A Novel-FE2 Method Based Fourier Macroscopic Model for Instability Phenomena of Long Fiber Reinforced Composites

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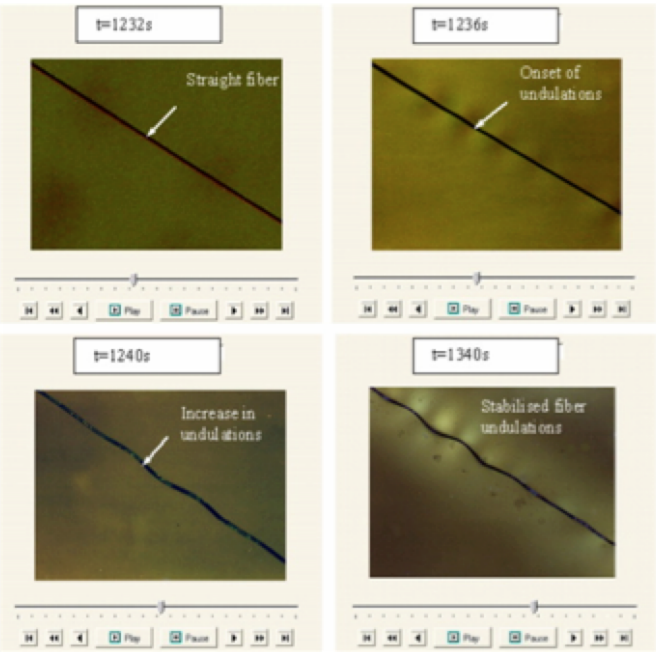
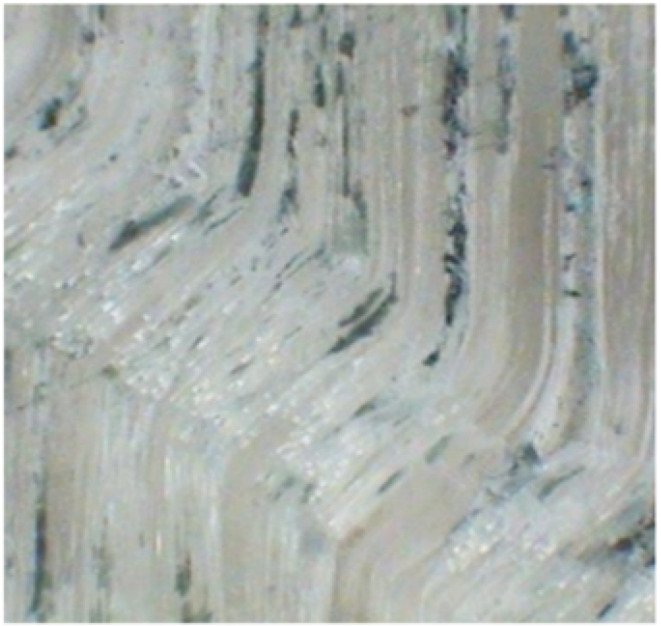
Compressive failure of long-fiber reinforced composites





Drapier et al., IJSS, 2007.

Research Methods:

1. FE2 Method
2. Asymptotic Numerical Method
3. Techniques of Fourier series with slowly varying coefficients

1. Kousnetzova et al., IJNME, 2002.
2. Damil et al., JMPS, 2010.
3. Cochelin et al., IJNME, 1994.
4. Liu et al., IJSS, 2012.

Finite Element Square Method

1. Capture the microscopic information compared to the classical homogenization technique.
2. Save lots of DOFs compared to the full microscopic model.
3. Still heavy because each Gauss point needs a RVE whose mesh should be fine enough to describe the local details, e.g., micro-buckling.
4. Difficult to pilot the non-linear calculations when there exist lots of bifurcation curves around the useful one, often requires special imperfection.

Fourier-related envelope model

1. Transform the fast oscillation unknown fields to slowly variable fields.
2. Improve computational efficiency.
3. Make it easy to pilot the non-linear analysis.

Asymptotic Numerical Method

1. Inverse one stiffness matrix in one non-linear step.
2. Very efficient in the case of strong non-linear calculations.

Research Methods:

1. **FE2 Method**
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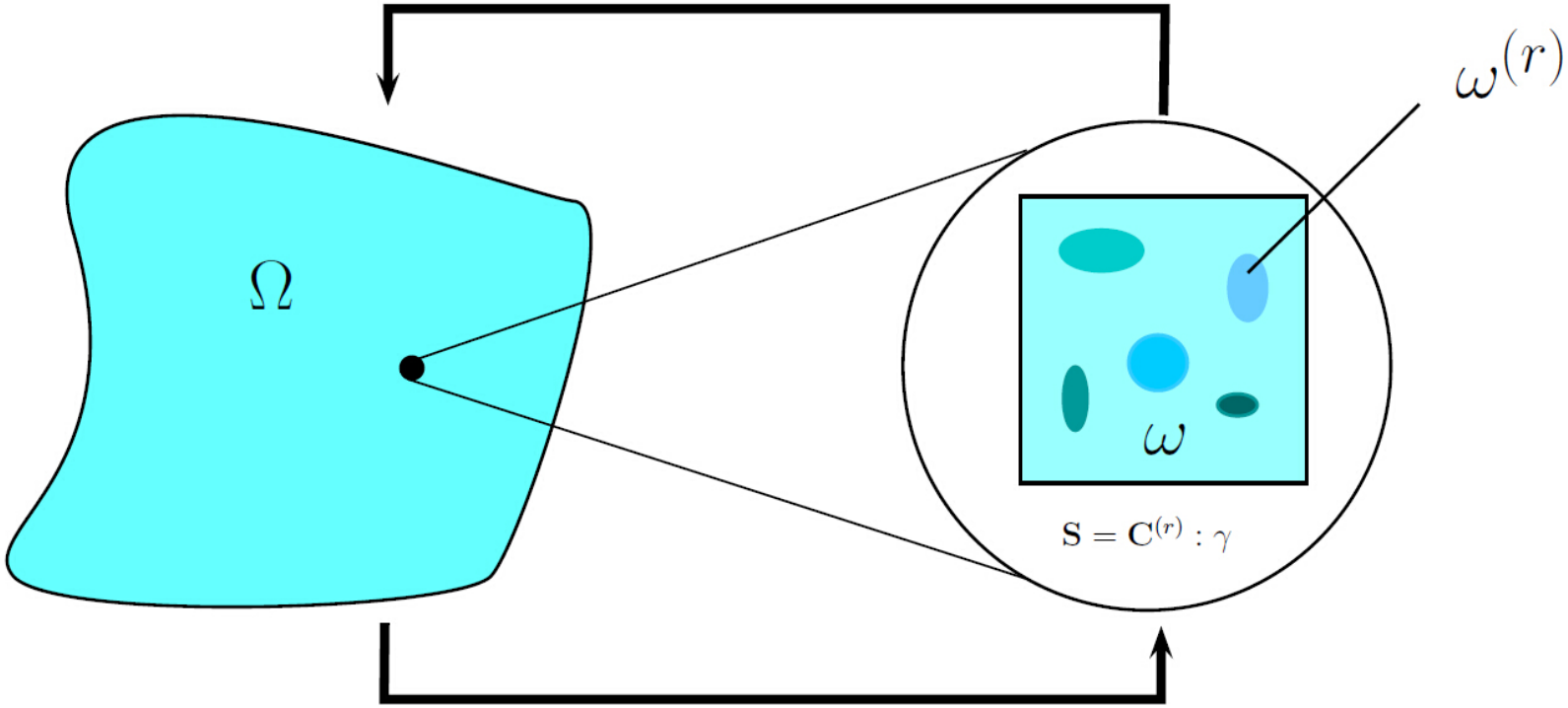
Exchange Information Between Two Scales

$$\overline{\mathbf{P}} = \frac{1}{\omega}\int_\omega \mathbf{P}(\mathbf{X})d\omega$$



$$\overline{\mathbf{F}} = \frac{1}{\omega}\int_\omega \mathbf{F}(\mathbf{X})d\omega$$

$$\mathbf{u}^+ - \mathbf{u}^- = (\overline{\mathbf{F}} - \mathbf{I}) \cdot (\mathbf{X}^+ - \mathbf{X}^-) \; on \; \partial\omega$$

Nezamabadi S., Potier-Ferry M., Zahrouni H., Yvonnet J., 2015. Compos. Struct.

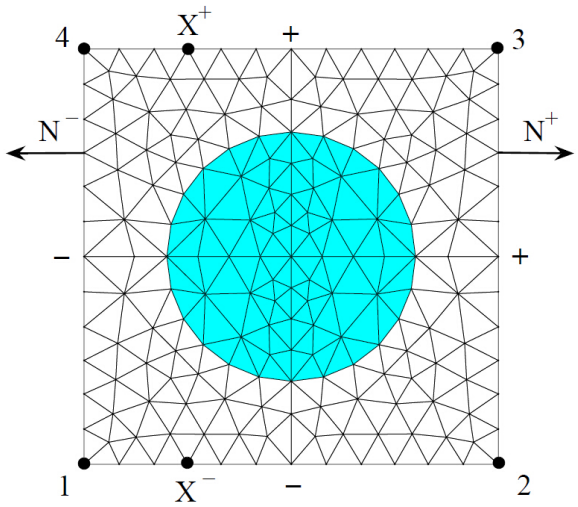
Macroscopic

$$\begin{cases} \int_{\Omega} \overline{\mathbf{P}} : \delta\overline{\mathbf{F}}\, d\Omega = \int_{\partial\Omega_{\mathbf{f}}} \mathbf{f} \cdot \delta\overline{\mathbf{u}}\ d\Gamma \\ \overline{\mathbf{S}} = \overline{\mathbf{C}} : \overline{\gamma} \\ \overline{\mathbf{P}} = \overline{\mathbf{F}} \cdot \overline{\mathbf{S}} \\ \overline{\gamma} = \frac{1}{2}(\overline{\mathbf{F}}^t \cdot \overline{\mathbf{F}} - \mathbf{I}) \\ \overline{\mathbf{F}} = \mathbf{I} + \nabla\overline{\mathbf{u}} \\ + \text{B.C} \end{cases}$$

Microscopic

$$\begin{cases} \int_{\omega} \mathbf{P} : \delta\mathbf{F}\, d\omega = \mathbf{0} \\ \mathbf{S} = \mathbf{C}^{(\mathbf{r})} : \gamma \\ \mathbf{P} = \mathbf{F} \cdot \mathbf{S} \\ \gamma = \frac{1}{2}(\mathbf{F}^t \cdot \mathbf{F} - \mathbf{I}) \\ \mathbf{F} = \mathbf{I} + \nabla\mathbf{u} \end{cases}$$

$\mathbf{u}^+ - \mathbf{u}^- = (\overline{\mathbf{F}} - \mathbf{I}) \cdot (\mathbf{X}^+ - \mathbf{X}^-)$ on $\partial\omega$



Marco to Micro

$$\begin{cases} \mathbf{u} = \mathbf{A} : \overline{\mathbf{F}} \\ \mathbf{F} = \mathbf{A}_{,\mathbf{X}} : \overline{\mathbf{F}} \end{cases}$$

Micro to Marco

$$\begin{cases} \mathbf{P} = \mathbf{F} \cdot \mathbf{S} = \mathbf{H}^r : \mathbf{F} = \mathbf{H}^r : \mathbf{A}_{,\mathbf{X}} : \overline{\mathbf{F}} = \mathbf{L} : \overline{\mathbf{F}} \\ \overline{\mathbf{P}} = \dfrac{1}{\omega} \displaystyle\int_\omega \mathbf{P}(\mathbf{X}) d\omega = \dfrac{1}{\omega} \displaystyle\int_\omega \mathbf{L} : \overline{\mathbf{F}} d\omega = \overline{\mathbf{L}} : \overline{\mathbf{F}} \end{cases}$$

u can be divided into four parts for $2D$ problems

$$\mathbf{u} = \overline{\mathbf{F}}^{(11)}\widetilde{\mathbf{u}}^{(11)} + \overline{\mathbf{F}}^{(12)}\widetilde{\mathbf{u}}^{(12)} + \overline{\mathbf{F}}^{(21)}\widetilde{\mathbf{u}}^{(21)} + \overline{\mathbf{F}}^{(22)}\widetilde{\mathbf{u}}^{(22)}$$

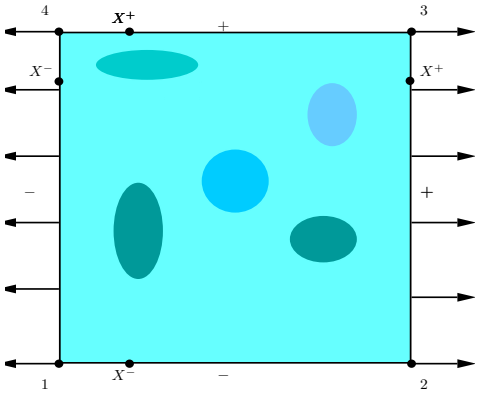
$\widetilde{u}$ can be given by four kinds of boundary conditions:

$$\begin{cases} \mathscr{L}(\widetilde{\mathbf{u}}^{(ij)}, \delta\mathbf{u}) = 0, in\ \omega. \\ \widetilde{\mathbf{u}}^{(ij)+} - \widetilde{\mathbf{u}}^{(ij)-} = \mathbf{X}^{(ij)+} - \mathbf{X}^{(ij)-}, on\ \partial\omega. \end{cases}$$

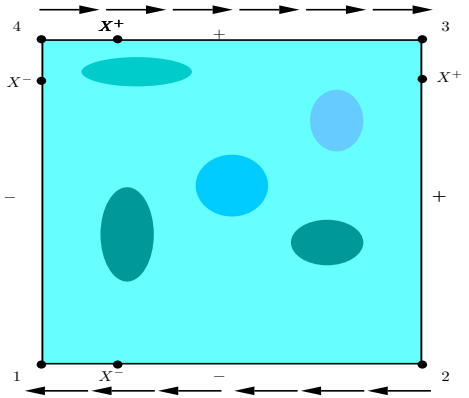
$$\mathbf{X}^{(11)} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} (\mathbf{X}^+ - \mathbf{X}^-) \qquad \mathbf{X}^{(12)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} (\mathbf{X}^+ - \mathbf{X}^-)$$

$$\mathbf{X}^{(21)} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} (\mathbf{X}^+ - \mathbf{X}^-) \qquad \mathbf{X}^{(22)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} (\mathbf{X}^+ - \mathbf{X}^-)$$

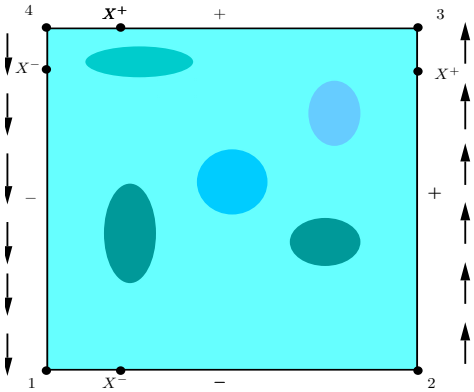
Exchange Information Between Two Scales



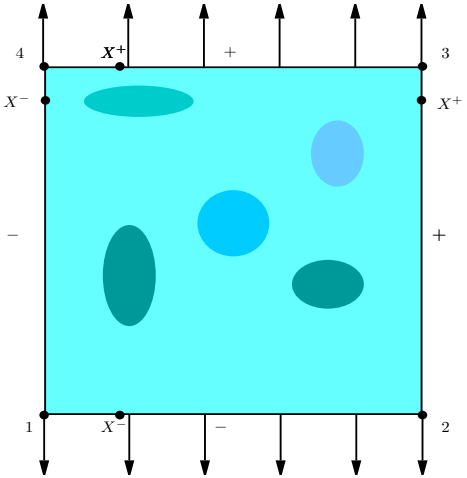
$$\mathbf{x}^{(11)} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} (\mathbf{x}^+ - \mathbf{x}^-)$$



$$\mathbf{x}^{(12)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} (\mathbf{x}^+ - \mathbf{x}^-)$$



$$\mathbf{x}^{(21)} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} (\mathbf{x}^+ - \mathbf{x}^-)$$



$$\mathbf{x}^{(22)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} (\mathbf{x}^+ - \mathbf{x}^-)$$

Research Methods:

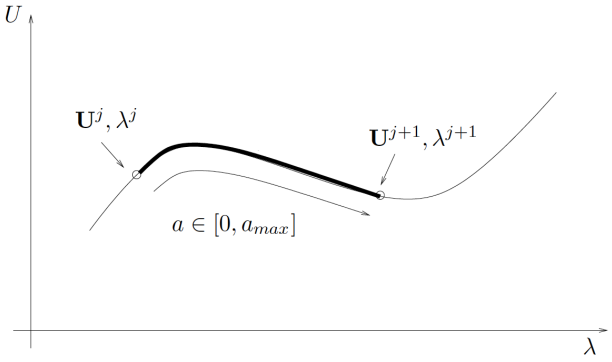
1. FE2 Method
2. **Asymptotic Numerical Method**
3. Techniques of Fourier series with slowly varying coefficients

Solver for Nonlinear Problem

Adapt the asymptotic numerical method (ANM) to solve the nonlinear equations.

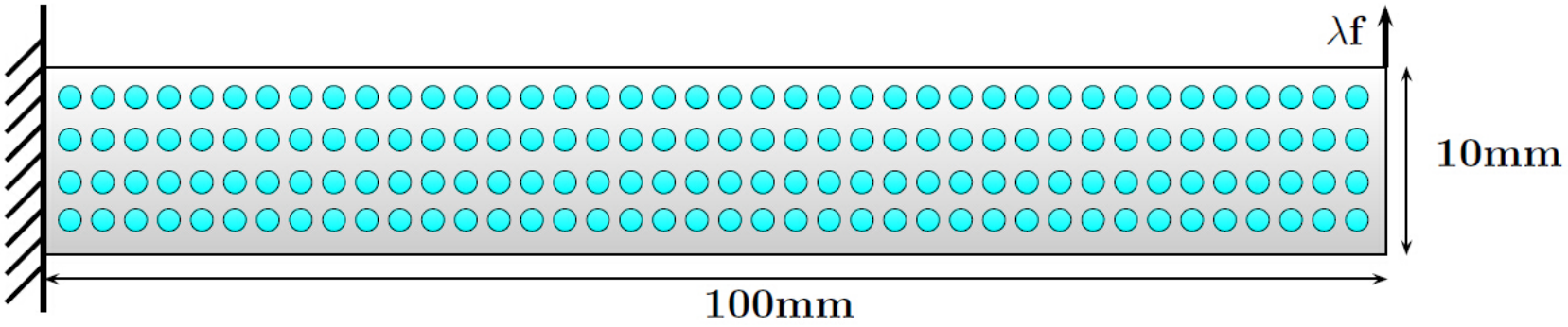
Strengths: 1. Every step of iteration to solve one inversion

2. Step length self adaption



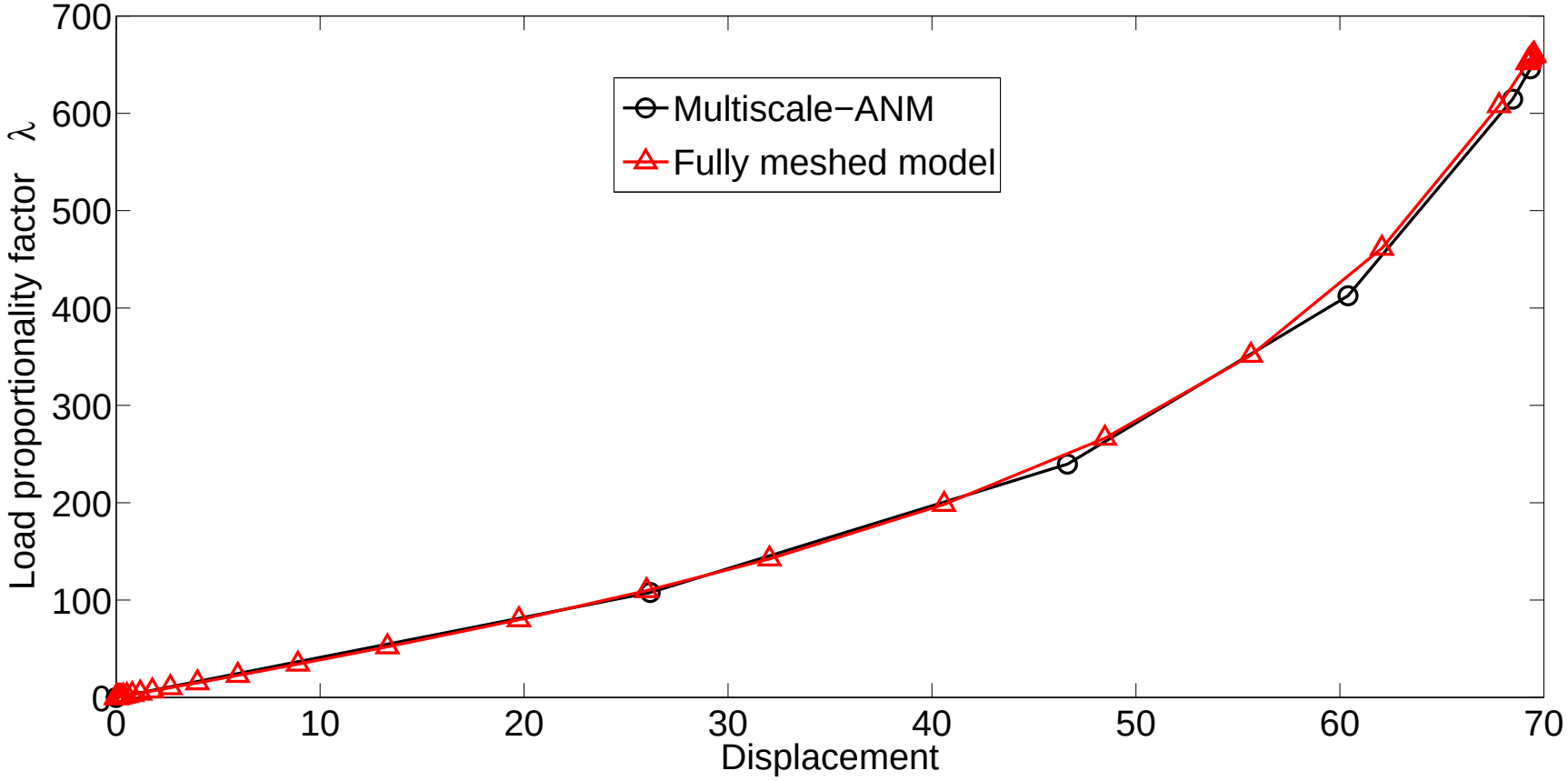
$$\begin{cases} \mathbf{U}^{j+1} = \mathbf{U}^j + a^p \mathbf{U}_p \\ \lambda^{j+1} = \lambda^j + a^p \lambda_p \end{cases} \quad p \in [1, n]$$

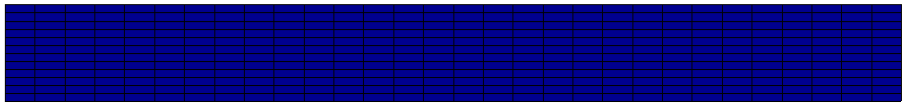
Cochelin B, Damil N, Potier-Ferry M.2007.

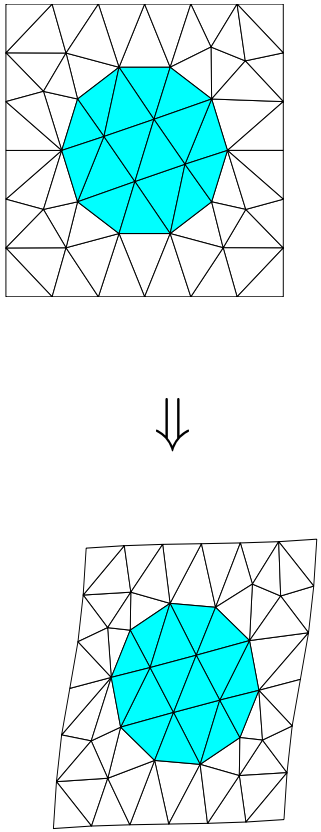

λf
10mm
100mm

$\mathbf{FE}^2$method, DOF: 177012

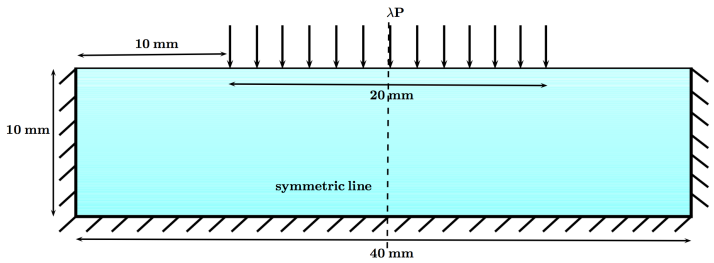
Fully meshed model, DOF: 910810

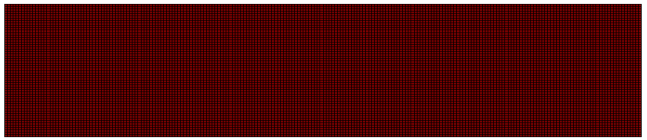


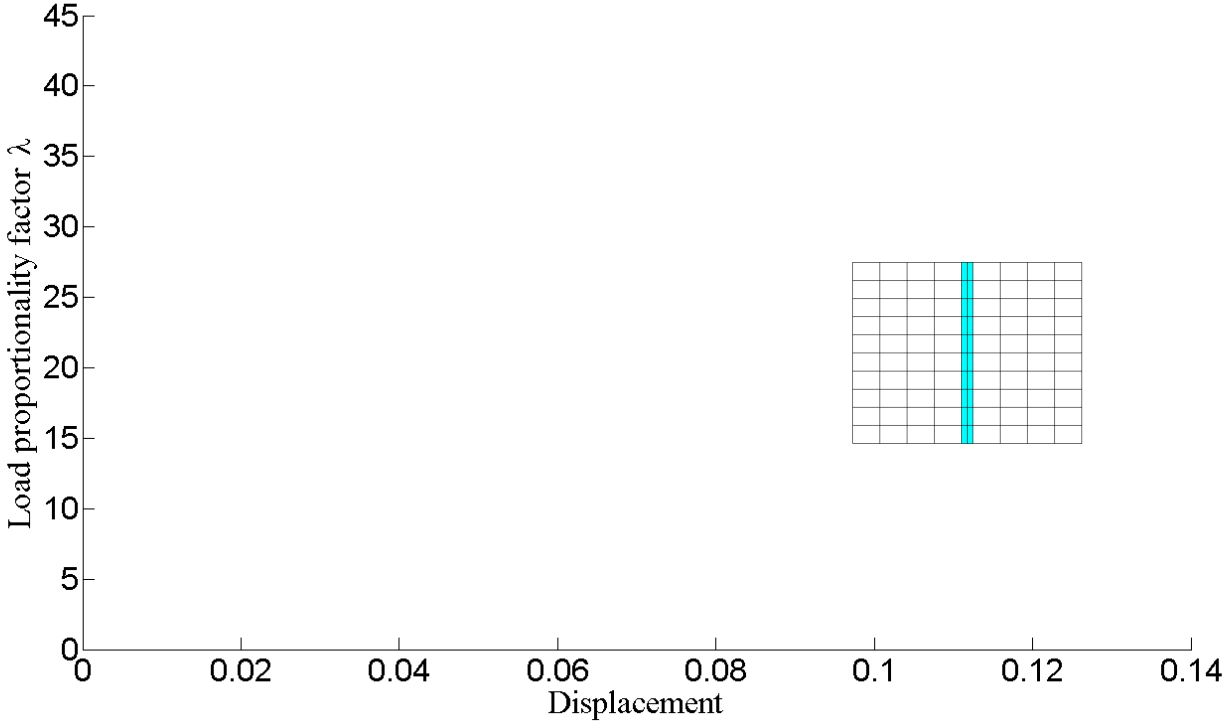




Micro-buckling of long fiber reinforced composites







Research Methods:

1. FE2 Method
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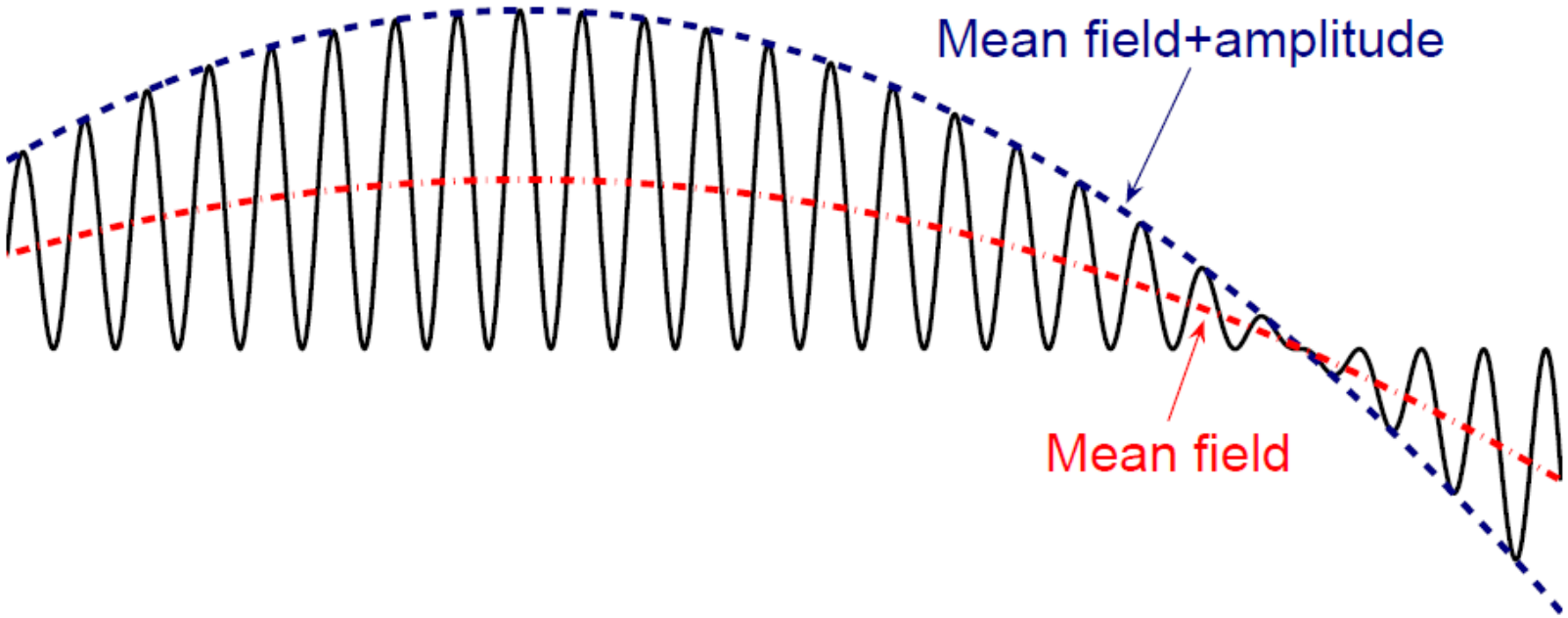
3. **Techniques of Fourier series with slowly varying coefficients**

Aim

Build a model coupling the mean field and the envelope of the oscillation.



Damil et al., JMPS, 2010

Principle

Fourier series with slowly varying coefficients

$$U(x) = \sum_{j=-\infty}^{+\infty} U_j(x) e^{jiqx}$$

$U(x)$ including the displacement vector, its gradient, strain and stress tensors, is expanded in the form of a Fourier series. The wave number q is defined as $q = \frac{\pi}{L} q_0$, in which, L is the length of the RVE (fiber expansion axis), and q_0 is a chosen wave length ($q_0 \in N, q_0 \geq 2$). The $U_j(X)$ represents the envelope for the $j^{th}(j = 0, 1...)$ order harmonic, vary more slowly than the harmonic function over a period $[x, x + \frac{2\pi}{q}]$, here we keep the order j up to 1.

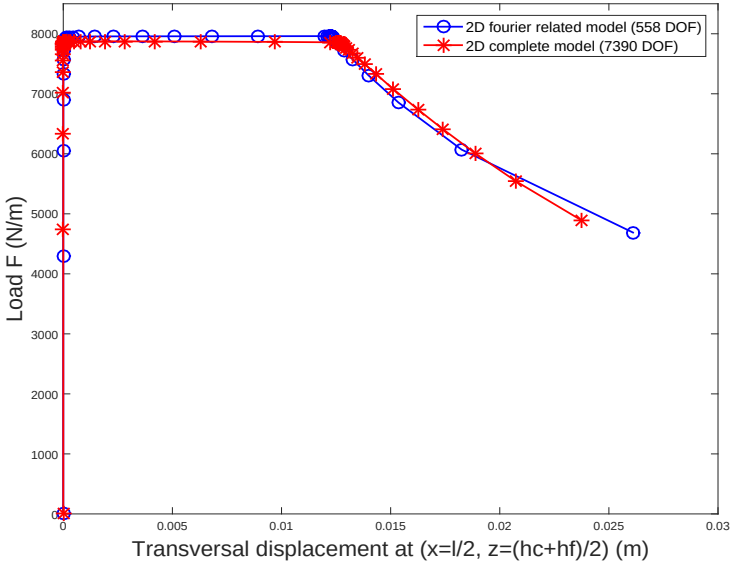
Virtual work and constitutive law of the RVE

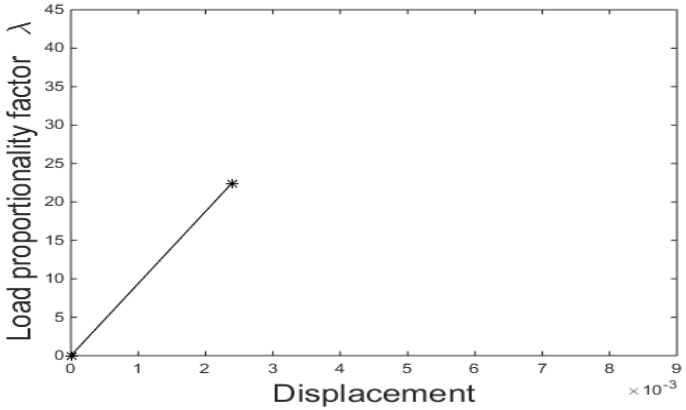
$$\begin{cases} \delta P_{int} = \int_0^L \sum_{j=-1}^{+1} \{\delta\gamma_{-j}\}\{S_j\}dx \\ = \int_0^L \{\delta\gamma_0\}\{S_0\} + 2\{\delta\gamma_{1R}\}\{S_{1R}\} + 2\{\delta\gamma_{1I}\}\{S_{1I}\}dx \\ [D]^{-1}\{S_0\} = \{\gamma_0\} = [H]\{\theta_0\} + \frac{1}{2}[A_0]\{\theta_0\} + [A_{1R}]\{\theta_{1R}\} + [A_{1I}]\{\theta_{1I}\} \\ [D]^{-1}\{S_{1R}\} = \{\gamma_{1R}\} = [H]\{\theta_{1R}\} + [A_0]\{\theta_{1R}\} \\ [D]^{-1}\{S_{1I}\} = \{\gamma_{1I}\} = [H]\{\theta_{1I}\} + [A_0]\{\theta_{1I}\} \end{cases}$$

For the microscopic scale, the resolution of the equations below is performed by using Asymptotic Numerical Method:

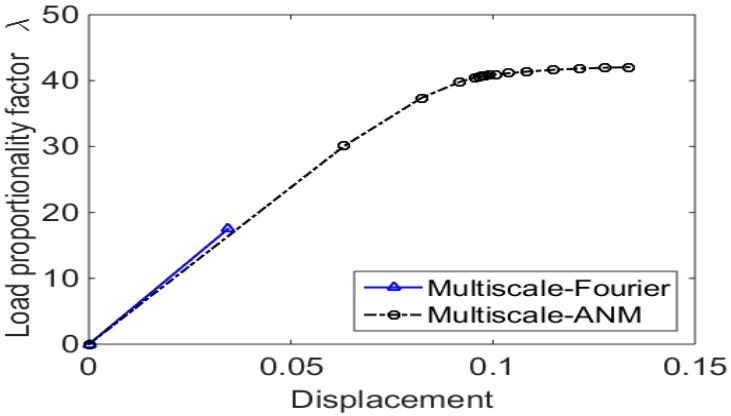
$$\begin{cases} \int_\omega {}^t\{\delta\gamma_{gen}\}\{S_{gen}\}d\omega = \lambda \int_{\partial\omega} {}^t\{\delta u\}\{F\} \\ \{\gamma_{gen}\} = [H_{gen}]\{\theta_{gen}\} + \frac{1}{2}[A(\theta_{gen})]\{\theta_{gen}\} \\ \{S_{gen}\} = [D_{gen}]\{\gamma_{gen}\} \end{cases}$$

Global-local coupling instability of sandwich composite beam, 2D fourier based model (558 DOFs) VS 2D complete model (7390 DOFs)

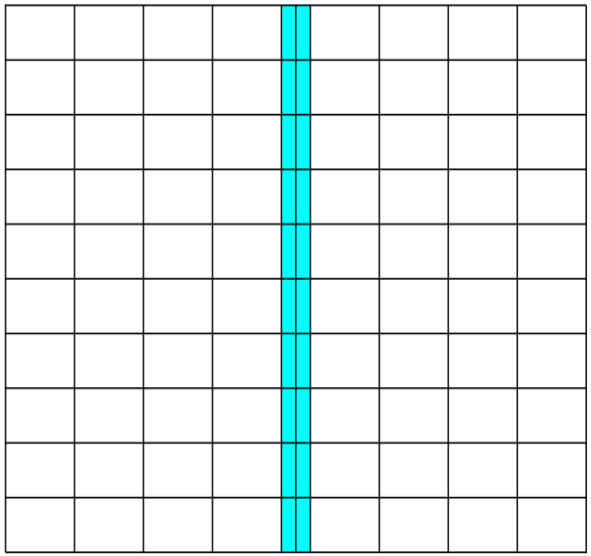




Microscopic Bifurcation Curve (RVE)



Macroscopic Nonlinear Response
(RVE: 198 DOFs VS 640 DOFs)



Conclusions:

1. Macroscopic compressive failure comes from the microscopic fiber buckling.
2. Multiscale model based on FE2 theory meet a good agreement with the results of commercial finite element codes, which shows that accurate results can be obtained with a reduced computational cost.
3. Multiscale model based on FE2 theory and Fourier slowly varying technique can detect the bifucation of the microscale.

Acknowledgements

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