLE expansions
- ❑ 2160 - 17226 DOFs

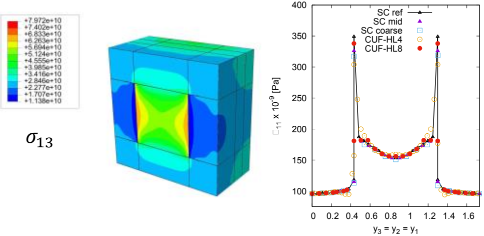
Challenges:

- ➢ Variation of phases in all directions
- ➢ Stress concentration in corners

Homogenization



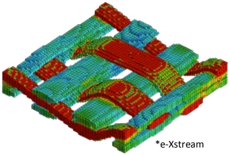
Dehomogenization

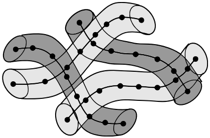


Model	SC refined	SC mid	SC coarse	HL4	HL8
Homogenization	138.65	18.23	3.21	1.17	10.78
Dehomogenization	120.02	41.08	14.62	3.96	6.71
Total	258.67	59.31	17.82	5.13	17.49

Conclusions and future work

- Assessment of the model: MSG/CUF coupling can be a highly efficient tool for the micromechanics analysis of periodically heterogeneous materials
- The accuracy of the micromechanic analysis is controlled by the polynomial order of the expansions: no need of iterative refinements of the mesh
- Mapping of the exact geometry of the components through the bending function method
- Fibers and inclusions can be modelled by only a single domain over the cross-section of the beam: great reduction of the complexity of the model with no loss of accuracy
- Multiscale analysis: high-order beams for macro, meso and micro scales
- Future developments: more complex SG, woven fabrics, multifield analysis (electric, thermal, magnetic), damage.





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Thank you for the attention, any questions?