Node-Dependent Kinematic Finite Element Models With Legendre Polynomial Expansions for the Analysis of Composite Beams

G. Li, A.G. de Miguel, A. Pagani, E. Zappino, and E. Carrera

Department of Mechanical and Aerospace Engineering
Politecnico di Torino

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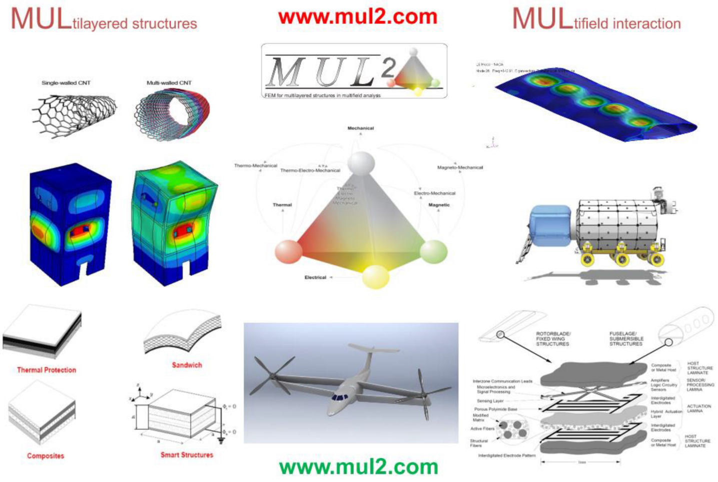
MUL^2 - MULtilayered structures & MULtifield effects



Overview

1. Carrera Unified Formulation (CUF) for refined 1D models;
2. Node-dependent kinematic 1D FEM models;
3. Hierarchical Legendre Expansions as section functions of refined 1D models;
4. Numerical examples;
5. Conclusions.

How CUF Works: A Higher-order Beam Theory

– Displacement description

$$\begin{cases} u = F_1 u_1 & +F_2 u_2 & +F_3 u_3 & +F_4 u_4 & +F_5 u_5 & +F_6 u_6 + \cdots \\ v = F_1 v_1 & +F_2 v_2 & +F_3 v_3 & +F_4 v_4 & +F_5 v_5 & +F_6 v_6 + \cdots \\ w = F_1 w_1 & +F_2 w_2 & +F_3 w_3 & +F_4 w_4 & +F_5 w_5 & +F_6 w_6 + \cdots \end{cases}$$

– F_τ

$$F_1 = 1,$$
$$F_2 = x, \quad F_3 = z,$$
$$F_4 = x^2, \quad F_5 = xz, \quad F_6 = z^2,$$
$$\ldots$$

– FEM discretization

$$\boldsymbol{u}(x,y,z) = N_i(y)\boldsymbol{u}_i(x,z) = N_i(y)F_\tau(x,z)\boldsymbol{U}_{i\tau}$$

– PVD

$$\boldsymbol{u}(x,y,z) = N_i(y)F_\tau(x,z)\,\boldsymbol{U}_{i\tau} \qquad \delta\boldsymbol{u}(x,y,z) = N_j(y)F_s(x,z)\,\delta\boldsymbol{U}_{js}$$

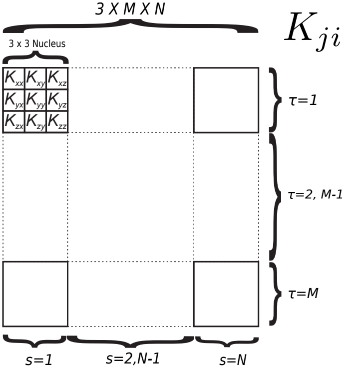
$$\delta L_{int} = \int_V \delta\boldsymbol{\varepsilon}^T\boldsymbol{\sigma}dV = \int_V \delta\boldsymbol{u}^T\boldsymbol{b}^T\boldsymbol{C}\boldsymbol{b}\boldsymbol{u}dV$$

$$= \delta\boldsymbol{U}_{js}^T \cdot \int_V F_s N_j \boldsymbol{b}^T\boldsymbol{C}\boldsymbol{b} N_i F_\tau dV \cdot \boldsymbol{U}_{i\tau} = \delta\boldsymbol{U}_{js}^T \cdot \boldsymbol{K}_{ij\tau s} \cdot \boldsymbol{U}_{i\tau}$$

$$\delta L_{ext} = \int_V \delta\boldsymbol{u}^T\boldsymbol{p}dV = \delta\boldsymbol{U}_{js}^T \cdot \int_V N_j F_s \boldsymbol{p}dV = \delta\boldsymbol{U}_{js}^T \cdot \boldsymbol{P}_{js}$$

$$\boldsymbol{K}_{ij\tau s} = \int_V N_j F_s \boldsymbol{b}^T\boldsymbol{C}\boldsymbol{b}F_\tau N_i dV$$

$\boldsymbol{K}_{ij\tau s}$: Fundamental Nucleus

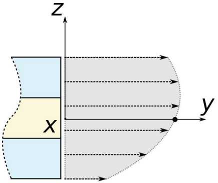


Section Functions for Refined 1D Models

$$\boldsymbol{u}(x,y,z) = \{u,v,w\}^T = F_\tau(x,z)\boldsymbol{u}_\tau(y)$$

ESL model adopting Taylor Expansions (TE)

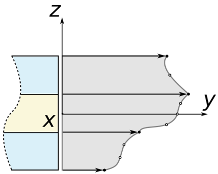
$$\boldsymbol{u}(x,y,z) = \sum_{\tau=1}^{N} x^{i_\tau} z^{j_\tau} \boldsymbol{u}_\tau(y)$$



Note:
– F_τ are defined on the whole cross-section domain;
– DOFs: mathematical weighting factors.

LW model employing Lagrange Expansions (LE)

$$\boldsymbol{u}^k(x,y,z) = \sum_{\tau=1}^{N} L_\tau(x,z)\boldsymbol{u}^k_\tau(y)$$



Note:
– F^k_τ defined on each layer section domain;
– All DOFs are physically meaningful.

Carrera, E., Cinefra, M., Petrolo, M. and Zappino, E.
Finite element analysis of structures through unified formulation.
John Wiley & Sons, 2014.

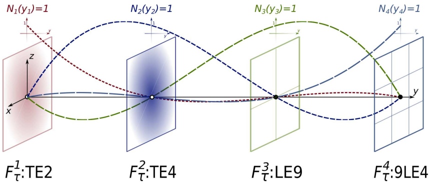
Beam Elements with Node-Dependent Kinematics (NDK)

– 1D Models with NDK

$$\boldsymbol{u} = F_\tau(x,z)N_i(y)\boldsymbol{u}_{i\tau}$$

$$\Downarrow$$

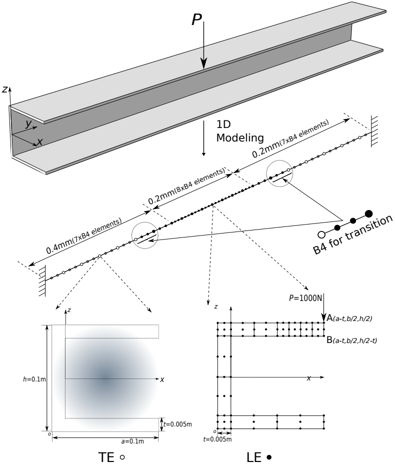
$$\boldsymbol{u} = F^{i}_\tau(x,z)N_i(y)\boldsymbol{u}_{i\tau}$$



A B4 element with NDK.

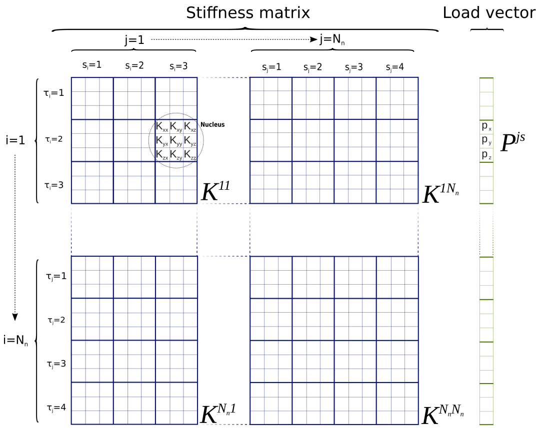
- Variable LW/ESL nodal capabilities;
- Modeling of patches;
- Global-local analysis.

– Application: Local kinematic refinement



Beam Elements with Node-Dependent Kinematics (NDK)

– Assembly approach



Assembly of stiffness matrix for 1D elements adopting NDK

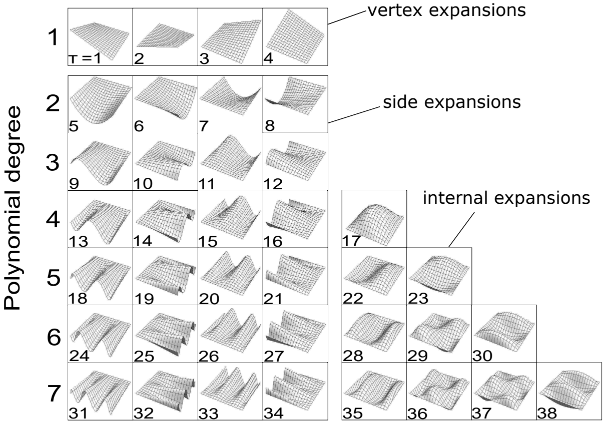
Carrera, E., and E. Zappino.
One-dimensional finite element formulation with node-dependent kinematics.
Computers & Structures 192 (2017): 114-125.

Carrera, E., Filippi, M., Pagani, A. and Zappino, E.
Node-dependent kinematics, refined zig-zag and multi-line beam theories for the analysis of composite structures.
In 58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, p. 0425. 2017.

Zappino, E., Carrera, E. and Li, G.
Free Vibration Analysis of Beams with Piezo-Patches Using a One-Dimensional Model with Node-Dependent Kinematics.
DEMEASS VIII, 2017.

Carrera, E., Cinefra, M., Li, G. and Zappino, E.
Node-Dependent Kinematic One-dimensional FEM Models for the Analysis of Beams with Piezo-patches.
Eccomas, 2017.

Hierarchical Legendre Expansions as section functions of 1D elements



HLE as shape functions for 2D $p-$version elements

Szabó, B. and Babuška, I.
Finite Element Analysis.
John Wiley & Sons, 1991.

Szabó, B., Düster, A. and Rank, E.
The p-version of the finite element method.
Encyclopedia of computational mechanics (2004).

Pagani, A., De Miguel, A.G., Petrolo, M. and Carrera, E.
Analysis of laminated beams via Unified Formulation and Legendre polynomial expansions.
Composite Structures, 156(2016), pp.78-92.

Carrera, E., De Miguel, A.G. and Pagani, A.
Hierarchical theories of structures based on Legendre polynomial expansions with finite element applications.
International Journal of Mechanical Sciences 120 (2017): 286-300.

Hierarchical Legendre Expansions as section functions of 1D elements

- Vertex expansions:

$$F_i(r,s) = \frac{1}{4}(1 - r_i r)(1 - s_i s) \qquad \tau = 1, 2, 3, 4$$

- Side expansions:

$$F_\tau(r,s) = \frac{1}{2}(1 - s)\phi_p(r) \qquad \tau = 5, 9, 13, 18, \cdots$$

$$F_\tau(r,s) = \frac{1}{2}(1 + r)\phi_p(s) \qquad \tau = 6, 10, 14, 19, \cdots$$

$$F_\tau(r,s) = \frac{1}{2}(1 + s)\phi_p(r) \qquad \tau = 7, 11, 15, 20, \cdots$$

$$F_\tau(r,s) = \frac{1}{2}(1 - r)\phi_p(s) \qquad \tau = 8, 14, 16, 21, \cdots$$

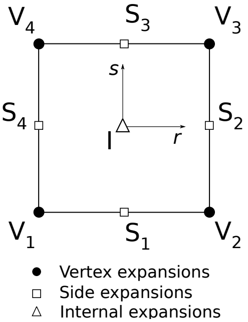
- Internal expansions:

$$F_\tau(r,s) = \phi_p(r)\phi_q(r) \qquad p, q \geqslant 4$$

- *Basis functions:

$$\phi_p(s) = \sqrt{\frac{2p-1}{2}} \int_{-1}^{s} L_{p-1}(x)dx = \frac{L_p(s) - L_{p-2}(s)}{\sqrt{4p-2}} \qquad p = 2, 3, \cdots$$

Note: $L_p(s)$ are Legendre polynomials.

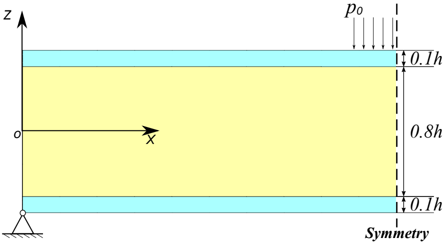


Szabó, B., Düster, A. and Rank, E.
The *p*-version of the finite element method.
Encyclopedia of computational mechanics (2004).

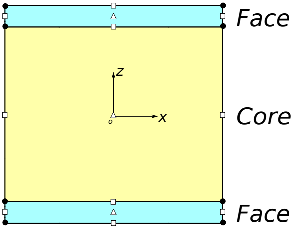
Simply supported sandwich beam under local pressure

Material properties used on the sandwich plate, $a/h = 5$

	E_{11}[GPa]	E_{22}[GPa]	E_{33}[GPa]	ν_{12}	ν_{13}	ν_{23}	G_{12}[GPa]	G_{13}[GPa]	G_{23}[GPa]
Face	131.1	6.9	6.9	0.32	0.32	0.49	3.588	3.088	2.3322
Core	0.2208×10^{-3}	0.2001×10^{-3}	2.76	0.99	0.00003	0.00003	16.56×10^{-3}	0.5451	0.4554



Geometry and loading of the sandwich beam



Mesh on the cross-section

Wenzel, C., P. Vidal, M. DÒttavio, and O. Polit.
Coupling of heterogeneous kinematics and Finite Element approximations applied to composite beam structures.
Composite Structures 116 (2014): 177-192.

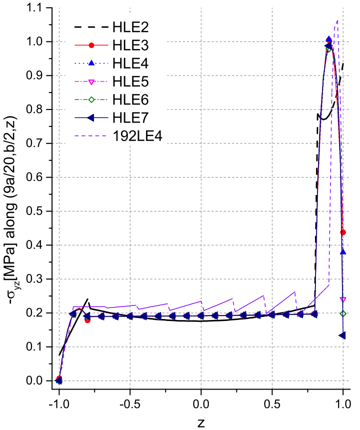
Simply supported sandwich beam under local pressure

Stress evaluation on the sandwich beam with 20 B4 elements

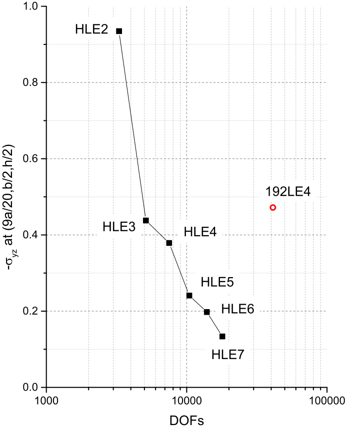
Kinematics	$-\sigma_{yy}$[MPa] $(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$-\sigma_{yz}$[MPa] $(\frac{9a}{20},\frac{b}{2},\frac{9h}{20})$	$-\sigma_{zz}$[MPa] $(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	DOFs
HLE2	18.05	0.7823	1.020	3294
HLE3	18.78	1.005	1.039	5124
HLE4	18.75	1.007	0.9913	7503
HLE5	18.72	0.9787	0.9858	10431
HLE6	18.73	0.9786	0.9838	13908
HLE7	18.73	1.015	0.9828	17934
192LE4	22.77	0.2823	0.4720	41175
Zappino et al. 2D	18.15	–	0.9989	37479
ABAQUS 3D	18.15	1.009	1.001	620787

Zappino, E., Li, G., Pagani, A. and Carrera, E.
Global-local analysis of laminated plates by node-dependent kinematic finite elements with variable ESL/LW capabilities.
Composite Structures 172 (2017): 1-14.

Simply supported sandwich beam under local pressure

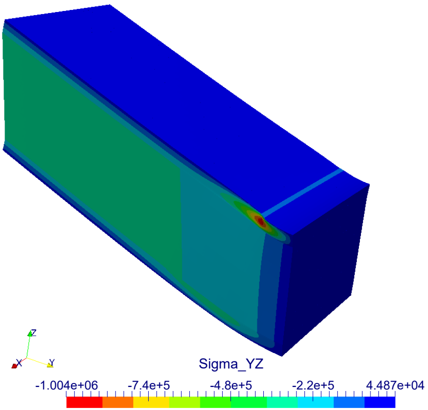


Variation of σ_{yz} along $(\frac{9a}{20}, \frac{b}{2}, z)$

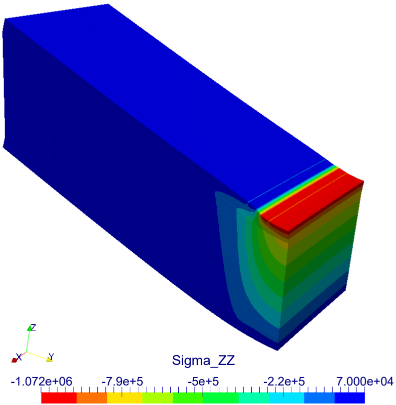


DOFs vs σ_{yz} on the top surface

Simply supported sandwich beam under local pressure

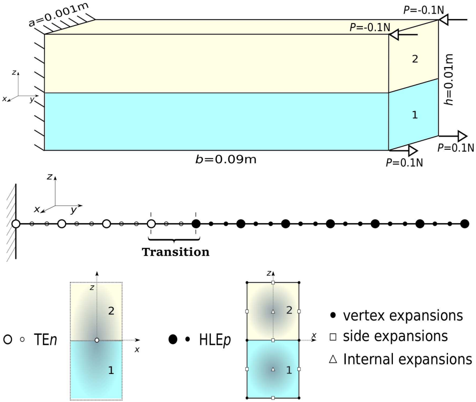


Contour plot of σ_{yz}



Contour plot of σ_{zz}

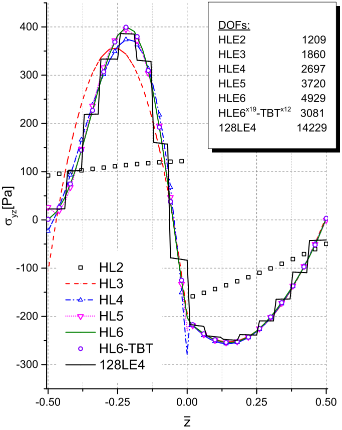
A two-layered beam



Loading and geometry (not to scale)

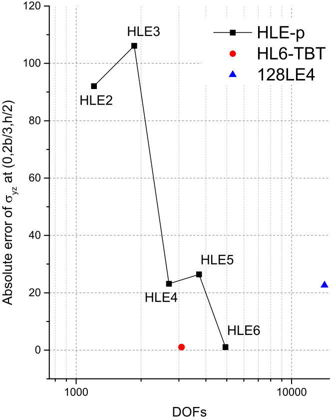
Carrera, E., Filippi, M., Mahato, P.K. and Pagani, A.
Accurate static response of single-and multi-cell laminated box beams.
Composite Structures 136 (2016): 372-383.

A two-layered beam



Variation of σ_{yz} *along* $(0, \frac{2b}{3}, \bar{z})$

Note: TBT – Timoshenko Beam Theory



DOFs vs absolute error of σ_{yz} *at* $(0, \frac{2b}{3}, -\frac{h}{2})$

Conclusions

1. Node-dependent kinematics provide a solution to integrate the accuracy of the LW models and the low computational cost of the ESL models to obtain optimal beam models;
2. Hierarchical Legendre Expansions are extended to one-dimensional node-dependent kinematic finite elements;
3. Based on CUF, the proposed models are compact in form without using any *ad hoc* coupling;
4. The proposed models are computationally efficient in the analysis of multi-layered slender structures;
5. The presented approach allows the local kinematic refinement be carried out without modifying the FEM mesh.

Acknowledgment

- FULLComp: FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures
- Funded by the European Commission under a *Marie Sklodowska-Curie* Innovative Training Networks grant for European Training Networks (Grant agreement No. 642121).
- The full spectrum of the design of composite structures will be dealt with, such as manufacturing, health-monitoring, failure, modeling, multiscale approaches, testing, prognosis, and prognostic.
- Research activities are aimed at engineering fields such as aeronautics, automotive, mechanical, wind energy and space.
- www.fullcomp.net

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