Analysis of curved composite structures through refined 1D finite elements with aerospace applications

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The FULLCOMP project

- FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures
- The FULLCOMP project is funded by the European Commission under a Marie Skłodowska-Curie Innovative Training Networks grant for European Training Networks (ETN).

- The FULLCOMP partners are:
  1. Politecnico di Torino (Italy) - Coordinator
  2. University of Bristol (UK)
  3. ENSMA (Bordeaux, France)
  4. Leibniz Universitaet Hannover (Germany)
  5. University of Porto (Portugal)
  6. University of Washington (USA)
  7. RMIT (Melbourne, Australia)
  8. Luxembourg Institute of Technology
  9. Elan-Ausy (Hamburg, Germany)

- 12 PhD students
- Full spectrum of design of composite structures: manufacturing, health-monitoring, failure, modeling, multiscale approaches, testing, prognosis and prognostic.

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Overview

1. From classical beam theories to the unified formulation
2. Non-local hierarchical theories of structure
3. Cross-sectional mapping
4. Component-wise analysis for composite structures
5. Conclusions and future developments
From classic to refined kinematics

Euler-Bernoulli beam theory

\[ u_x(x, y, z) = u_{x_1}(y) \]
\[ u_y(x, y, z) = u_{y_1}(y) - x \frac{\partial u_{x_1}(y)}{\partial y} - z \frac{\partial u_{z_1}(y)}{\partial y} \]
\[ u_z(x, y, z) = u_{z_1}(y) \]

Timoshenko beam theory

\[ u_x(x, y, z) = u_{x_1}(y) \]
\[ u_y(x, y, z) = u_{y_1}(y) + x \phi_z(y) - z \phi_x(y) \]
\[ u_z(x, y, z) = u_{z_1}(y) \]

Saint Venant beam theory

\[ u_x(x, y, z) = u_{x_1}(y) - z \theta(y) \]
\[ u_y(x, y, z) = u_{y_1}(y) + x \phi_z(y) - z \phi_x(y) + \psi(x, z) \frac{\partial \theta(y)}{\partial y} \]
\[ u_z(x, y, z) = u_{z_1}(y) + x \theta(y) \]

Washizu’s statement:
"For a complete removal of the inconsistency and an improvement of the accuracy of the beam theory" -> enrich beam kinematics with higher-order terms
Beam kinematics through the unified formulation

\[ \mathbf{u}(x, y, z) = F_\tau(x, z) \mathbf{u}_\tau(y) \]

\[ u_x(x, y, z) = F_1(x, z) u_{x1}(y) + F_2(x, z) u_{x2}(y) + F_3(x, z) u_{x3}(y) + \ldots + F_M(x, z) u_{xM}(y) \]

\[ u_y(x, y, z) = F_1(x, z) u_{y1}(y) + F_2(x, z) u_{y2}(y) + F_3(x, z) u_{y3}(y) + \ldots + F_M(x, z) u_{yM}(y) \]

\[ u_z(x, y, z) = F_1(x, z) u_{z1}(y) + F_2(x, z) u_{z2}(y) + F_3(x, z) u_{z3}(y) + \ldots + F_M(x, z) u_{zM}(y) \]

\[ \tau = 1, \ldots, M \text{ (number of expansion terms)} \]

Classical models, such as Euler or Timoshenko, can be obtained as particular cases of the linear models.
CUF stiffness matrix

\[ \mathbf{u} = \mathbf{F}_\tau(x, z)N_i(y)\, \mathbf{u}_{\tau i} \quad (1D) \]

\[ \delta L_{\text{int}} = \delta \mathbf{u}_{\tau i}^T \mathbf{K}^{ij\tau s} \mathbf{u}_{sj} \]

\[ \delta L_{\text{ext}} = \mathbf{P} \delta \mathbf{u}^T \]

**fundamental nucleus**

\[ K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_\Omega \mathbf{F}_{\tau, x} \mathbf{F}_{s, x} \, d\Omega \int_I N_i N_j \, dy + \ldots \]

- \( i, j \) - Shape function indexes.
- \( \tau, s \) - Expansion function indexes

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**Assembly Technique**

\( \tau-s \) block

\( i-j \) block

Global stiffness matrix

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Hierarchical Legendre Expansions, HLE

**Vertex polynomials**

\[ F_\tau = \frac{1}{4} (1 - r_\tau r)(1 - s_\tau s) \]

**Side polynomials**

\[ F_\tau = \frac{1}{2} (1 - s) \phi_p (r) \]

**Internal polynomials**

\[ F_\tau = \phi_{pr} (r) \phi_{ps} (s) \quad pr + ps = p \]

- generalized displacements unknowns, hierarchical kinematics
- cross-section discretization

E.g. HL4 = 4\textsuperscript{th} order expansion, 17\times3 DOFs
Cross-sectional mapping

- HLE models: coarse section discretizations, large domains
- Need of representing the exact geometry of curved domains
- Blending function method → non isoparametric expansions (i.e. the mapping functions, $Q$, do not correspond with the expansion functions, $F_\tau$)

\[
\begin{align*}
    x_2(\eta) &= a_x + b_x \eta + c_x \eta^2 + d_x \eta^3 \\
    z_2(\eta) &= a_z + b_z \eta + c_z \eta^2 + d_z \eta^3
\end{align*}
\]

\[
\begin{align*}
    Q_x &= F_\tau(\xi, \eta) X_\tau + \left( x_2(\eta) - \left( \frac{1 - \eta}{2} X_2 + \frac{1 + \eta}{2} X_3 \right) \right) \frac{1 + \xi}{2} \\
    Q_z &= F_\tau(\xi, \eta) Z_\tau + \left( z_2(\eta) - \left( \frac{1 - \eta}{2} Z_2 + \frac{1 + \eta}{2} Z_3 \right) \right) \frac{1 + \xi}{2}
\end{align*}
\]

isoparametric term  \hspace{2cm} blending function term
3D-like modelling using CUF beams

Geometrically exact curved sections

3D-like boundary conditions

Thin-walled structures

Composites
Wing structure

Airfoil NACA 2415

L = 1766 N/m

Aluminum: $E = 75 \text{ Gpa}$, $\nu = 0.33$

HLE mapped model

Shell-like displacement solutions

<table>
<thead>
<tr>
<th>Model</th>
<th>$u_z$ m</th>
<th>$\sigma_{yy}$ MPa</th>
<th>$\sigma_{yz}$ MPa</th>
<th>DOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>HL3</td>
<td>0.214</td>
<td>30.044</td>
<td>-</td>
<td>3720</td>
</tr>
<tr>
<td>HL5</td>
<td>0.220</td>
<td>31.646</td>
<td>-</td>
<td>7905</td>
</tr>
<tr>
<td>HL8</td>
<td>0.223</td>
<td>32.000</td>
<td>-</td>
<td>17670</td>
</tr>
<tr>
<td>MSC Nastran</td>
<td>0.225</td>
<td>32.301</td>
<td>8.761</td>
<td>395280</td>
</tr>
</tbody>
</table>
3D-like stress solutions

\[ \sigma_{xx}, \quad \sigma_{yy}, \quad \sigma_{zz} \]

\[ \sigma_{xy}, \quad \sigma_{xz}, \quad \sigma_{yz} \]
The Component-Wise Approach

- Only 1D beam elements are used to model all components.
- No need of reference surfaces. This might be useful in a CAD-FEM interface scenario.
- Straightforward refining of the model on particular zones of interest.

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Curved sandwich

\[ p = 10 \text{kN/m}^2 \]

\[ L = 1 \text{ m}, \quad h = w = 40 \text{ mm}, \quad t_f = 1 \text{ mm}, \quad t_c = 5 \text{ mm} \]

Faces: \( E = 75 \text{ GPa}, \quad \nu = 0.33 \)

Core: \( E = 0.1063 \text{ GPa}, \quad \nu = 0.32 \)

\[ \sigma_{yy} \times 10^{-7} \text{ Pa} \]

\[ \sigma_{yz} \times 10^{-6} \text{ Pa} \]

MSC Nastran solutions

<table>
<thead>
<tr>
<th>Model</th>
<th>( u_z \times 10^2 \text{ m} )</th>
<th>( \sigma_{yy} \times 10^{-7} \text{ Pa} )</th>
<th>( \sigma_{yz} \times 10^{-6} \text{ Pa} )</th>
<th>DOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>-3.715</td>
<td>3.353</td>
<td>4.324</td>
<td>166617</td>
</tr>
<tr>
<td>Shell</td>
<td>-3.900</td>
<td>3.496</td>
<td>2.721</td>
<td>71000</td>
</tr>
</tbody>
</table>

HLE model solutions

<table>
<thead>
<tr>
<th>Model</th>
<th>( u_z \times 10^2 \text{ m} )</th>
<th>( \sigma_{yy} \times 10^{-7} \text{ Pa} )</th>
<th>( \sigma_{yz} \times 10^{-6} \text{ Pa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HL2</td>
<td>-3.032</td>
<td>3.421</td>
<td>-6.551</td>
</tr>
<tr>
<td>HL3</td>
<td>-3.711</td>
<td>3.307</td>
<td>4.484</td>
</tr>
<tr>
<td>HL4</td>
<td>-3.712</td>
<td>3.293</td>
<td>4.466</td>
</tr>
<tr>
<td>HL5</td>
<td>-3.712</td>
<td>3.288</td>
<td>4.276</td>
</tr>
<tr>
<td>HL6</td>
<td>-3.712</td>
<td>3.290</td>
<td>4.256</td>
</tr>
<tr>
<td>HL7</td>
<td>-3.712</td>
<td>3.298</td>
<td>4.308</td>
</tr>
<tr>
<td>HL8</td>
<td>-3.712</td>
<td>3.297</td>
<td>4.321</td>
</tr>
</tbody>
</table>
Fiber-matrix cell

F = 0.1 N
L = 4 mm

HLE mapped model
1D elements

<table>
<thead>
<tr>
<th>model</th>
<th>$u_z \times 10^2$ [mm]</th>
<th>$\sigma_{yy} \times 10^{-8}$ [Pa]</th>
<th>$\sigma_{yy} \times 10^{-8}$ [Pa]</th>
<th>$\sigma_{yz} \times 10^{-8}$ [Pa]</th>
<th>DOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE (N=5)</td>
<td>-7.795</td>
<td>9.327</td>
<td>7.090</td>
<td>-2.375</td>
<td>7623</td>
</tr>
<tr>
<td>LE (12L9+8L6)</td>
<td>-7.933</td>
<td>9.450</td>
<td>7.046</td>
<td>-2.500</td>
<td>7533</td>
</tr>
<tr>
<td>HLE (5HL5)</td>
<td>-7.773</td>
<td>9.383</td>
<td>7.070</td>
<td>-2.746</td>
<td>6603</td>
</tr>
</tbody>
</table>

E [GPa]  $\nu$
Fiber    202.038  0.2128
Matrix  3.252    0.355
Cross-ply laminate

<table>
<thead>
<tr>
<th>Component</th>
<th>$E_L$</th>
<th>$E_T$</th>
<th>$E_Z$</th>
<th>$G_{LT}$</th>
<th>$G_{LZ}$</th>
<th>$G_{TZ}$</th>
<th>$\nu_{LT}$</th>
<th>$\nu_{LZ}$</th>
<th>$\nu_{TZ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber (ortho)</td>
<td>202.038</td>
<td>12.134</td>
<td>12.134</td>
<td>8.358</td>
<td>8.358</td>
<td>47.756</td>
<td>0.2128</td>
<td>0.2128</td>
<td>0.2704</td>
</tr>
<tr>
<td>Layer (ortho)</td>
<td>103.173</td>
<td>5.145</td>
<td>5.145</td>
<td>2.107</td>
<td>2.107</td>
<td>2.353</td>
<td>0.2835</td>
<td>0.2835</td>
<td>0.3124</td>
</tr>
<tr>
<td>Matrix (iso)</td>
<td>3.252</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2706 DOFs

35433 DOFs

14601 DOFs
Main Conclusions and Perspectives

1. Hierarchical theories of structure with mapping capabilities extend the beam modelling to the accurate analysis of curved thin-walled structures.
2. The accuracy is mainly controlled by the polynomial order and refinement of the expansions over the cross-section.
3. The Component-Wise approach provides a tool for the efficient analysis of multi-component structures, such as fiber-reinforced composites.
4. Reduction of DOFs by one order of magnitude, or more, in comparison with commercial solid models for the same level of accuracy.

Ongoing developments and future extensions

- Multi-scale analysis of fiber-reinforced components.
- Extension of CUF to non-linear problems: progressive damage, plasticity, etc.
- Introduction of curved beams: extension of CUF beam models to a new range of structural geometries.
Thank you for the kind attention, any questions?
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