20th International Conference on Composite Structures (ICCS20)
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On the accuracy of the displacement-based Unified Formulation for modelling laminated composite beam structures

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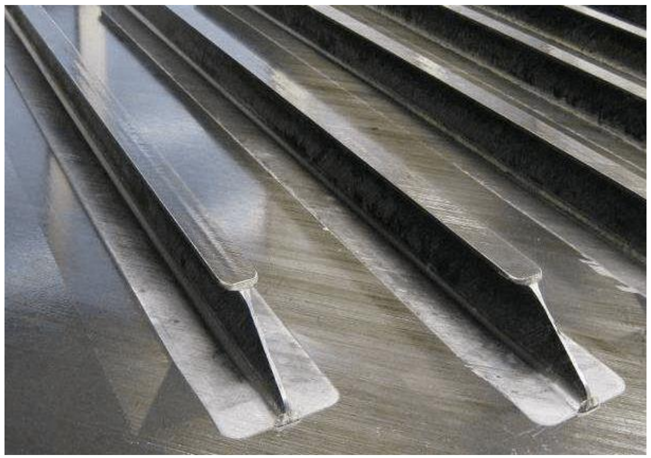
Outline

- Motivation & Objective
- Challenges in modelling laminated composite structures
- Carrera Unified Formulation (CUF) for refined 1D models
- Model Validation and Results
- Conclusions

Motivation & Objective

Goal

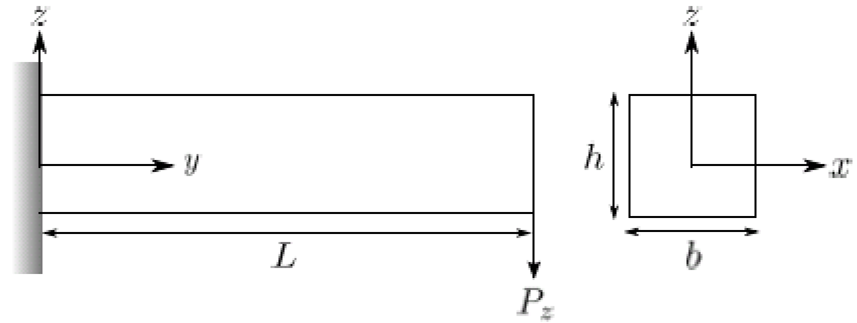
To provide a robust and efficient mathematical model for accurate stress predictions in laminated composite structures.



Challenges

Goal

To provide a robust and efficient mathematical model for accurate stress predictions in laminated composite structures.



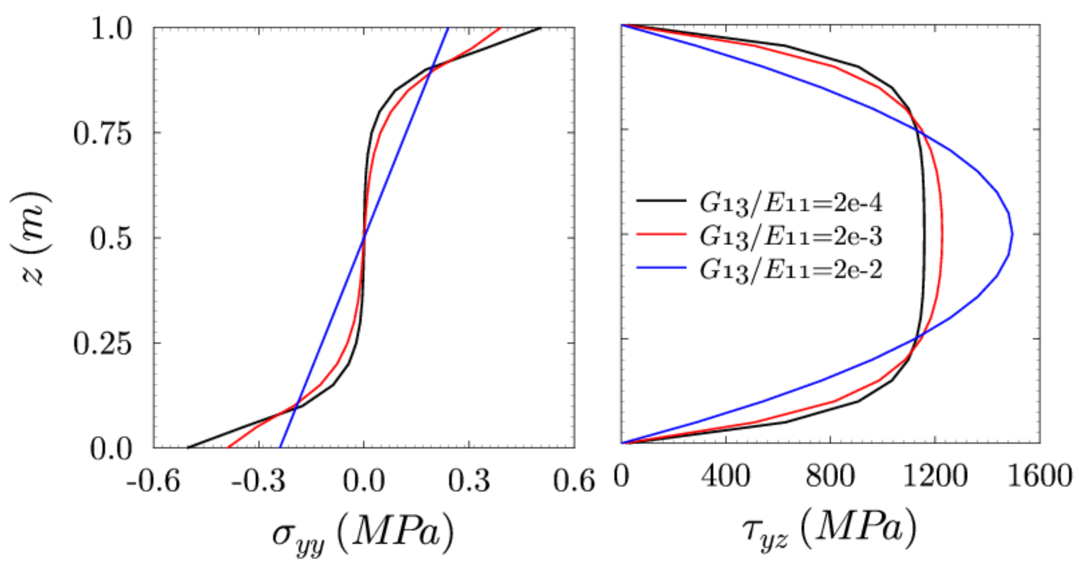
Typical modelling issues

- Low orthotropy ratio (G_{13}/E_{11})

  - Structure becomes more flexible in transverse shear

  - Higher-order distortions of the cross-section

  - Channelling of the axial stress towards the surface



Challenges

Goal

To provide a robust and efficient mathematical model for accurate stress predictions in laminated composite structures.

Typical modelling issues

- Low orthotropy ratio (G_{13}/E_{11})
- Static inconsistencies at clamped ends

Reddy's third order theory

$$u_x = u_0 + z\theta + z^2\zeta + z^3\xi$$

$$u_z = w_0$$

$$u_x = u_0 + z\theta - \frac{4}{3t^2}z^3\left(w_{0,x} + \theta\right)$$

$$u_z = w_0$$

Presence of Kirchhoff rotations over-constrains the model for clamped BCs for thick laminates.

Same concept is applied in other higher-order theories available in the literature

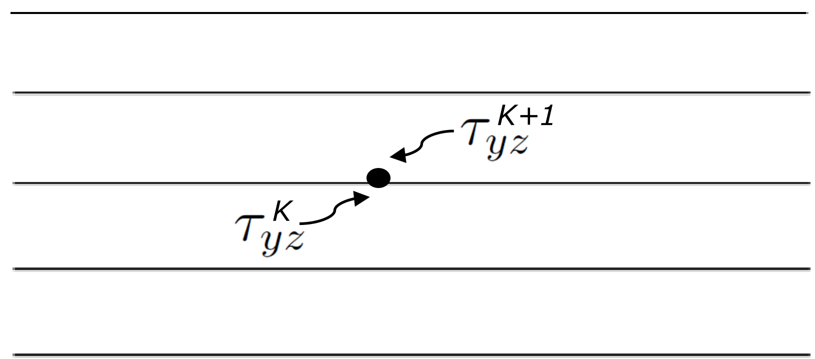
Challenges

Goal

To provide a robust and efficient mathematical model for accurate stress predictions in laminated composite structures.

Typical modelling issues

- Low orthotropy ratio (G_{13}/E_{11})
- Static inconsistencies at clamped ends
- Interlaminar Continuity (IC) condition on displacements and transverse stresses



C^0-continuous displacements u_x, u_y and u_z and transverse stresses σ_{zz}, τ_{yz} and τ_{xz} along the thickness

Challenges

Goal

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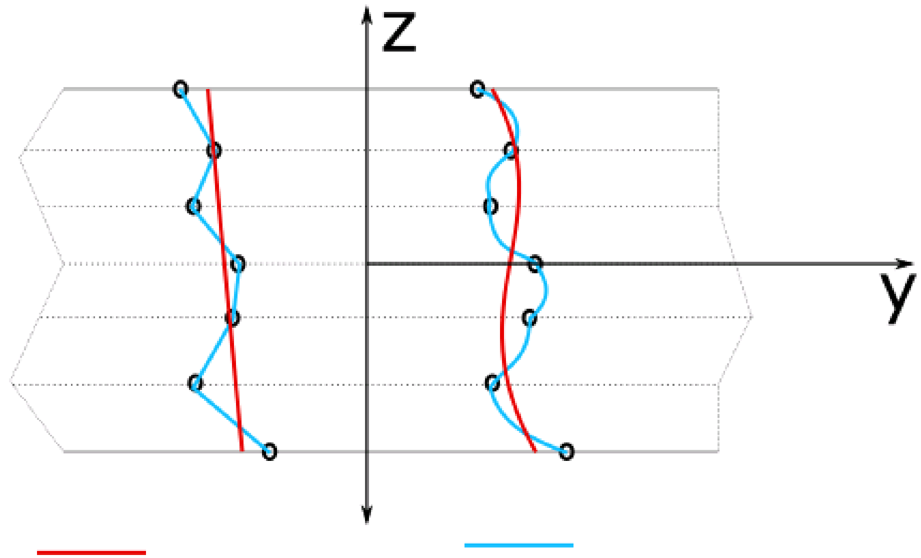
Typical modelling issues

- Low orthotropy ratio (G_{13}/E_{11})
- Static inconsistencies at clamped ends
- Interlaminar Continuity (IC) condition on displacements and transverse stresses
- Zig-zag effect due to transverse anisotropy

  - Slope of the displacement fields must be different at the layer interfaces.

Linear and higher-order displacement fields



Without ZZ functions
C^1-continuous displacements

With ZZ functions
C^1-discontinuous displacements

Proposed Solution

Goal

To provide a robust and efficient mathematical model for accurate stress predictions in laminated composite structures.

3D FE

CUF

HR3-RZT[1]

Typical modelling issues

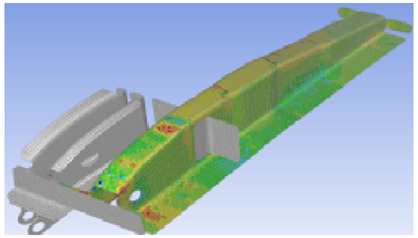
- Low orthotropy ratio (G_{13}/E_{11})
- Static inconsistencies at clamped ends
- Interlaminar Continuity (IC) condition on displacements and transverse stresses
- Zig-zag effect due to transverse anisotropy

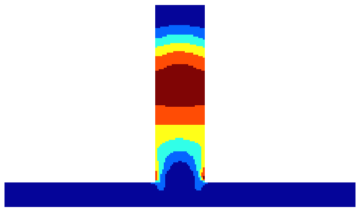
How to address?

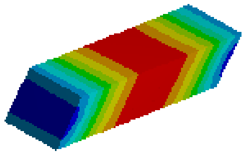
- Higher-order models (order should be problem dependent)
- In-plane displacement fields must not contain Kirchhoff rotations
- Displacement-based: Stress recovery or a Mixed formulation
- Layer-wise approach or ESL with zig-zag functions

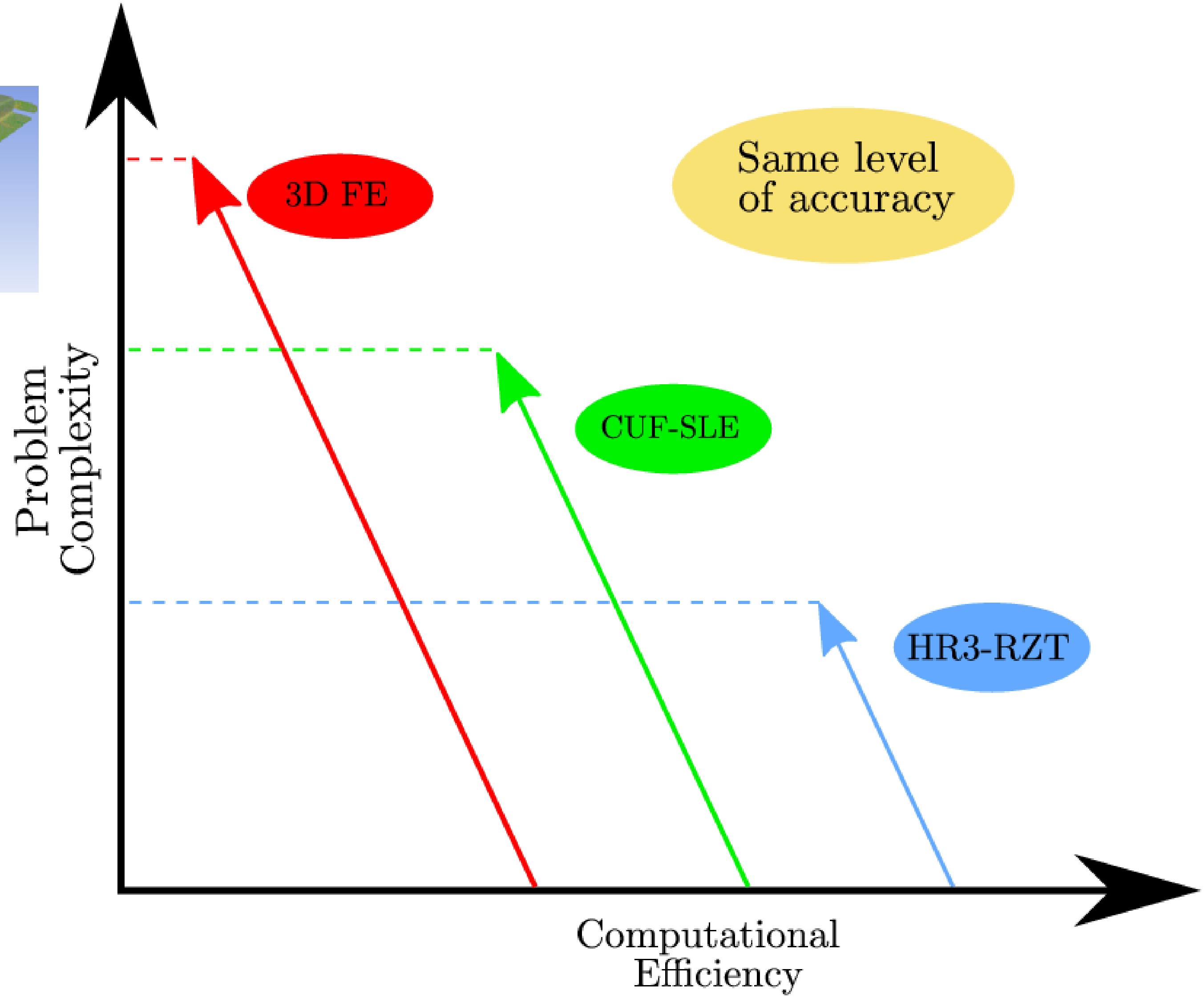
[1] ***RMJ Groh and PM Weaver.*** *On displacement-based and mixed-variational ESL theories for modelling highly heterogeneous laminated beams. Int. J. of Solids & Struct., 2015.*

Proposed Solution



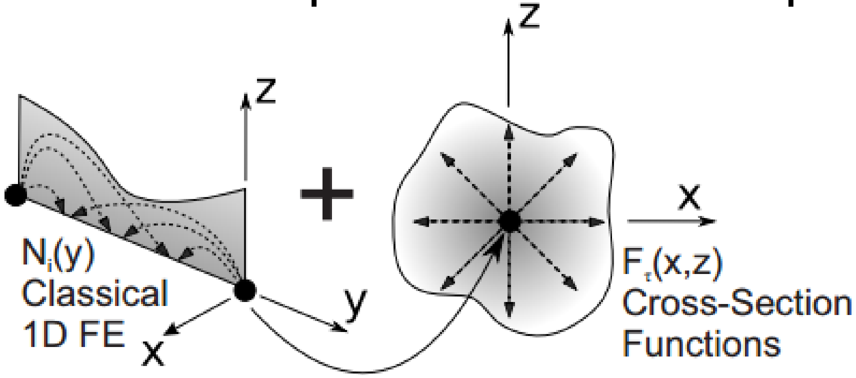






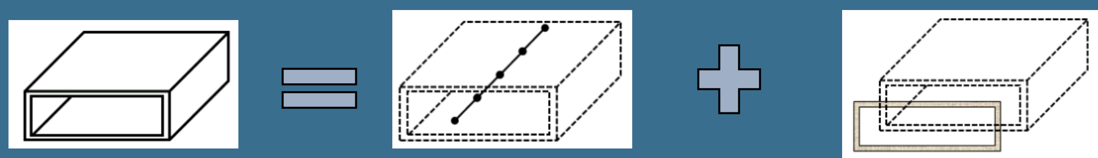
Carrera Unified Formulation - Overview

Computational expansion of the displacement field



Ref: ***E, Carrera.*** Finite Element Analysis of Structures through Unified Formulation. *Wiley Publications, 2014.*

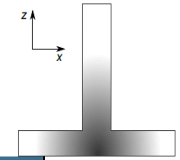
3D structure | 1D FE model | Cross-section expansion



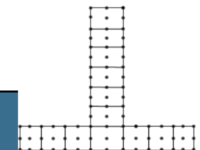
$$u(x,y,z) = N_i(y)\, F_\zeta(x,z)\, u_{\zeta i}$$
$$v(x,y,z) = N_i(y)\, F_\zeta(x,z)\, v_{\zeta i}$$
$$w(x,y,z) = N_i(y)\, F_\zeta(x,z)\, w_{\zeta i}$$

$N_i(y)$ - 1D shape functions (Linear, quadratic, cubic)

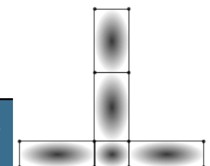
Taylor-like polynomial expansions (TE)



Lagrange polynomial expansions (LE)



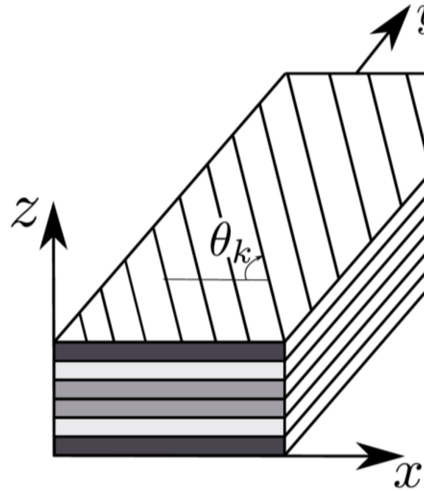
Serendipity Lagrange expansions (SLE)



Transverse Stress Recovery

3D indefinite equilibrium equations

$$\begin{aligned}\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} &= 0,\\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} &= 0,\\ \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= 0.\end{aligned}$$

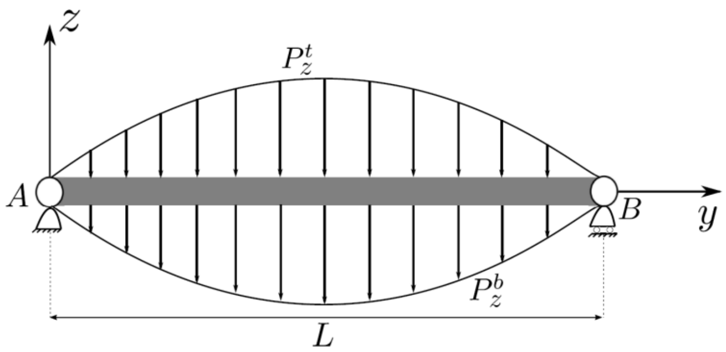


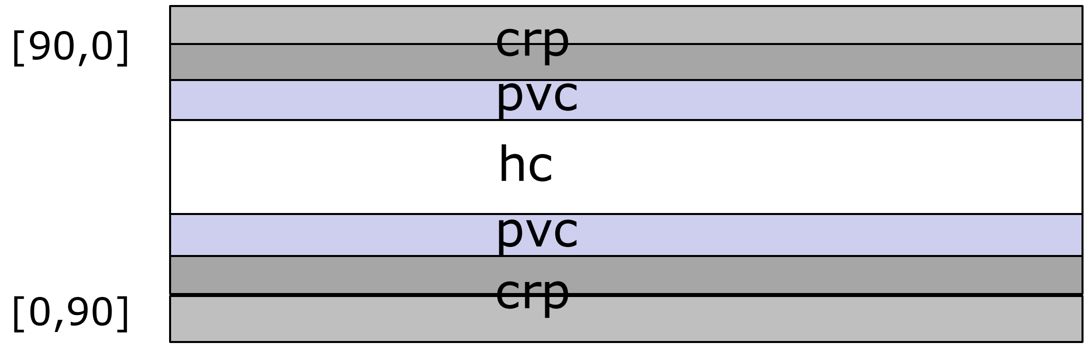
$$\sigma_{zz}^k(z) = \sigma_{zz_b}^k - \int_{z_k}^{z} \left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} \right) dz$$

stress value in the k-layer

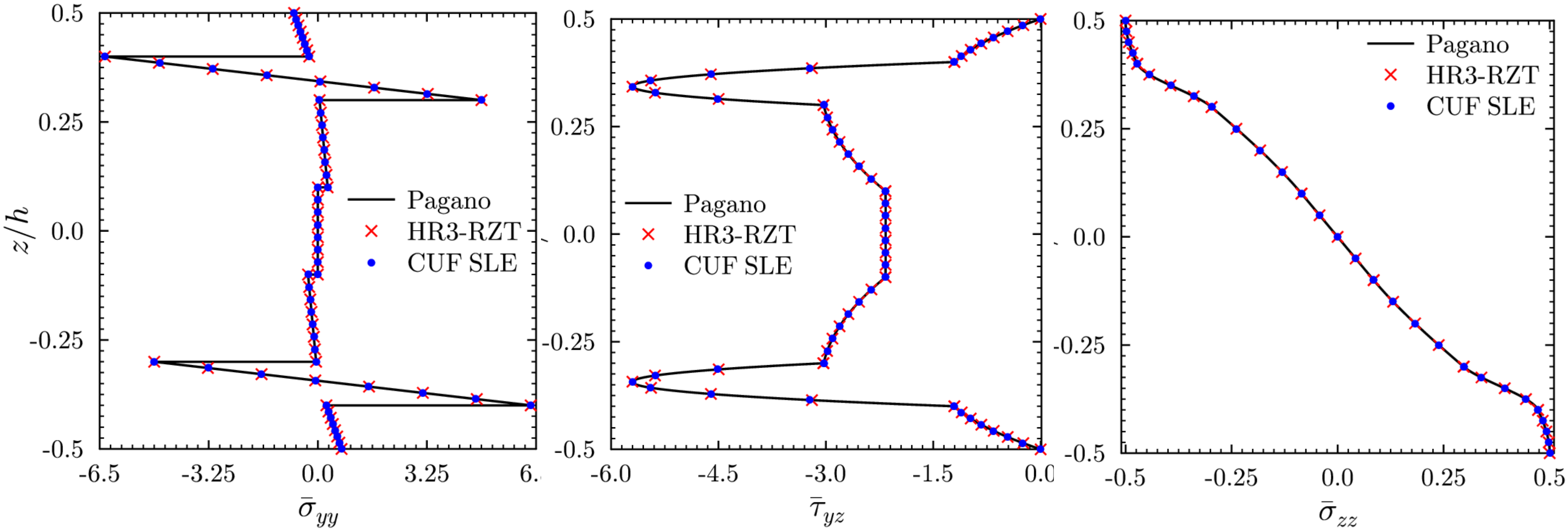
stress value at the bottom of the k-layer

Model Validation

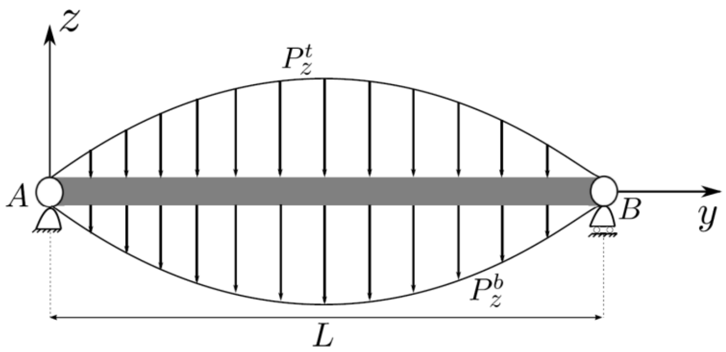


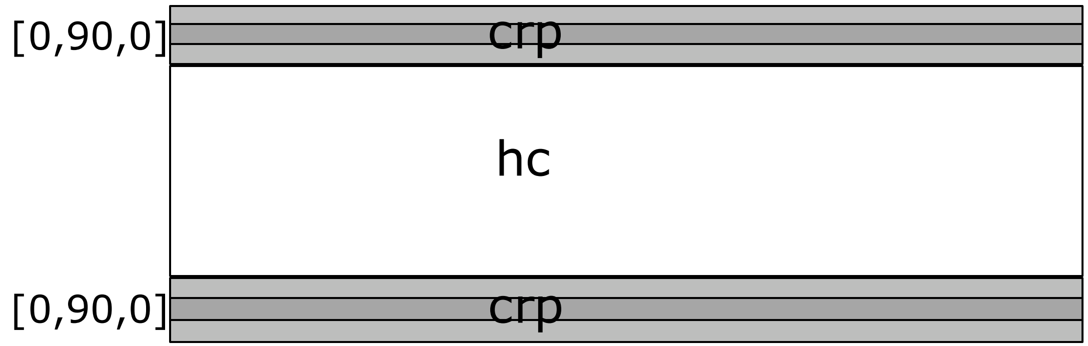


Through-thickness stress distribution at mid-span



Model Validation





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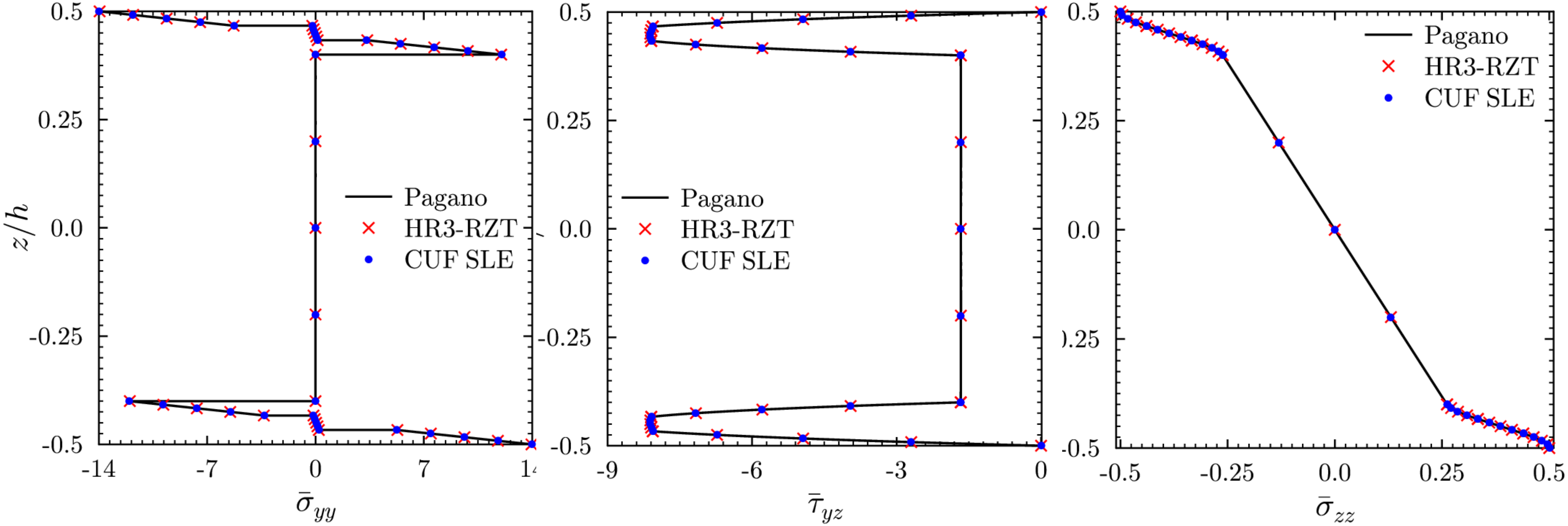
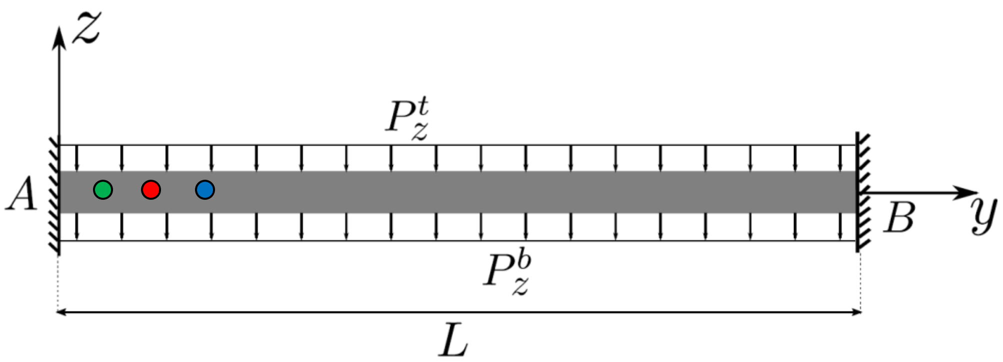


Numerical Results



crp [90]
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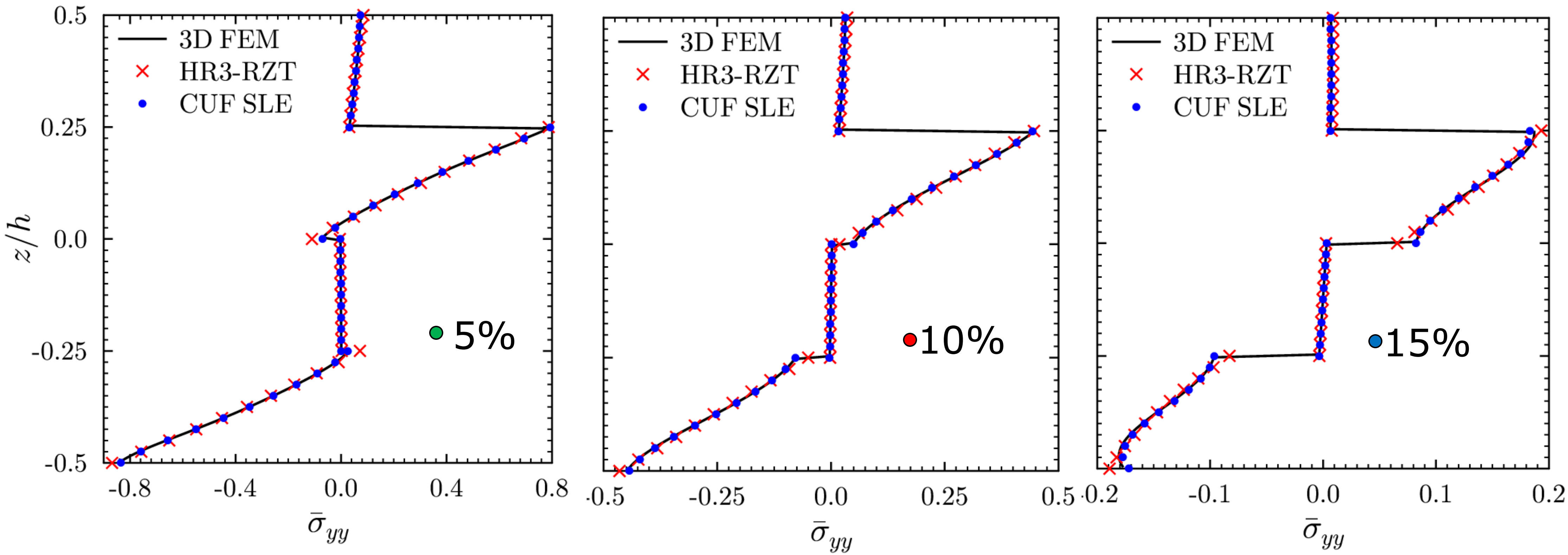

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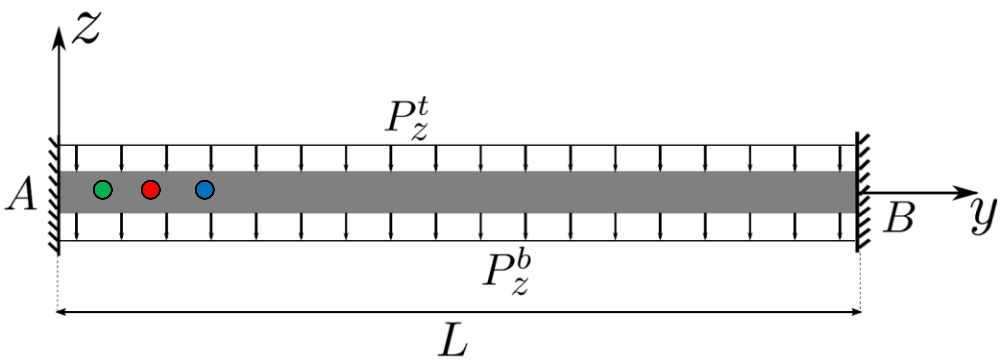

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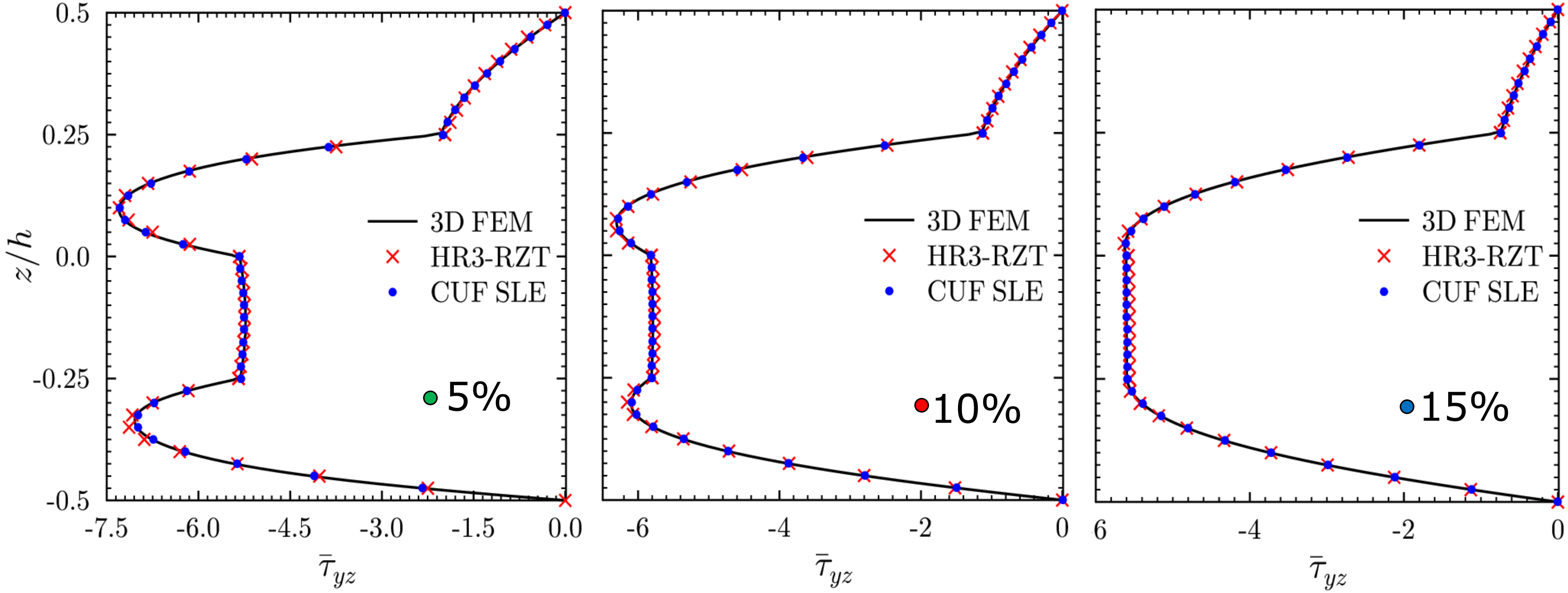

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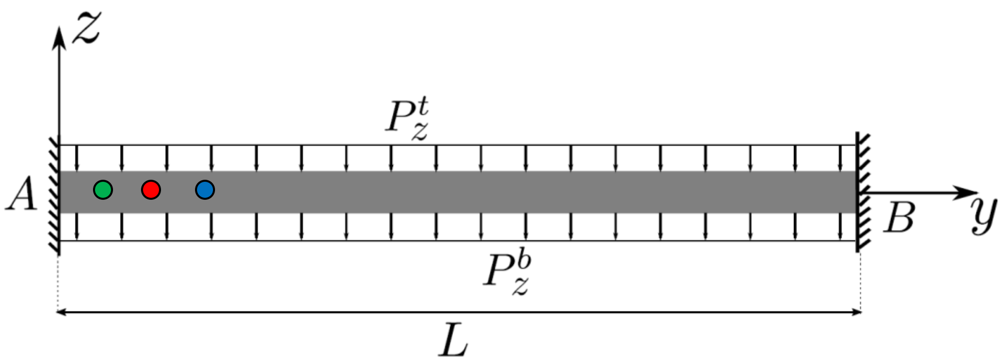

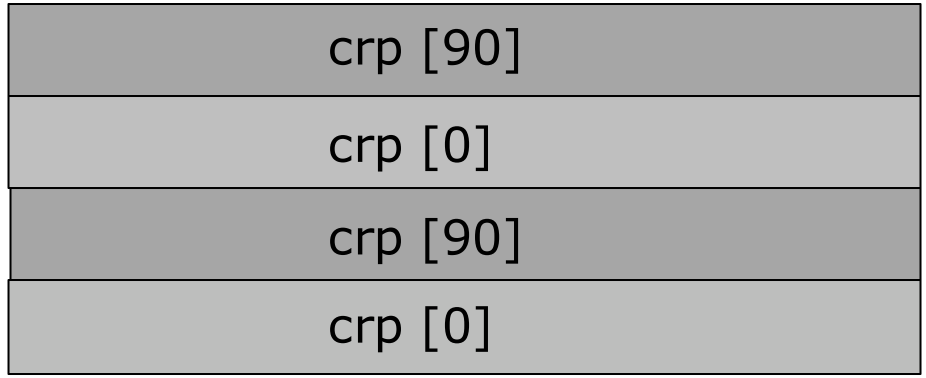

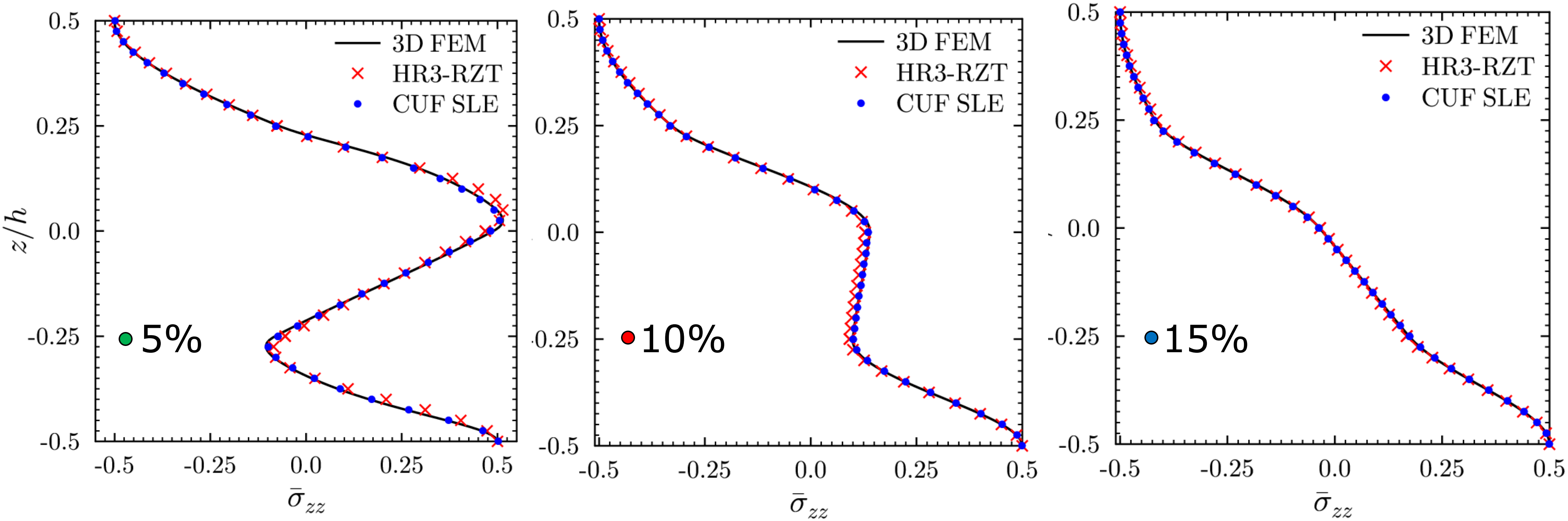

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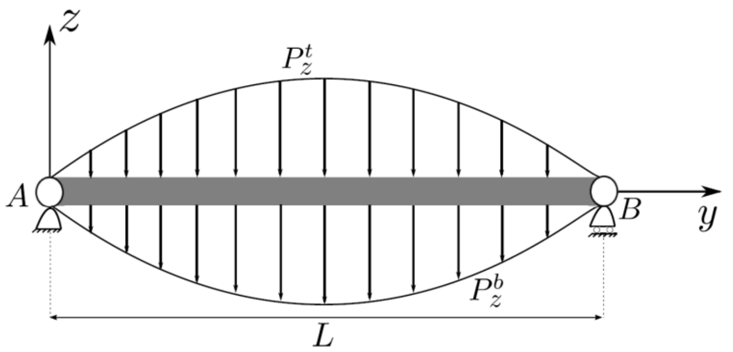

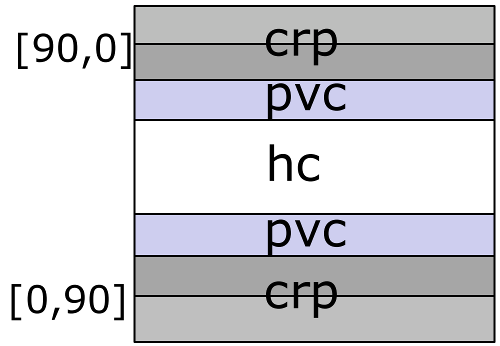

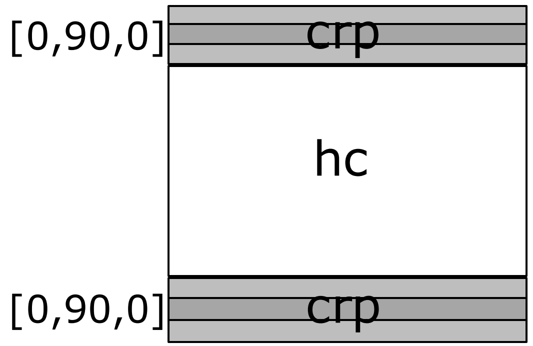

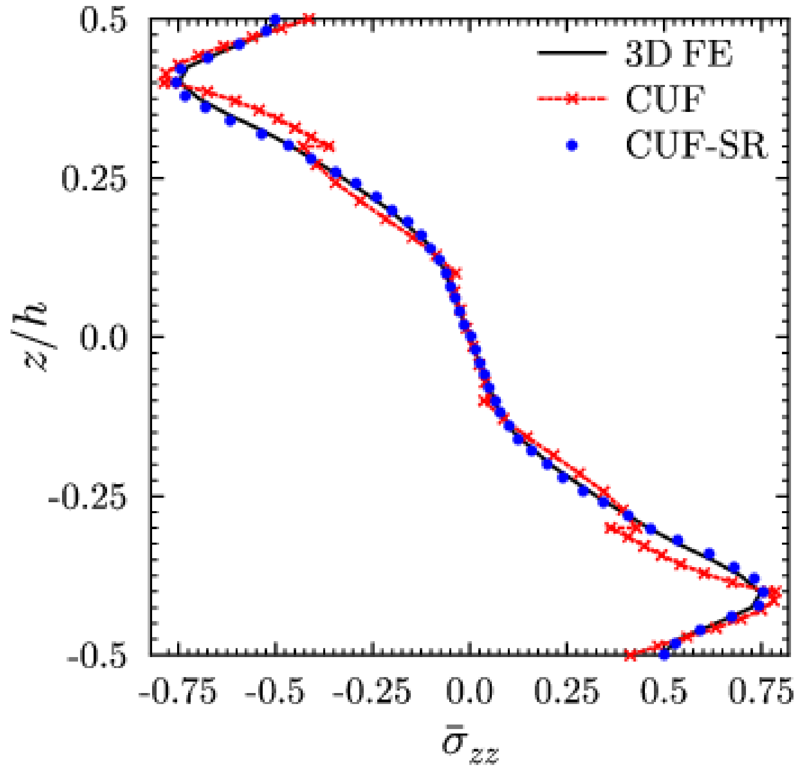

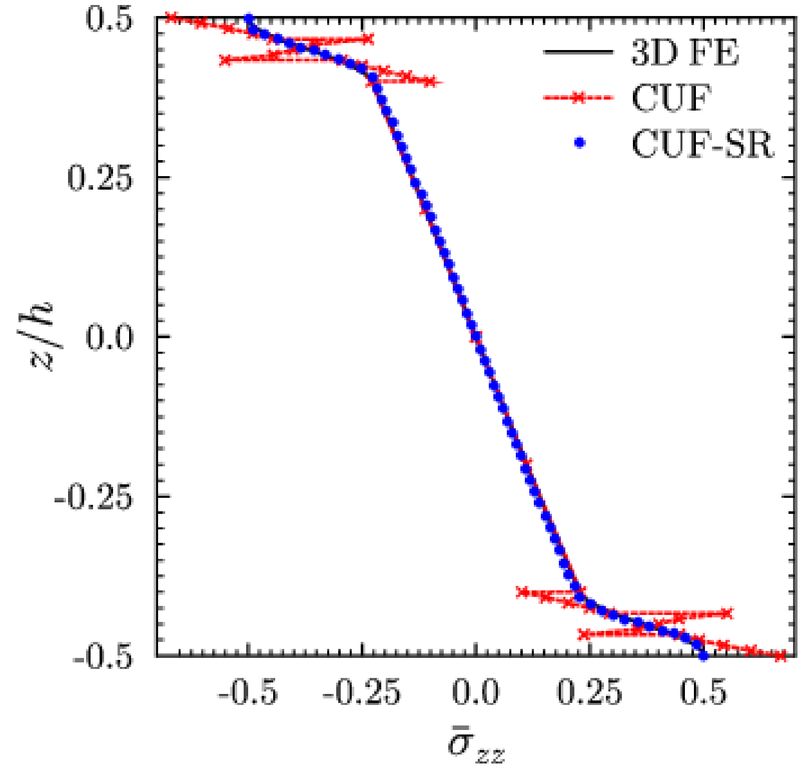

Conclusions

- CUF-SLE model predicts 3D stress fields accurately and the results are in excellent agreement with Pagano's 3D elasticity solution.
- The present approach being displacement-based, to ensure the accurate prediction of transverse stresses, a posteriori stress recovery step is employed.
- The present approach has significant benefits, over the mixed formulation (HR3-RZT) as it is more general in terms of the variety of structure mechanics problems that can be solved.
- The proposed model accurately predicts the boundary layer effects that arise due to local variations in the 3D stresses towards clamped ends.
- The combination of accuracy and computational expense makes the Unified formulation, based on Serendipity Lagrange expansion model, an attractive basis for industrial design tools.

Acknowledgement

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For more details visit: www.fullcomp.net

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