PROGRESSIVE DAMAGE ANALYSIS OF COMPOSITE STRUCTURES VIA ONE-DIMENSIONAL CARRERA UNIFIED FORMULATIONS

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International Conference on Composite Structures, ICCS19
5-9 September 2016, Porto (Portugal)
MUL2 - Our Research Group

Marie Curie Project on Composites

www.fullcomp.net
The FULLCOMP project

- FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures

The FULLCOMP project is funded by the European Commission under a Marie Sklodowska-Curie Innovative Training Networks grant for European Training Networks (ETN).

The FULLCOMP partners are:

1. Politecnico di Torino (Italy) - Coordinator
2. University of Bristol (UK)
3. Ecole Nationale Superieure d’arts et Metiers (Bordeaux, France)
4. Leibniz Universitaet Hannover (Germany)
5. University of Porto (Portugal)
6. University of Washington (USA)
7. RMIT (Australia)
8. Luxembourg Institute of Technology
9. Elan-Ausy, Hamburg, (Germany)

FULLCOMP has recruited 12 PhD students who will work in an international framework to develop integrated analysis tools to improve the design of composite structures.

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www.fullcomp.net
Overview

1. Description of the Carrera Unified Formulation for refined models (CUF).

2. Main 1D CUF capabilities overview (1D Taylor- and Lagrange-based models).


4. Introduction to the Component-Wise approach (CW).

5. Progressive Damage Model

6. Numerical Results
Brief Overview of Beam Refinement Methods and Contributors

1. Shear correction factors (Timoshenko, Sokolnikoff, Cowper, Gruttmann, etc.).

2. Warping functions and Saint-Venant solutions (El Fatmi, Ladéveze, etc.).

3. Variational asymptotic method (Berdichevsky, Hodges, Yu, etc.).

4. Generalized beam theory (Schardt, Camotim, Silvestre, etc.).

5. Higher-order models (Washizu, Reddy, Kapania, Carrera, etc.).
1D Advanced Structural Models

Actual Wing

Our Model

1D Carrera Unified Formulation, CUF - FEM Version

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CARRERA UNIFIED FORMULATION - COMPONENT-WISE 1D MODELS - POLITECNICO DI TORINO (ITALY) - WWW.MUL2.COM
Fundamental Nucleus Equations by CUF

Finite Element Formulation (FEM)

\[ u = F_\tau(x, z) N_i(y) u_{\tau i}(1D) \]

\[ \delta L_{int} = \delta q_{\tau i}^T K_{ij}^{\tau s} q_{sj} \]

\[ \delta L_{line} = \delta q_{\tau i}^T M_{ij}^{\tau s} \ddot{q}_{sj} \]

\[ \delta L_{ext} = P \delta u^T \]

\[ i, j: \text{Shape function indexes (depend on the FE discretization).} \]

\[ \tau, s: \text{Expansion function indexes (depend on the model order).} \]

Assembly Technique

\[ K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_{\Omega} F_{\tau,x} F_{s,x} d\Omega \int_{I} N_i N_j dy + ... \]

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1D CUF Applications Overview - Books

- Thin-Walled and Reinforced Structures
- FGM Structures
- Aeroelasticity
- Biomechanics
- Analysis of Aerospace Structures via the Component-Wise Approach
- Analysis of Civil Structures via the Component-Wise Approach
- Component-Wise Approach for the Multiscale Analyses of Composites

- Buckling, Free Vibration and Dynamic Response Analysis
- Variable Kinematics Models
- Load Factors and Non-Structural Masses
- Multifield Analysis
- Composite Structures
- Axiomatic/Asymptotic Analyses and Best Theory Diagrams
- Rotors and Rotating Blades
- Nanostructures

FEM
2D and Smart Structures
1D

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The Taylor CUF 1D Models, TE

\[ u_x = u_{x_1} + x u_{x_2} + z u_{x_3} 
\]
\[ u_y = u_{y_1} + x u_{y_2} + z u_{y_3} \]
\[ u_z = u_{z_1} + x u_{z_2} + z u_{z_3} \]

Classical models, such as Timoshenko, can be obtained as particular cases of the linear models.

Assembly Technique

\[ K_{ij}^{s\tau} = \tilde{C}_{22} \int_{\Omega} F_{\tau,x} F_{s,x} d\Omega \int l N_i N_j dy + ... \]
Wave propagation in a Thin-Walled Cylinder

E. Carrera, A. Varello,
Dynamic Response of Thin-Walled Structures by Variable Kinematic One-Dimensional Models

N = 8 beam model, 6000 DOFs
Plastic beam under bending load

- Material model: Isotropic von-Mises plasticity
- Ideal elastic-perfect plastic stress-strain relationship
- Plastic strength factor is estimated
- Emphasizes the importance of refined higher order models

<table>
<thead>
<tr>
<th>Type</th>
<th>NDOF</th>
<th>( U_Z ) at yield load</th>
<th>( U_Z ) at limit load</th>
<th>Plastic Strength Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>-</td>
<td>2.080</td>
<td>4.622</td>
<td>1.5</td>
</tr>
<tr>
<td>ABQ - 3D</td>
<td>22,590</td>
<td>2.077</td>
<td>4.281</td>
<td>1.55</td>
</tr>
<tr>
<td>Taylor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EBBM</td>
<td>279</td>
<td>2.083</td>
<td>3.13</td>
<td>32.39</td>
</tr>
<tr>
<td>TBM</td>
<td>279</td>
<td>2.084</td>
<td>3.13</td>
<td>32.38</td>
</tr>
<tr>
<td>N = 1</td>
<td>279</td>
<td>2.084</td>
<td>3.13</td>
<td>32.38</td>
</tr>
<tr>
<td>N = 2</td>
<td>558</td>
<td>2.072</td>
<td>4.21</td>
<td>8.98</td>
</tr>
<tr>
<td>N = 3</td>
<td>930</td>
<td>2.072</td>
<td>4.42</td>
<td>4.80</td>
</tr>
</tbody>
</table>

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Lagrange-Based 1D Models, LE

\[ u_x = L_\tau u_{x_\tau} \]
\[ u_y = L_\tau u_{y_\tau} \]
\[ u_z = L_\tau u_{z_\tau} \]

L9 polynomials - Isoparametric

\[ L_\tau = \frac{1}{4} (r^2 + r_\tau)(s^2 + ...) \]

Cross-Section Elements

3D Geometry from CAD

LE Modeling

Beam element
Beam node
Lagrange node above the first beam node cross-section
Lagrange node above the second beam node cross-section
DOFs: pure displacements of each Lagrange node (3 DOFs per Lagrange node)

Lagrange nodes can be placed above the physical surface of the structure

Computational Model
The Component-Wise Approach

- Only displacements as unknowns.
- Each component of the structure is modeled via beams only.
- No need of reference surfaces. This might be useful in a CAD-FEM interface scenario.
- No need of homogenization techniques.

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Aircraft Wing

Shell-like modal shapes by means of a BEAM MODEL

Natural frequencies [Hz]

<table>
<thead>
<tr>
<th>Mode</th>
<th>1D-LE</th>
<th>SOLID</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.23</td>
<td>4.22</td>
</tr>
<tr>
<td>2</td>
<td>21.76</td>
<td>21.69</td>
</tr>
<tr>
<td>3</td>
<td>25.15</td>
<td>24.78</td>
</tr>
<tr>
<td>4</td>
<td>31.14</td>
<td>29.18</td>
</tr>
<tr>
<td>5</td>
<td>59.26</td>
<td>56.12</td>
</tr>
<tr>
<td>6</td>
<td>66.65</td>
<td>62.41</td>
</tr>
<tr>
<td>7</td>
<td>74.23</td>
<td>68.77</td>
</tr>
</tbody>
</table>

Computational costs

CUF LE $\approx 20,000$ DOFs

SOLID $\approx 190,000$ DOFs

E. Carrera, A. Pagani and M. Petrolo,
Component-wise Method
Applied to Vibration of Wing Structures,
doi:10.1115/1.4007849.
Thin-walled lipped channel beam

- Material model: Isotropic von-Mises plasticity
- Ideal elastic-perfect plastic stress-strain relationship

<table>
<thead>
<tr>
<th>Type</th>
<th>NDOF</th>
<th>$\lambda$ = 12.69</th>
<th>$\lambda$ = 40.94</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBT</td>
<td>3,222</td>
<td>13.63</td>
<td>77.6</td>
</tr>
<tr>
<td>ABAQUS - Shell</td>
<td>69,948</td>
<td>13.47</td>
<td>74.93</td>
</tr>
<tr>
<td>ABAQUS - 3D Brick</td>
<td>1,342,656</td>
<td>13.31</td>
<td>69.92</td>
</tr>
<tr>
<td>CUF - CW - (30-B4 elements)</td>
<td>23,751</td>
<td>13.17</td>
<td>73.22</td>
</tr>
<tr>
<td>14L9</td>
<td>40,131</td>
<td>13.29</td>
<td>75.95</td>
</tr>
<tr>
<td>44L9</td>
<td>72,891</td>
<td>13.32</td>
<td>78.34</td>
</tr>
</tbody>
</table>
Response based on **Continuum Damage Mechanics** and **Hashin’s Initiation criterion**

Three damage variables are introduced: $d_f$ (fiber), $d_m$ (matrix) and $d_s$ (shear) - Tension and Compression handled separately

$$d_l = \frac{\hat{\epsilon}_{l,f} (\hat{\epsilon}_l - \hat{\epsilon}_{l,0})}{\hat{\epsilon}_l (\hat{\epsilon}_{l,f} - \hat{\epsilon}_{l,0})} \quad \forall \ l \in [ft, fc, mc, mt]$$

Linear damage evolution law

$$\sigma = C(d_l) : \epsilon$$

Mesh objectivity obtained by introducing characteristic length scale

$$\hat{\epsilon}_{l,f} = \frac{2 \ G_{l,c}}{L_c X_l} \quad \forall \ l \in [ft, fc, mc, mt]$$

Viscous regularization scheme introduced to alleviate convergence issues

$$\dot{d}_l^\nu = \frac{1}{\eta_l} \left( d_l - d_l^\nu \right) \quad \forall \ l \in [ft, fc, mc, mt]$$
Failure Envelope

Tensile Fiber Mode

\[ \hat{\sigma}_{22} = X_T \]

Compressive Fiber Mode

\[ |\hat{\sigma}_{22}| = X_C \]

Tensile Matrix Mode \((\sigma_{11} + \hat{\sigma}_{33} > 0)\)

\[
\frac{1}{Y_T^2} \left( \hat{\sigma}_{11}^2 + \hat{\sigma}_{33}^2 \right) + \frac{1}{S_T^2} \left( \hat{\sigma}_{13}^2 - \hat{\sigma}_{11}\hat{\sigma}_{33} \right) + \frac{1}{S_L^2} \left( \hat{\sigma}_{12}^2 + \hat{\sigma}_{23}^2 \right) = 1
\]

Compressive Matrix Mode \((\sigma_{11} + \hat{\sigma}_{33} < 0)\)

\[
\frac{1}{Y_C} \left[ \left( \frac{Y_C}{2S_T} \right)^2 - 1 \right] (\hat{\sigma}_{11} + \hat{\sigma}_{33}) + \frac{1}{4S_T^2} \left( \hat{\sigma}_{11} + \hat{\sigma}_{33} \right)^2 + \frac{1}{S_T^2} \left( \hat{\sigma}_{13}^2 + \hat{\sigma}_{11}\hat{\sigma}_{33} \right)^2 + \frac{1}{S_L^2} \left( \hat{\sigma}_{12}^2 + \hat{\sigma}_{32}^2 \right)^2 = 1
\]

Nomenclature: 2 - Fiber direction, 1 & 3 - Transverse direction
\(\hat{\sigma}\) - effective stress
Failure Strength Estimation

1. Composite Laminate subjected to uni-axial tension
2. Laminate: HTA/6376-C
3. Each ply is modeled as a component (5 L9 elements)

Geometry of the beam

Material properties of HTA/6376-C

<table>
<thead>
<tr>
<th>$E_{11}$ (GPa)</th>
<th>$E_{22} = E_{33}$ (GPa)</th>
<th>$G_{12} = G_{13}$ (GPa)</th>
<th>$G_{23}$ (GPa)</th>
<th>$\nu_{12} = \nu_{13}$</th>
<th>$\nu_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>139</td>
<td>10</td>
<td>5.2</td>
<td>5.2</td>
<td>0.32</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Orthotropic damage initiation properties for HTA/6376-C

<table>
<thead>
<tr>
<th>$X_T$ (MPa)</th>
<th>$X_C$ (MPa)</th>
<th>$Y_T$ (MPa)</th>
<th>$Y_C$ (MPa)</th>
<th>$S_L$ (MPa)</th>
<th>$S_T$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2170</td>
<td>1600</td>
<td>73</td>
<td>290</td>
<td>83</td>
<td>83</td>
</tr>
</tbody>
</table>
Stacking Sequence: $[90/0]_{2s}$

Various stages of damage propagation at fixed end

- Fully virgin composite
- $90^0$ lamina damaged
- Fully damaged composite

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>$DDM^1$</th>
<th>CUF-CW</th>
<th>ABAQUS-2D shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Strength</td>
<td>1060</td>
<td>1107.6</td>
<td>1037</td>
<td>1088</td>
</tr>
<tr>
<td>DOF</td>
<td>-</td>
<td>900*</td>
<td>17,391</td>
<td>40,000</td>
</tr>
</tbody>
</table>

1. Moure et al., Damage evolution in open-hole laminated composite plates subjected to in-plane loads, 2015

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Stacking Sequence: \([90/0]_4\)

<table>
<thead>
<tr>
<th>Experimental</th>
<th>DDM</th>
<th>CUF-CW</th>
<th>ABAQUS-2D shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Strength (MPa)</td>
<td>1110</td>
<td>1118.1</td>
<td>1036.29</td>
</tr>
<tr>
<td>DOF</td>
<td>-</td>
<td>900*</td>
<td>27,621</td>
</tr>
</tbody>
</table>

1. Moure et al., Damage evolution in open-hole laminated composite plates subjected to in-plane loads, 2015

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Main Conclusions and Perspectives

- Accurate 3D stress fields obtained through CUF is successfully employed for damage propagation analysis
- Framework is able to predict the failure strength of composites (un-notched) within acceptable limits
- Computationally efficient higher-order non-linear finite element framework is developed

**Future extensions**

- Failure strength estimation of notched composites
- Component wise (fiber and matrix) damage propagation
- Extension of damage analysis for explicit scheme
- Inclusion of other damage initiation criteria
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