Analysis of curved composite structures through refined 1D finite elements with aerospace applications

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ASME International Mechanical Engineering Congress and Exposition
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The FULLCOMP project

- FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures
- The FULLCOMP project is funded by the European Commission under a Marie Sklodowska-Curie Innovative Training Networks grant for European Training Networks (ETN).
- The FULLCOMP partners are:
  1. Politecnico di Torino (Italy) - Coordinator
  2. University of Bristol (UK)
  3. ENSMA (Bordeaux, France)
  4. Leibniz Universitaet Hannover (Germany)
  5. University of Porto (Portugal)
  6. University of Washington (USA)
  7. RMIT (Melbourne, Australia)
  8. Luxembourg Institute of Technology
  9. Elan-Ausy (Hamburg,Germany)

- 12 PhD students
- Full spectrum of design of composite structures: manufacturing, health-monitoring, failure, modeling, multiscale approaches, testing, prognosis and prognostic.
- www.fullcomp.net

Overview

1. From classical beam theories to the unified formulation
2. Non-local hierarchical theories of structure
3. Cross-sectional mapping
4. Component-wise analysis for composite structures
5. Conclusions and future developments

From classic to refined kinematics

Euler-Bernoulli beam theory

$$\begin{aligned} u_x(x,y,z) &= u_{x_1}(y) \\ u_y(x,y,z) &= u_{y_1}(y) - x\,\frac{\partial u_{x_1}(y)}{\partial y} - z\,\frac{\partial u_{z_1}(y)}{\partial y} \\ u_z(x,y,z) &= u_{z_1}(y) \end{aligned}$$

Timoshenko beam theory

$$\begin{aligned} u_x(x,y,z) &= u_{x_1}(y) \\ u_y(x,y,z) &= u_{y_1}(y) + x\,\phi_z(y) - z\,\phi_x(y) \\ u_z(x,y,z) &= u_{z_1}(y) \end{aligned}$$

Saint Venant beam theory

$$\begin{aligned} u_x(x,y,z) &= u_{x_1}(y) - z\,\theta(y) \\ u_y(x,y,z) &= u_{y_1}(y) + x\,\phi_z(y) - z\,\phi_x(y) + \psi(x,z)\,\frac{\partial\theta(y)}{\partial y} \\ u_z(x,y,z) &= u_{z_1}(y) + x\,\theta(y) \end{aligned}$$

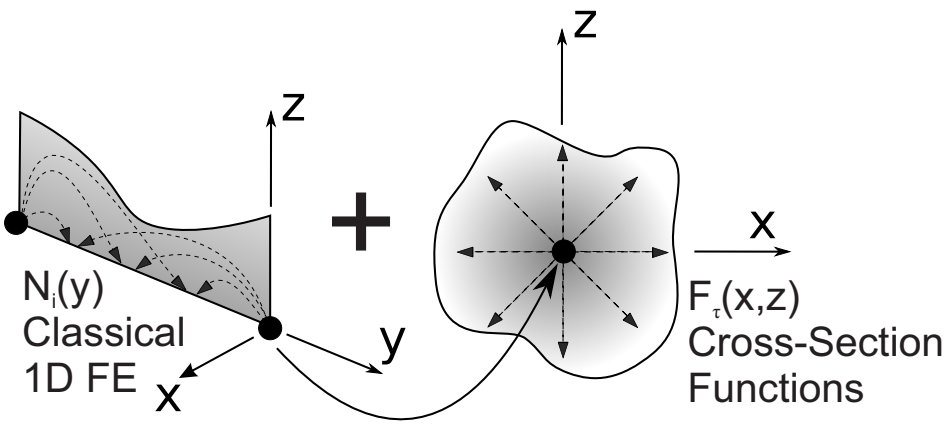
Washizu's statement:
"For a complete removal of the inconsistency and an improvement of the accuracy of the beam theory" -> enrich beam kinematics with higher-order terms

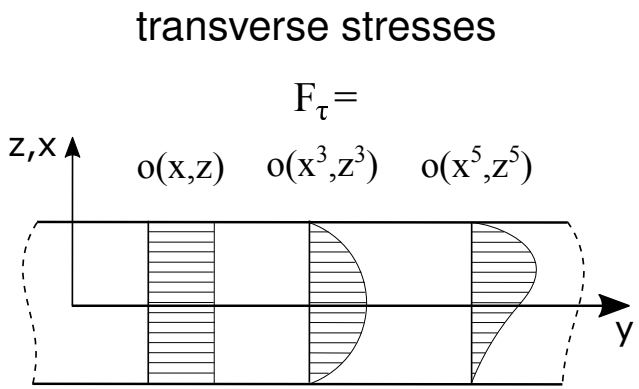
Beam kinematics through the unified formulation

$$\mathbf{u}(x,y,x) = F_\tau(x,z)\,\mathbf{u}_\tau(y)$$

$$u_x(x,y,z) = F_1(x,z)\,u_{x_1}(y) + F_2(x,z)\,u_{x_2}(y) + F_3(x,z)\,u_{x_3}(y) + ... + F_M(x,z)\,u_{x_M}(y)$$
$$u_y(x,y,z) = F_1(x,z)\,u_{y_1}(y) + F_2(x,z)\,u_{y_2}(y) + F_3(x,z)\,u_{y_3}(y) + ... + F_M(x,z)\,u_{y_M}(y)$$
$$u_z(x,y,z) = F_1(x,z)\,u_{z_1}(y) + F_2(x,z)\,u_{z_2}(y) + F_3(x,z)\,u_{z_3}(y) + ... + F_M(x,z)\,u_{z_M}(y)$$

$$\tau = 1, ..., M \ (\textit{number of expansion terms})$$





Classical models, such as Euler or Timoshenko, can be obtained as particular cases of the linear models.

CUF stiffness matrix

CUF displacement field

$$\mathbf{u} = F_\tau(x,z)N_i(y)\,\mathbf{u}_{\tau i} \quad (1D)$$

PVD - Static

$$\delta L_{int} = \delta \mathbf{u}_{\tau i}^T \mathbf{K}^{ij\tau s} \mathbf{u}_{sj}$$

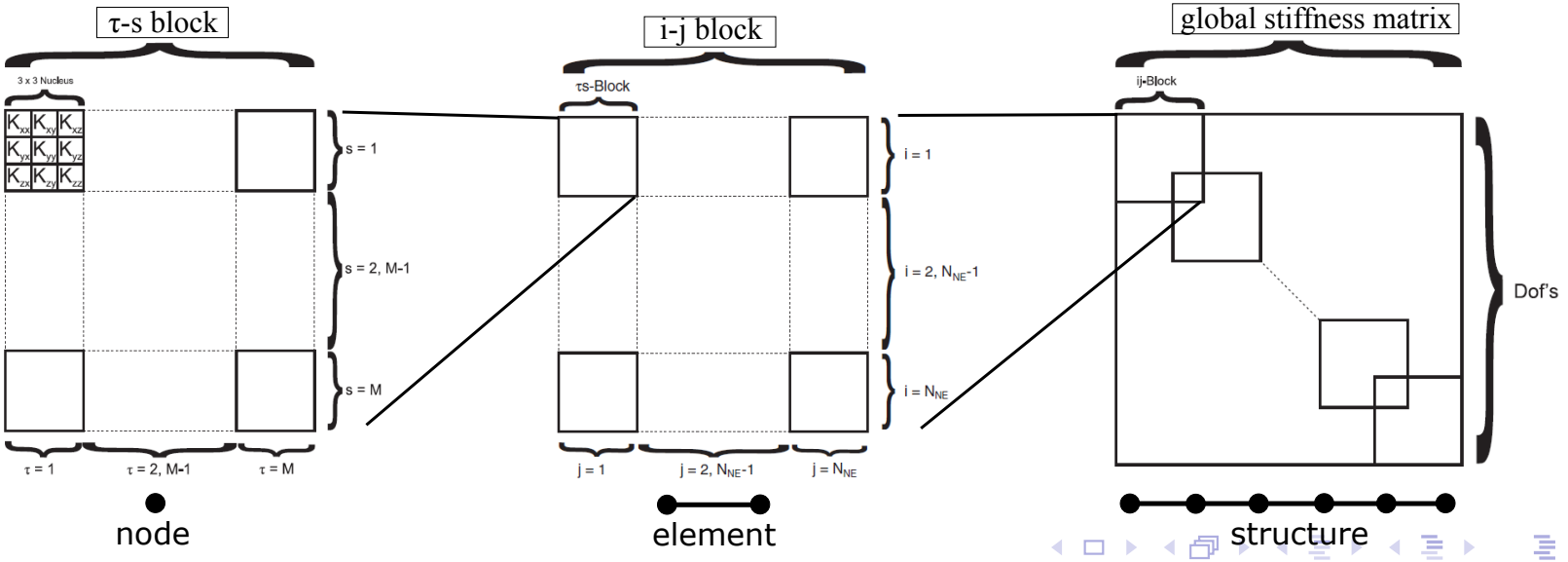
$$\delta L_{ext} = \mathbf{P}\delta \mathbf{u}^T$$

fundamental nucleus

$$K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_\Omega F_{\tau,x} F_{s,x}\, d\Omega \int_l N_i N_j dy + \ldots$$

- i, j - Shape function indexes.
- τ, s - Expansion function indexes

Assembly Technique



Hierarchical Legendre Expansions, HLE

Vertex polynomials

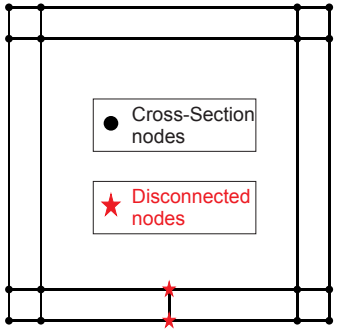
$$F_\tau = \frac{1}{4}(1 - r_\tau r)(1 - s_\tau s)$$

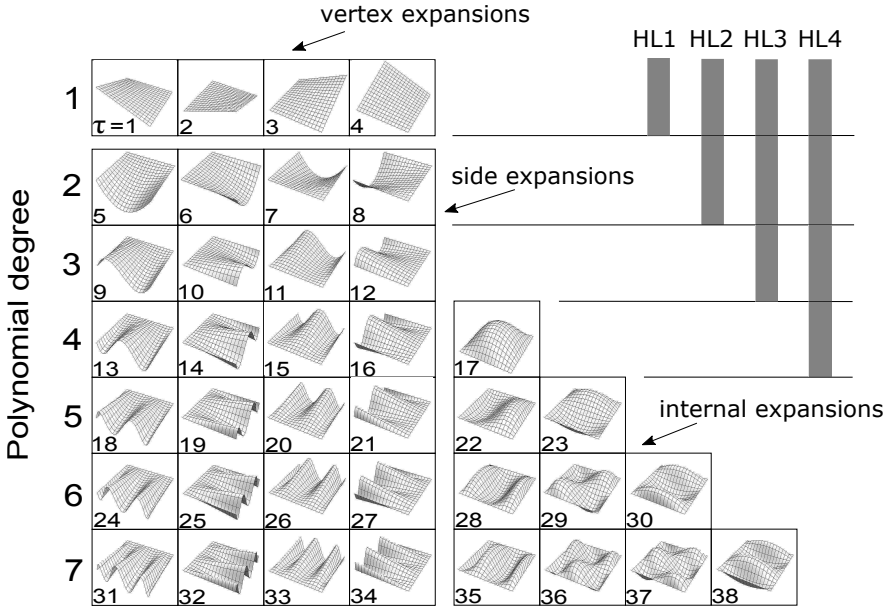
Side polynomials

$$F_\tau = \frac{1}{2}(1 - s)\phi_p(r)$$

Internal polynomials

$$F_\tau = \phi_{p_r}(r)\phi_{p_s}(s) \qquad p_r + p_s = p$$



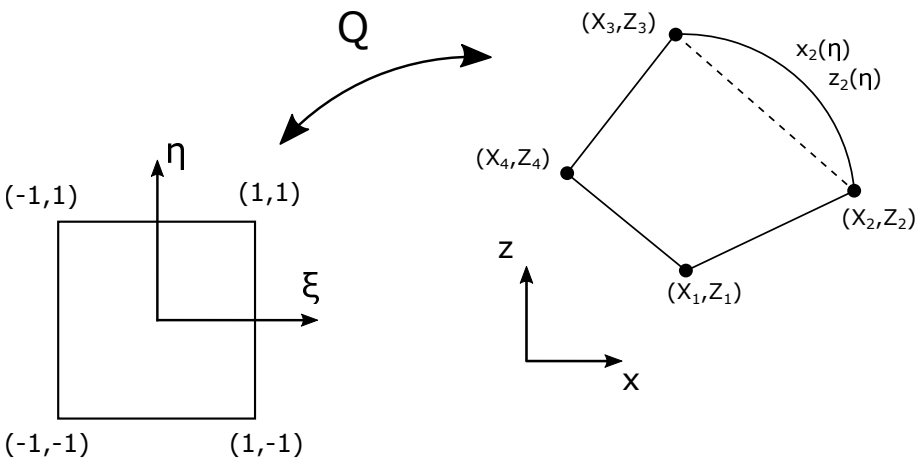


e.g. HL4 = 4th order expansion, 17×3 DOFs

- generalized displacements unknowns, hierarchical kinematics
- cross-section discretization

Cross-sectional mapping

- HLE models: coarse section discretizations, large domains
- Need of representing the exact geometry of curved domains
- Blending function method → non isoparametric expansions (i.e. the mapping functions, Q, do not correspond with the expansion functions, F_τ)

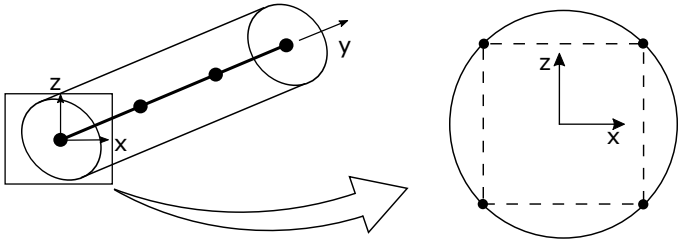


$$x_2(\eta) = a_x + b_x\eta + c_x\eta^2 + d_x\eta^3$$
$$z_2(\eta) = a_z + b_z\eta + c_z\eta^2 + d_z\eta^3$$

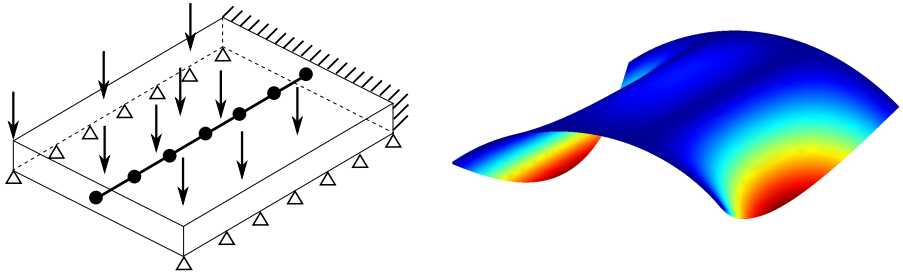
$\Downarrow$ Mapping functions

$$Q_x = F_\tau(\xi,\eta)X_\tau + \left(x_2(\eta) - \left(\frac{1-\eta}{2}X_2 + \frac{1+\eta}{2}X_3\right)\right)\frac{1+\xi}{2}$$
$$Q_z = \underbrace{F_\tau(\xi,\eta)Z_\tau}_{\textit{isoparametric term}} + \underbrace{\left(z_2(\eta) - \left(\frac{1-\eta}{2}Z_2 + \frac{1+\eta}{2}Z_3\right)\right)\frac{1+\xi}{2}}_{\textit{blending function term}}$$

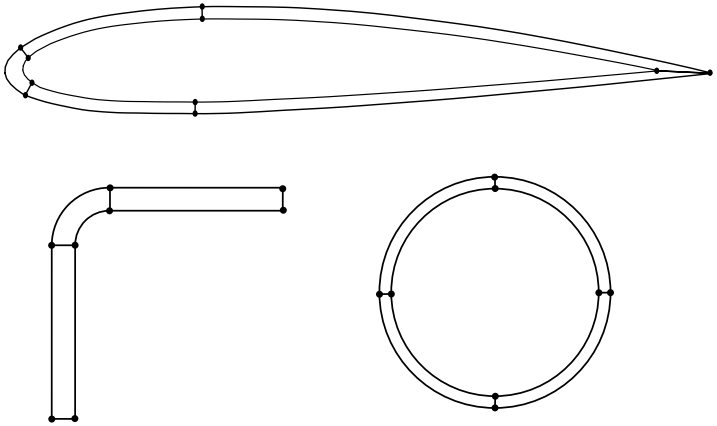
3D-like modelling using CUF beams



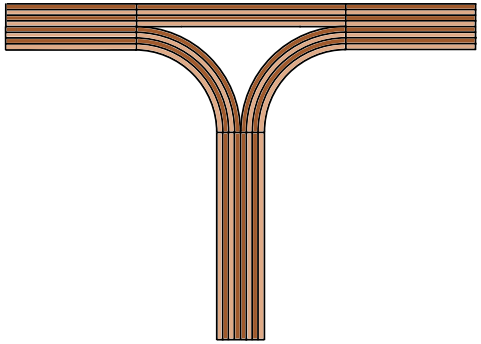
Geometrically exact curved sections



3D-like boundary conditions



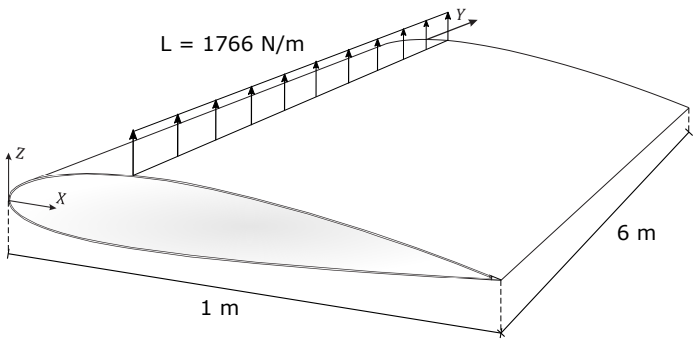
Thin-walled structures



Composites

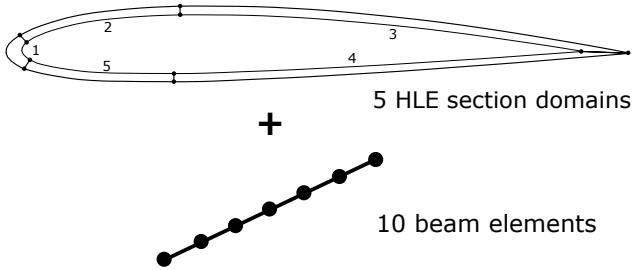
Wing structure

Airfoil NACA 2415

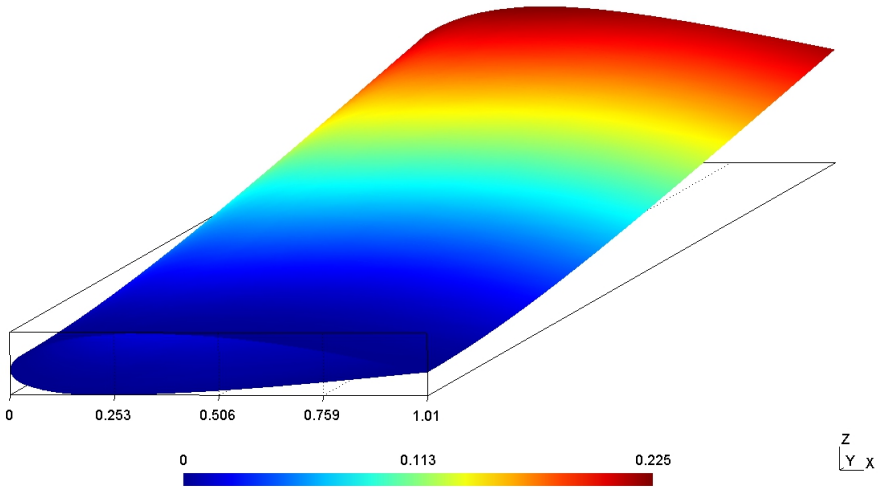


- Aluminum: E = 75 Gpa, ν = 0.33

HLE mapped model

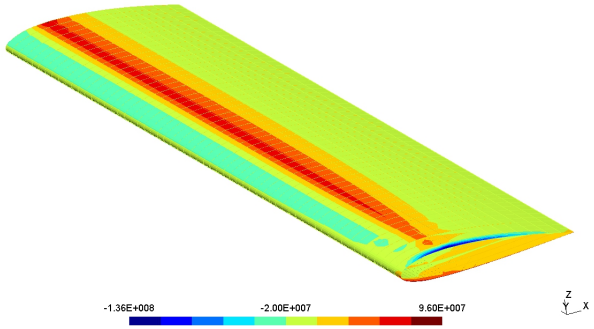


Shell-like displacement solutions

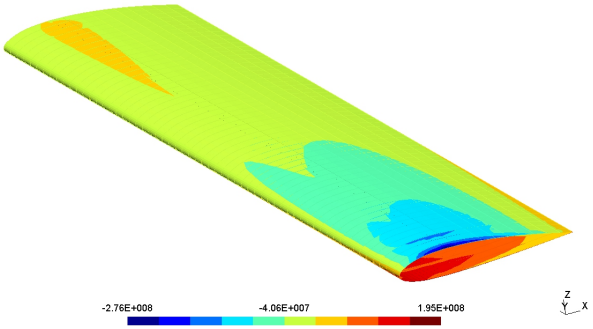


Model	u_z m Load point, $y = L$	σ_{yy} MPa Load point, $y = L/2$	σ_{yz} MPa Leading edge, $y = L/2$ m	DOFs
HL3	0.214	30.044	9.686	3720
HL5	0.220	31.646	8.609	7905
HL8	0.223	32.000	8.839	17670
MSC Nastran	0.225	32.301	8.761	395280

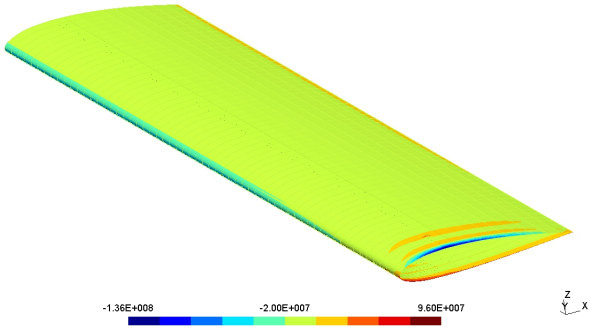
3D-like stress solutions



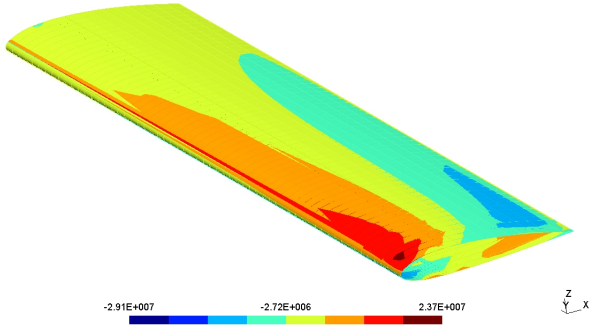
σ_{xx}



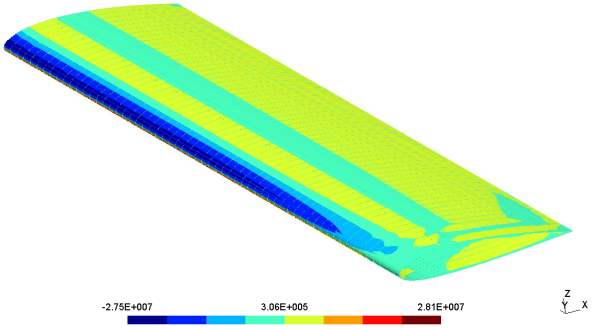
σ_{yy}



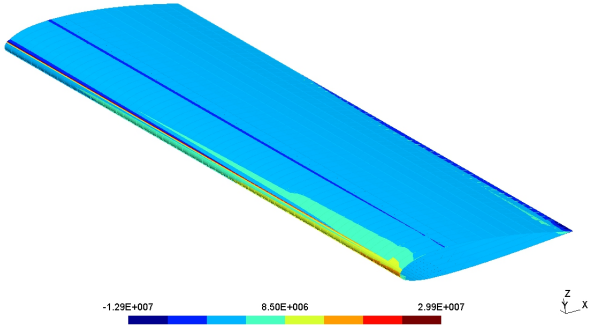
σ_{zz}



σ_{xy}

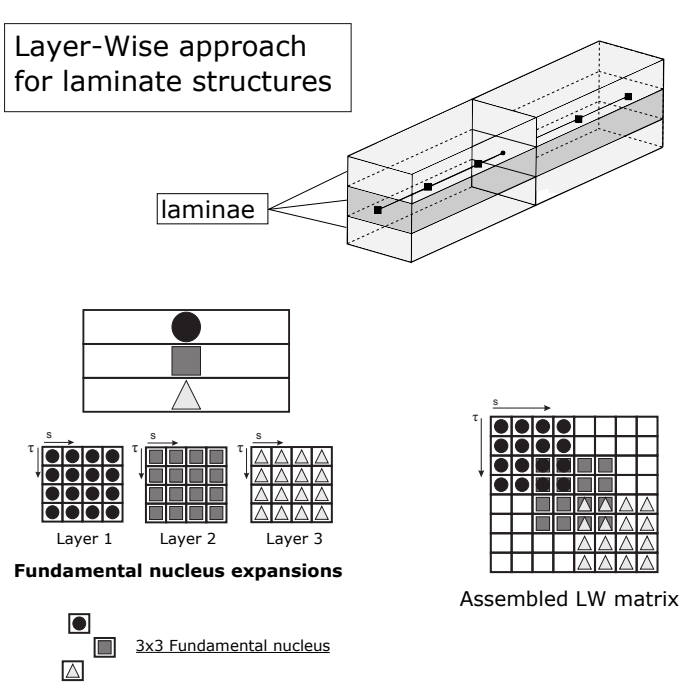


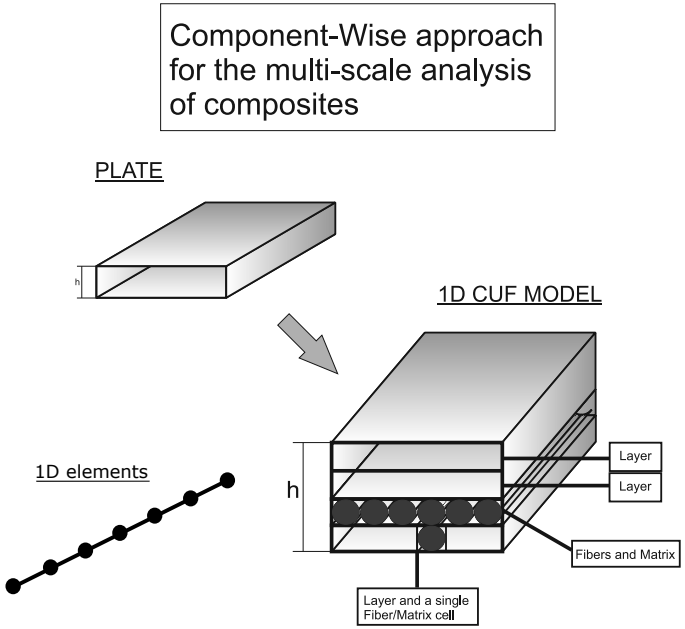
σ_{xz}



σ_{yz}

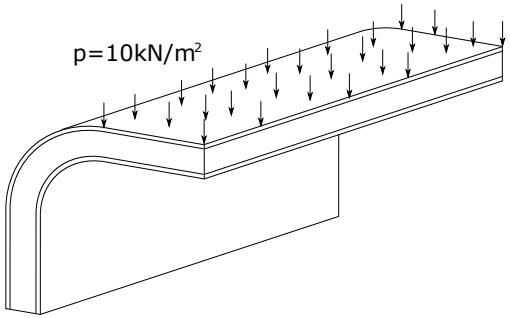
The Component-Wise Approach



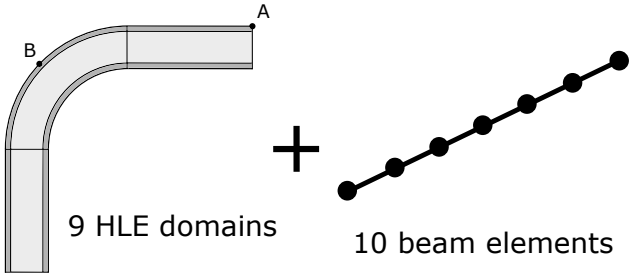


- Only 1D beam elements are used to model all components.
- No need of reference surfaces. This might be useful in a CAD-FEM interface scenario.
- Straightfoward refining of the model on particular zones of interest.

Curved sandwhich

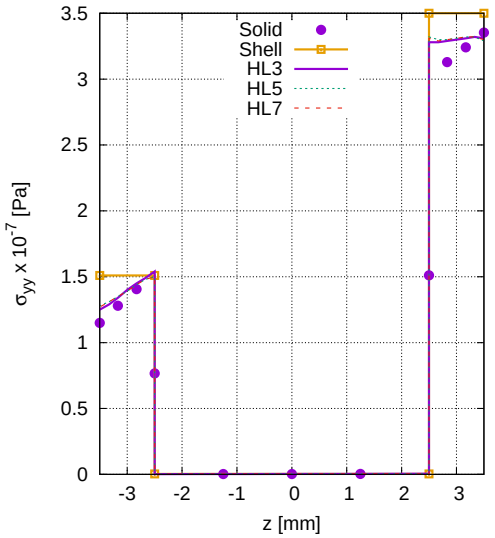


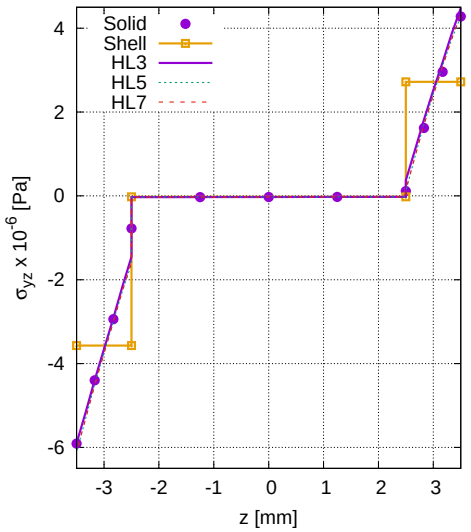
L = 1 m, h = w = 40 mm,
t_f = 1 mm, t_c = 5 mm



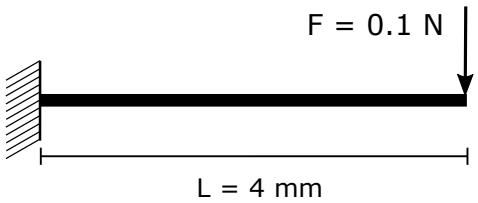
- Faces: E = 75 GPa, ν = 0.33
- Core: E = 0.1063 GPa, ν = 0.32

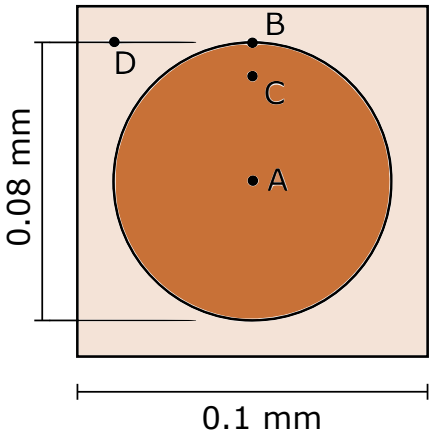
model	$u_z \times 10^2$ m Point A	$\sigma_{yy} \times 10^{-7}$ Pa point B	$\sigma_{yz} \times 10^{-6}$ Pa Point B	DOFs
		MSC Nastran solutions		
Solid	-3.715	3.353	4.324	166617
Shell	-3.900	3.496	2.721	71000
		HLE model solutions		
HL2	-3.032	3.421	-6.551	3720
HL3	-3.711	3.307	4.484	952
HL4	-3.712	3.293	4.466	9021
HL5	-3.712	3.288	4.276	12927
HL6	-3.712	3.290	4.256	17670
HL7	-3.712	3.298	4.308	23250
HL8	-3.712	3.297	4.321	29667



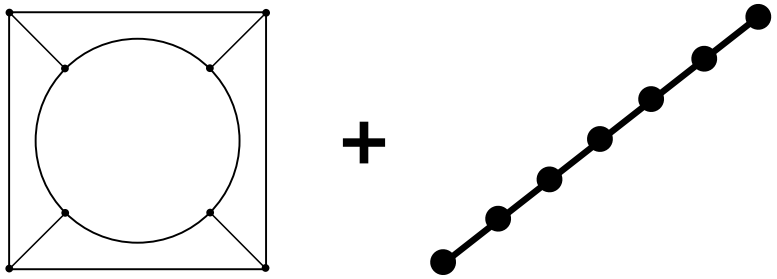


Fiber-matrix cell

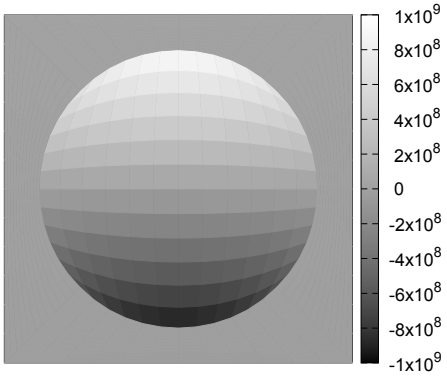




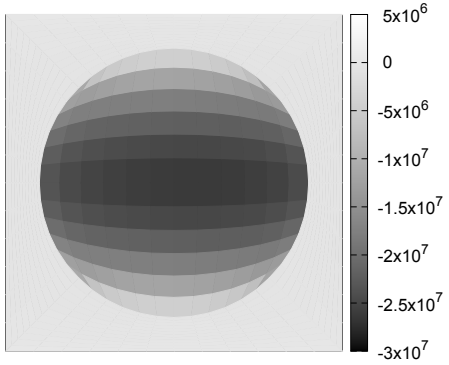
	E [GPa]	ν
Fiber	202.038	0.2128
Matrix	3.252	0.355



model	$u_z \times 10^2$ [mm] Point A, y = L	$\sigma_{yy} \times 10^{-8}$ [Pa] Point B, y= L/2	$\sigma_{yy} \times 10^{-8}$ [Pa] Point C, y= L/2	$\sigma_{yz} \times 10^{-8}$ [Pa] Point D, y= L/2	DOFs
MSC Nastran	-7.818	9.492	7.094	-2.383	268215
TE (N=5)	-7.795	9.327	7.090	-2.375	7623
LE (12L9+8L6)	-7.933	9.450	7.046	-2.500	7533
HLE (5HL5)	-7.773	9.383	7.070	-2.746	6603



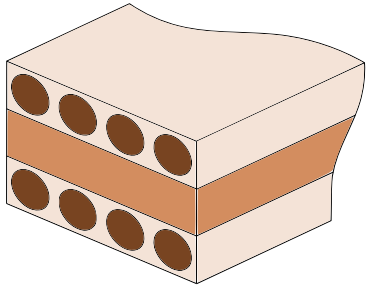
σ_{yy}



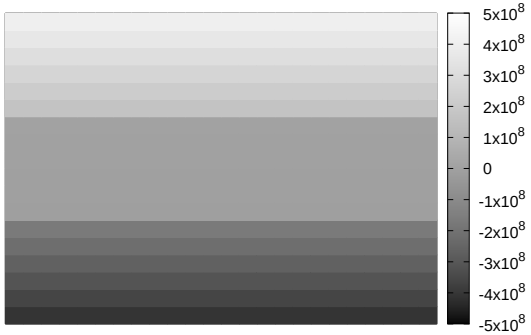
σ_{yz}

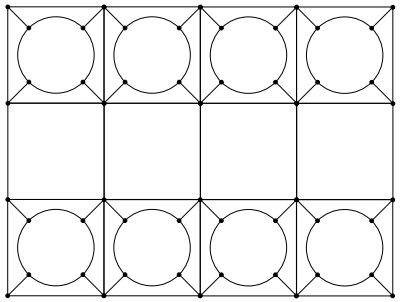
Cross-ply laminate

Component	E_L	E_T	E_z	G_{LT}	G_{Lz}	G_{Tz}	ν_{LT}	ν_{Lz}	ν_{Tz}
Fiber (ortho)	202.038	12.134	12.134	8.358	8.358	47.756	0.2128	0.2128	0.2704
Layer (ortho)	103.173	5.145	5.145	2.107	2.107	2.353	0.2835	0.2835	0.3124
Matrix (iso)		3.252			1.200			0.355	

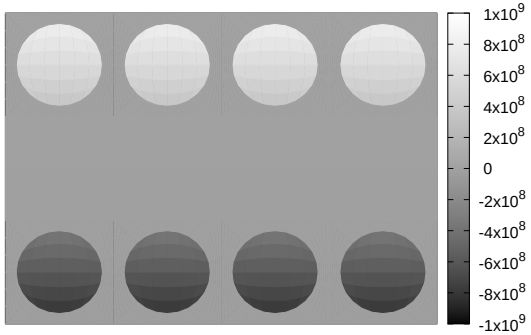


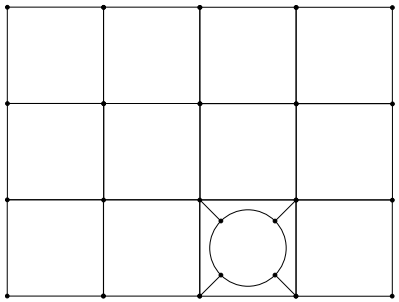
2706 DOFs



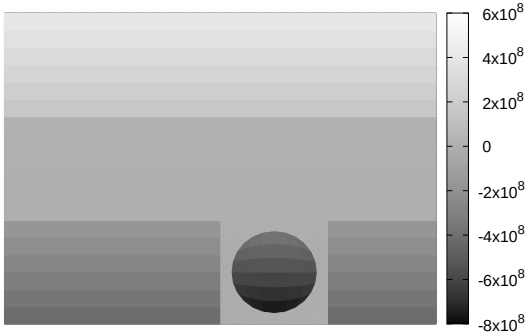


35433 DOFs





14601 DOFs



Main Conclusions and Perspectives

1. Hierarchical theories of structure with mapping capabilities extend the beam modelling to the accurate analysis of curved thin-walled structures.
2. The accuracy is mainly controlled by the polynomial order and refinement of the expansions over the cross-section.
3. The Component-Wise approach provides a tool for the efficient analysis of multi-component structures, such as fiber-reinforced composites.
4. Reduction of DOFs by one order of magnitude, or more, in comparison with commercial solid models for the same level of accuracy.

Ongoing developments and future extensions

- Multi-scale analysis of fiber-reinforced components.
- Extension of CUF to non-linear problems: progressive damage, plasticity, etc.
- Introduction of curved beams: extension of CUF beam models to a new range of structural geometries.

Thank you for the kind attention,

any questions?

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