PROGRESSIVE DAMAGE ANALYSIS OF COMPOSITE STRUCTURES VIA ONE-DIMENSIONAL CARRERA UNIFIED FORMULATIONS

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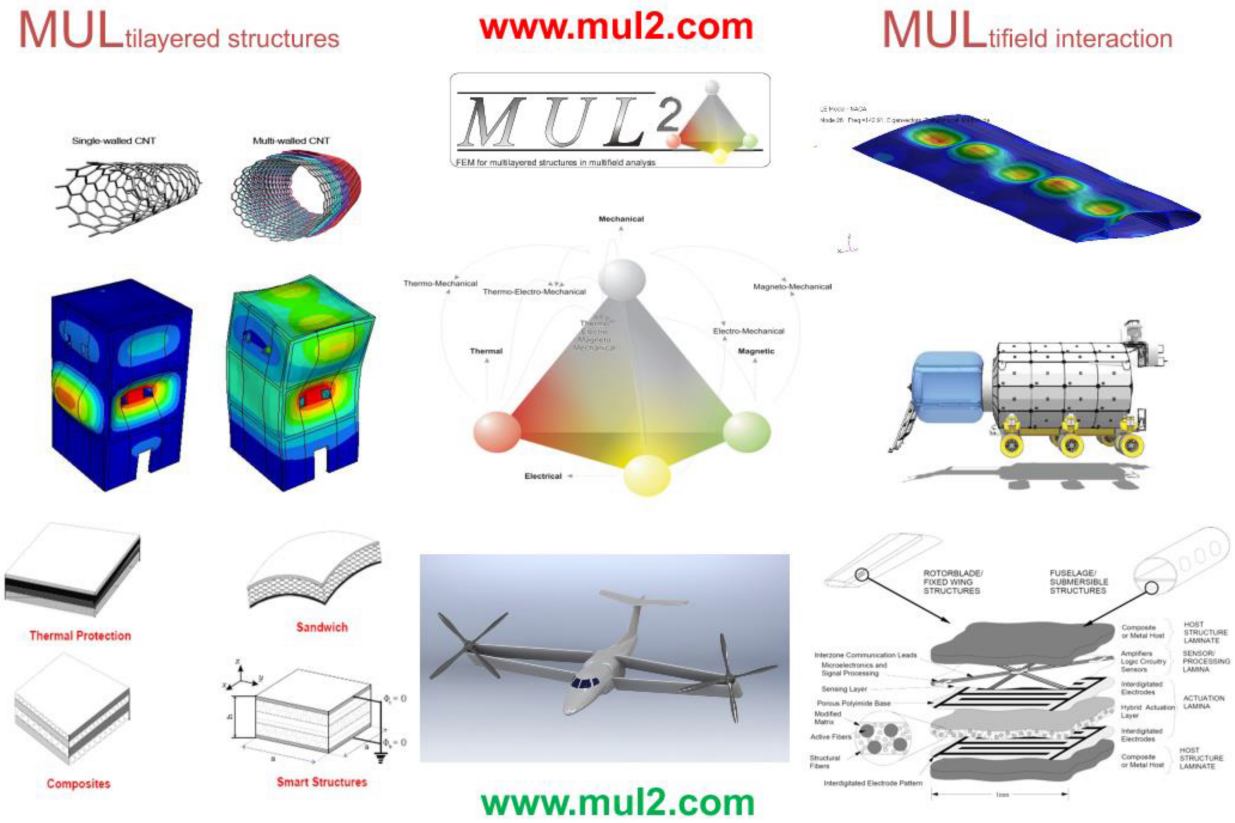
International Conference on Composite Structures, ICCS19
5-9 September 2016, Porto (Portugal)

MUL2 - Our Research Group



Marie Curie Project on Composites

www.fullcomp.net

The FULLCOMP project

- FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures
- The FULLCOMP project is funded by the European Commission under a Marie Sklodowska-Curie Innovative Training Networks grant for European Training Networks (ETN).
- The FULLCOMP partners are:
  1. Politecnico di Torino (Italy) - Coordinator
  2. University of Bristol (UK)
  3. Ecole Nationale Superieure d'arts et Metiers (Bordeaux, France)
  4. Leibniz Universitaet Hannover (Germany)
  5. University of Porto (Portugal)
  6. University of Washington (USA)
  7. RMIT (Australia)
  8. Luxembourg Institute of Technology
  9. Elan-Ausy, Hamburg, (Germany)
- FULLCOMP has recruited 12 PhD students who will work in an international framework to develop integrated analysis tools to improve the design of composite structures.
- The full spectrum of the design of composite structures will be dealt with, such as manufacturing, health-monitoring, failure, modeling, multiscale approaches, testing, prognosis, and prognostic. The FULLCOMP research activity is aimed at many engineering fields, e.g. aeronautics, automotive, mechanical, wind energy, and space.
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Overview

1. Description of the Carrera Unified Formulation for refined models (CUF).
2. Main 1D CUF capabilities overview (1D Taylor- and Lagrange-based models).
3. Numerical examples dealing with different applications (aerospace structures, composites, Bio Structures).
4. Introduction to the Component-Wise approach (CW).
5. Progressive Damage Model
6. Numerical Results

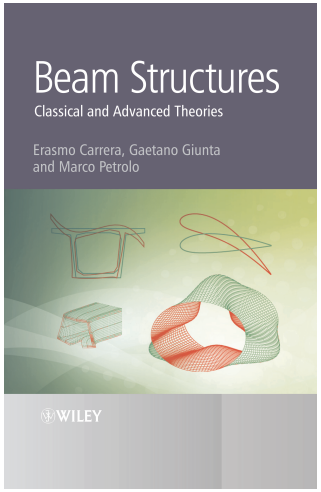
Brief Overview of Beam Refinement Methods and Contributors

1. Shear correction factors (Timoshenko, Sokolnikoff, Cowper, Gruttmann, etc.).
2. Warping functions and Saint-Venant solutions (El Fatmi, Ladéveze, etc.).
3. Variational asymptotic method (Berdichevsky, Hodges, Yu, etc.).
4. Generalized beam theory (Schardt, Camotim, Silvestre, etc.).
5. Higher-order models (Washizu, Reddy, Kapania, Carrera, etc.).

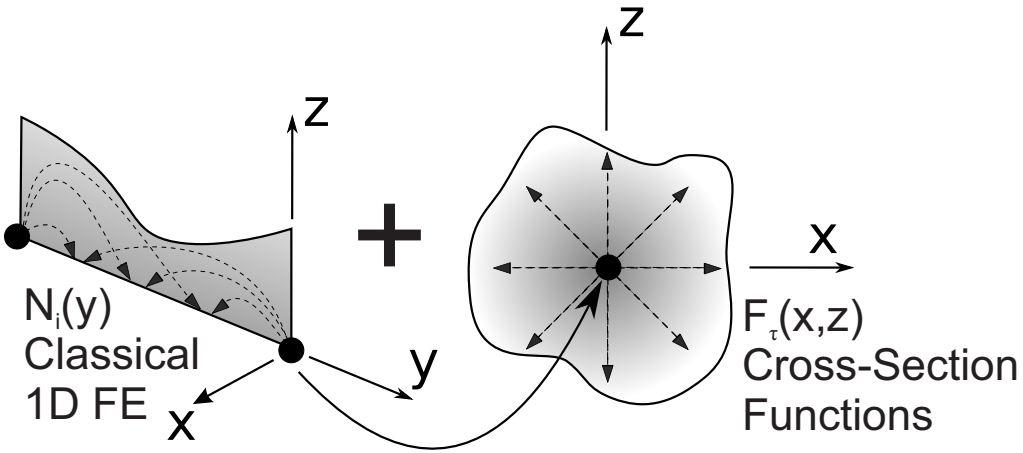
1D Advanced Structural Models

Actual Wing

Our Model



1D Carrera Unified Formulation, CUF - FEM Version



Fundamental Nucleus Equations by CUF

Finite Element Formulation (FEM)

$$\mathbf{u} = F_\tau(x,z) N_i(y)\, \mathbf{u}_{\tau i}(1D)$$

$$\delta L_{int} = \delta \mathbf{q}_{\tau i}^T \mathbf{K}^{ij\tau s} \mathbf{q}_{sj}$$

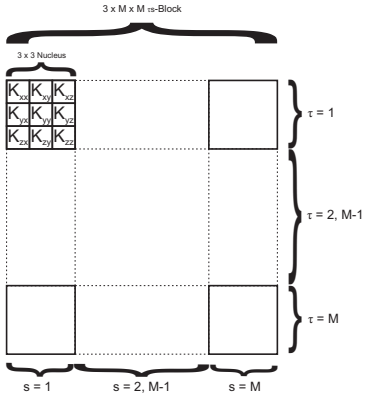
$$\delta L_{ine} = \delta \mathbf{q}_{\tau i}^T \mathbf{M}^{ij\tau s} \ddot{\mathbf{q}}_{sj}$$

$$\delta L_{ext} = \mathbf{P} \delta \mathbf{u}^T$$

i, j: Shape function indexes (depend on the FE discretization).

τ, s: Expansion function indexes (depend on the model order).

Assembly Technique



Fundamental Nucleus

$$K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_\Omega F_{\tau,x} F_{s,x}\, d\Omega \int_l N_i N_j\, dy + \ldots$$

1D CUF Applications Overview - Books

Thin-Walled and Reinforced Structures

Buckling, Free Vibration and Dynamic Response Analysis

Composite Structures

FGM Structures

Variable Kinematics Models

Axiomatic/Asymptotic Analyses and Best Theory Diagrams

Aeroelasticity

Load Factors and Non-Structural Masses

Rotors and Rotating Blades

Biomechanics

Multifield Analysis

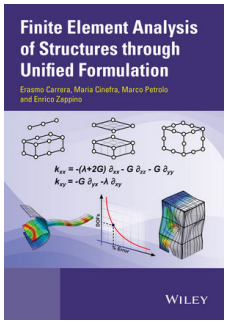
Nanostructures

Analysis of Aerospace Structures via the Component-Wise Approach

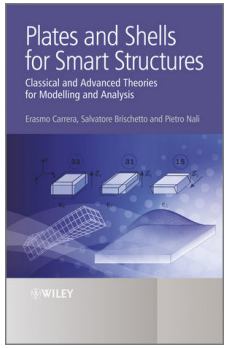
Analysis of Civil Structures via the Component-Wise Approach

Component-Wise Approach for the Multiscale Analyses of Composites

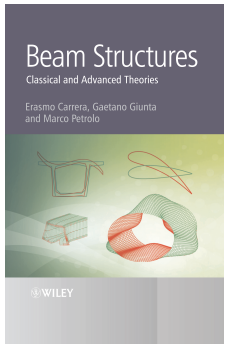
FEM



2D and Smart Structures



1D

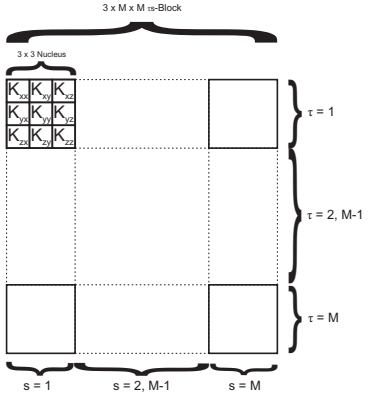


The Taylor CUF 1D Models, TE

$$
\begin{array}{lcccc}
u_x = & u_{x_1} & +x\ u_{x_2} + z\ u_{x_3} & +x^2\ u_{x_4} + xz\ u_{x_5} + z^2\ u_{x_6} + \ldots \\
u_y = & u_{y_1} & +x\ u_{y_2} + z\ u_{y_3} & +x^2\ u_{y_4} + xz\ u_{y_5} + z^2\ u_{y_6} + \ldots \\
u_z = & u_{z_1} & +x\ u_{z_2} + z\ u_{z_3} & +x^2\ u_{z_4} + xz\ u_{z_5} + z^2\ u_{z_6} + \ldots \\
 & \underbrace{\phantom{u_{x_1}}}_{\substack{N=0 \\ \tau=1 \\ 3\ \ DOFs}} & \underbrace{\phantom{+x\ u_{x_2} + z\ u_{x_3}}}_{\substack{N=1 \\ \tau=2,\tau=3 \\ 9\ \ DOFS}} & \underbrace{\phantom{+x^2\ u_{x_4} + xz\ u_{x_5} + z^2\ u_{x_6}}}_{\substack{N=2 \\ \tau=4,\tau=5,\tau=6 \\ 18\ \ DOFs}}
\end{array}
$$

Classical models, such as Timoshenko, can be obtained as particular cases of the linear models.

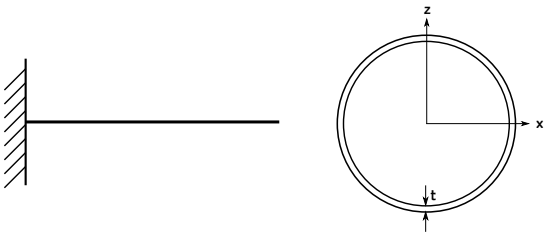
Assembly Technique

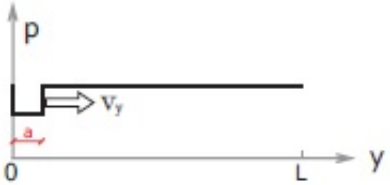


Fundamental Nucleus

$$K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_{\Omega} F_{\tau,x} F_{s,x}\, d\Omega \int_{l} N_i N_j\, dy + \ldots$$

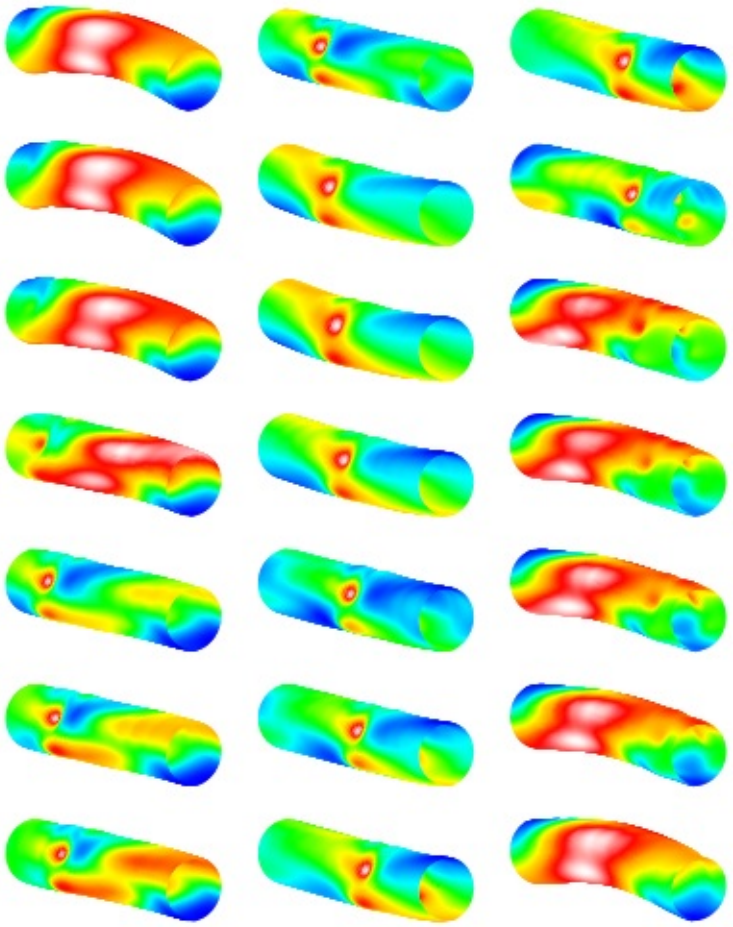
Wave propagation in a Thin-Walled Cylinder



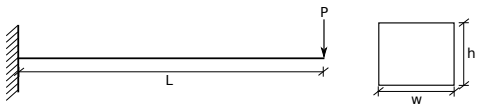


E. Carrera, A. Varello,
Dynamic Response of Thin-Walled Structures by Variable Kinematic One-Dimensional Models
Journal of Sound and Vibration, 331(24), pp. 5268-5282, 2012.

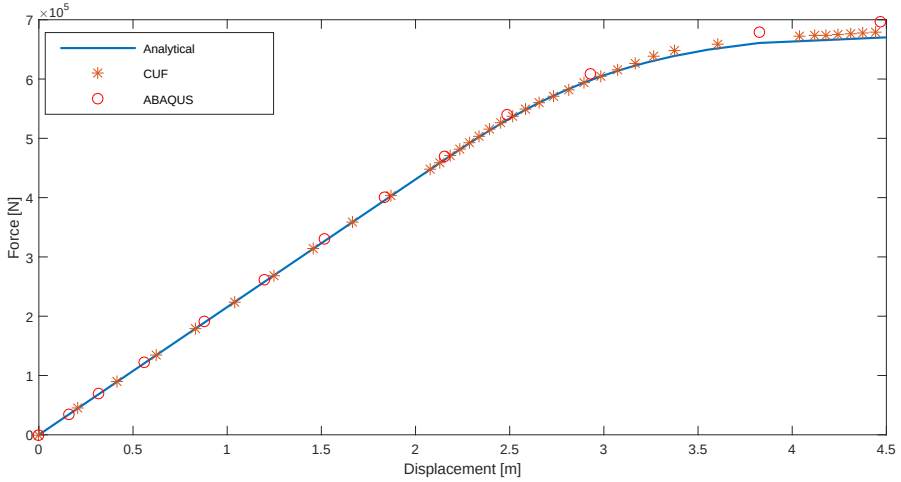
N = 8 beam model, 6000 DOFs



Plastic beam under bending load



- Material model : Isotropic von-Mises plasticity
- Ideal elastic-perfect plastic stress-strain relationship
- Plastic strength factor is estimated
- Emphasizes the importance of refined higher order models



Type	NDOF	U_Z at yield load		U_Z at limit load		Plastic Strength Factor	
		Value [m]	Error [%]	Value [m]	Error [%] [%]	Value	Error [%] [%]
Analytical	-	2.080	-	4.622	-	1.5	-
ABQ - 3D	22,590	2.077	0.14	4.281	7.38	1.55	3.57
Taylor							
EBBM	279	2.083	0.16	3.13	32.39	-	-
TBM	279	2.084	0.18	3.13	32.38	-	-
N = 1	279	2.084	0.18	3.13	32.38	-	-
N = 2	558	2.072	0.40	4.21	8.98	1.59	6.00
N = 3	930	2.072	0.39	4.42	4.80	1.52	1.33

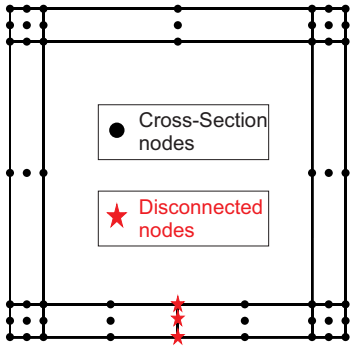
Lagrange-Based 1D Models, LE

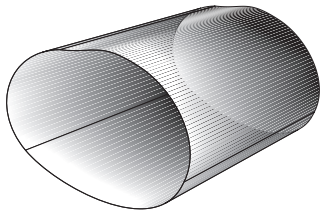
$$u_x = L_\tau \; u_{x_\tau}$$
$$u_y = L_\tau \; u_{y_\tau}$$
$$u_z = L_\tau \; u_{z_\tau}$$

L9 polynomials - Isoparametric

$$L_\tau = \frac{1}{4}(r^2 + r \; r_\tau)(s^2 + ...$$

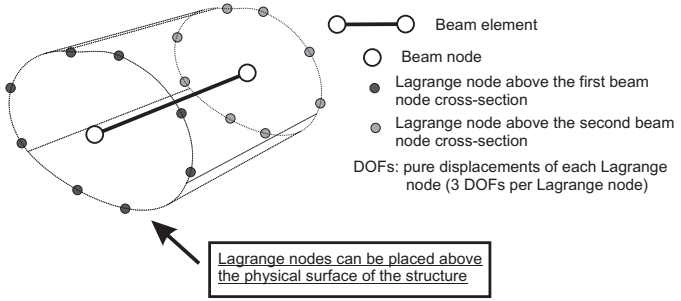
Cross-Section Elements



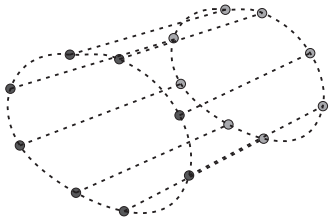


3D Geometry from CAD

LE Modeling

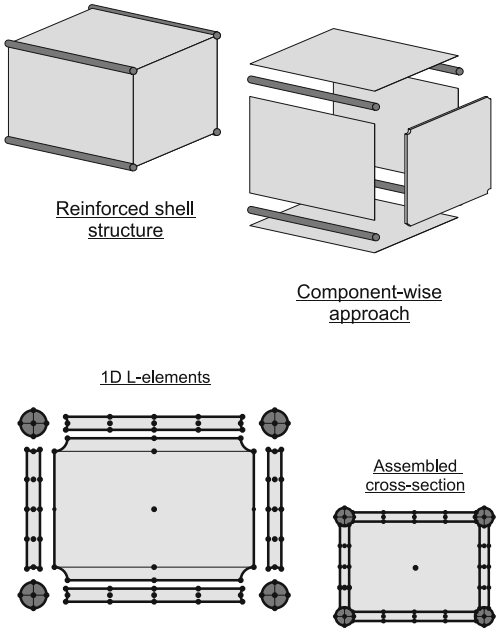


Computational Model

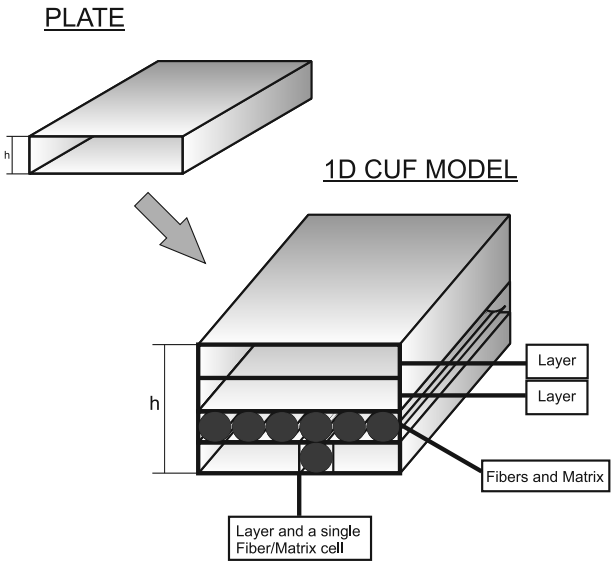


The Component-Wise Approach

Component-Wise approach for typical aircraft structures

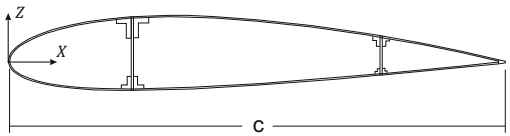


Component-Wise approach for the multi-scale analysis of composites



- Only displacements as unknowns.
- Each component of the structure is modeled via beams only.
- No need of reference surfaces. This might be useful in a CAD-FEM interface scenario.
- No need of homogenization techniques.

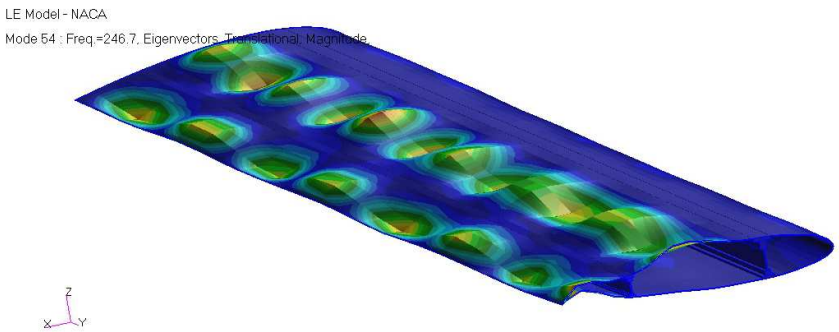
Aircraft Wing



E. Carrera, A. Pagani and M. Petrolo,
Component-wise Method Applied to Vibration of Wing Structures,
Journal of Applied Mechanics, 80(4), 2013.
doi:10.1115/1.4007849.

Shell-like modal shapes by means of a BEAM MODEL

Mode 54 - 246.7 Hz



Natural frequencies [Hz]

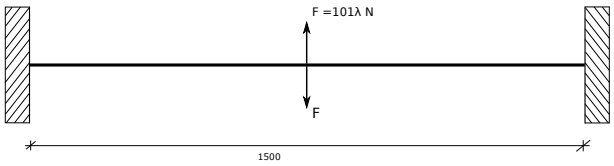
Mode	1D-LE	SOLID
1	4.23	4.22
2	21.76	21.69
3	25.15	24.78
4	31.14	29.18
5	59.26	56.12
6	66.65	62.41
7	74.23	68.77

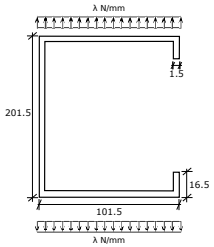
Computational costs

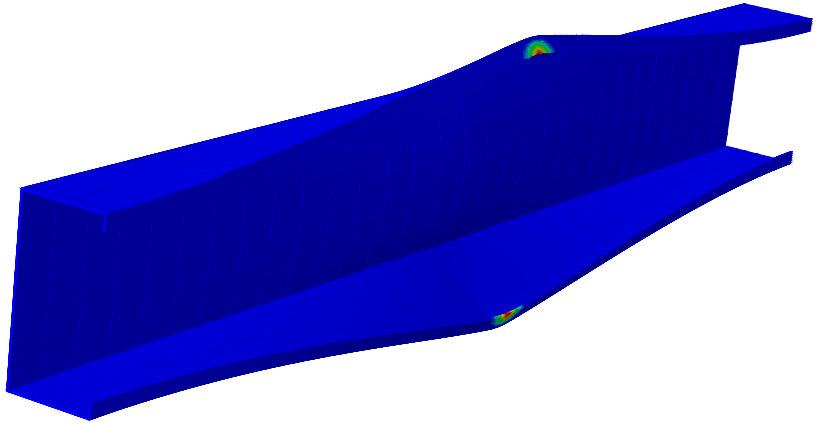
CUF LE ≈ **20,000** DOFs

SOLID ≈ **190,000** DOFs

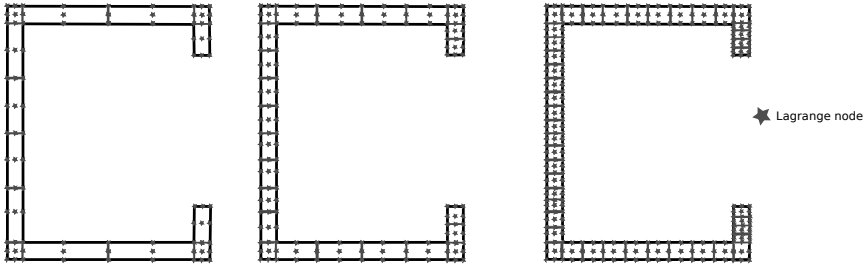
Thin-walled lipped channel beam







- Material model :
  Isotropic von-Mises plasticity
- Ideal elastic-perfect plastic stress-strain relationship



14 L9 24 L9 44 L9

Type	NDOF	Displacement	
		$\lambda = 12.69$	$\lambda = 40.94$
GBT	3,222	13.63	77.6
ABAQUS - Shell	69,948	13.47	74.93
ABAQUS - 3D Brick	1,342,656	13.31	69.92
CUF - CW - (30-B4 elements)			
14L9	23,751	13.17	73.22
24L9	40,131	13.29	75.95
44L9	72,891	13.32	78.34

1. Response based on **Continuum Damage Mechanics** and **Hashin's Initiation criterion**
2. Three damage variables are introduced : d_f (fiber), d_m (matrix) and d_s (shear) - Tension and Compression handled separately

$$d_l = \frac{\hat{\epsilon}_{l,f}\left(\hat{\epsilon}_l - \hat{\epsilon}_{l,0}\right)}{\hat{\epsilon}_l\left(\hat{\epsilon}_{l,f} - \hat{\epsilon}_{l,0}\right)} \quad \forall\, l \in [ft, fc, mc, mt]$$

3. Linear damage evolution law

$$\boldsymbol{\sigma} = \mathbf{C}(d_l) : \boldsymbol{\epsilon}$$

4. Mesh objectivity obtained by introducing characteristic length scale

$$\hat{\epsilon}_{l,f} = \frac{2\, G_{l,c}}{L_c\, X_l} \quad \forall\, l \in [ft, fc, mc, mt]$$

5. Viscous regularization scheme introduced to alleviate convergence issues

$$\dot{d}_l^v = \frac{1}{\eta_l}\left(d_l - d_l^v\right) \quad \forall\, l \in [ft, fc, mc, mt]$$

Failure Envelope

Tensile Fiber Mode

$$\hat{\sigma}_{22} = X_T$$

Compressive Fiber Mode

$$|\hat{\sigma}_{22}| = X_C$$

Tensile Matrix Mode $(\sigma_{11} + \hat{\sigma}_{33} > 0)$

$$\frac{1}{Y_T^2}\left(\hat{\sigma}_{11}^2 + \hat{\sigma}_{33}^2\right) + \frac{1}{S_T^2}\left(\hat{\sigma}_{13}^2 - \hat{\sigma}_{11}\hat{\sigma}_{33}\right) + \frac{1}{S_L^2}\left(\hat{\sigma}_{12}^2 + \hat{\sigma}_{23}^2\right) = 1$$

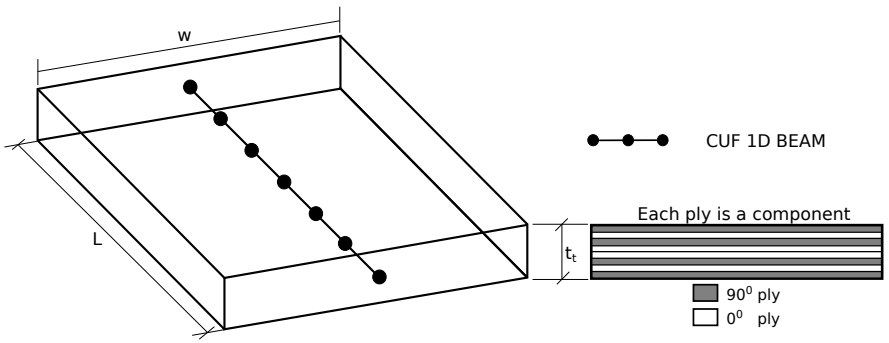
Compressive Matrix Mode $(\hat{\sigma}_{11} + \hat{\sigma}_{33} < 0)$

$$\frac{1}{Y_C}\left[\left(\frac{Y_C}{2S_T}\right)^2 - 1\right](\hat{\sigma}_{11} + \hat{\sigma}_{33}) + \frac{1}{4S_T^2}\left(\hat{\sigma}_{11} + \hat{\sigma}_{33}\right)^2 + \frac{1}{S_T^2}\left(\hat{\sigma}_{13}^2 + \hat{\sigma}_{11}\hat{\sigma}_{33}\right)^2 + \frac{1}{S_L^2}\left(\hat{\sigma}_{12}^2 + \hat{\sigma}_{32}^2\right)^2 = 1$$

Nomenclature : 2 - Fiber direction , 1 & 3 - Transverse direction
$\hat{\sigma}$ - effective stress

Failure Strength Estimation

1. Composite Laminate subjected to uni-axial tension
2. Laminate : HTA/6376-C
3. Each ply is modeled as a component (5 L9 elements)



Geometry of the beam

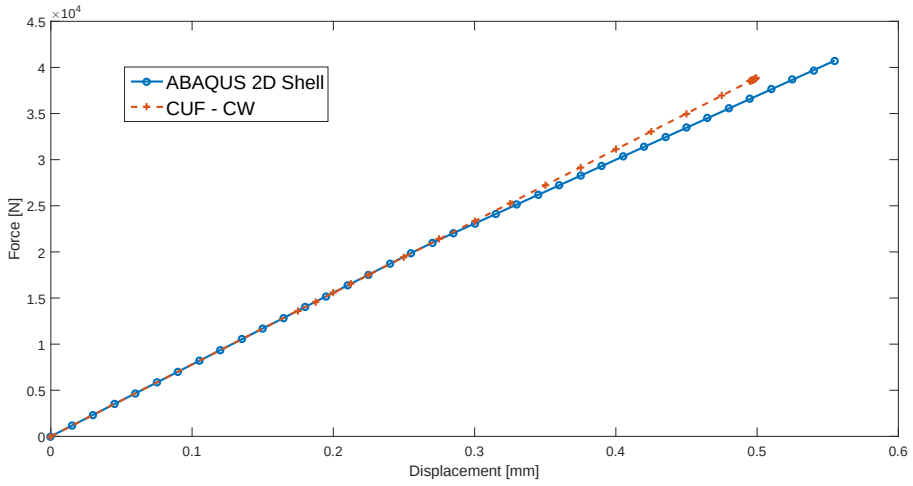
Material properties of HTA /6376-C

E_{11} (GPa)	$E_{22} = E_{33}$ (GPa)	$G_{12} = G_{13}$ (GPa)	G_{23}(GPa)	$\nu_{12} = \nu_{13}$	ν_{23}
139	10	5.2	5.2	0.32	0.51

Orthotropic damage initiation properties for HTA /6376-C

X_T (MPa)	X_C (MPa)	Y_T (MPa)	Y_C (MPa)	S_L (MPa)	S_T (MPa)
2170	1600	73	290	83	83

Stacking Sequence : $[90/0]_{2s}$



Various stages of damage propagation at fixed end

Fully virgin composite

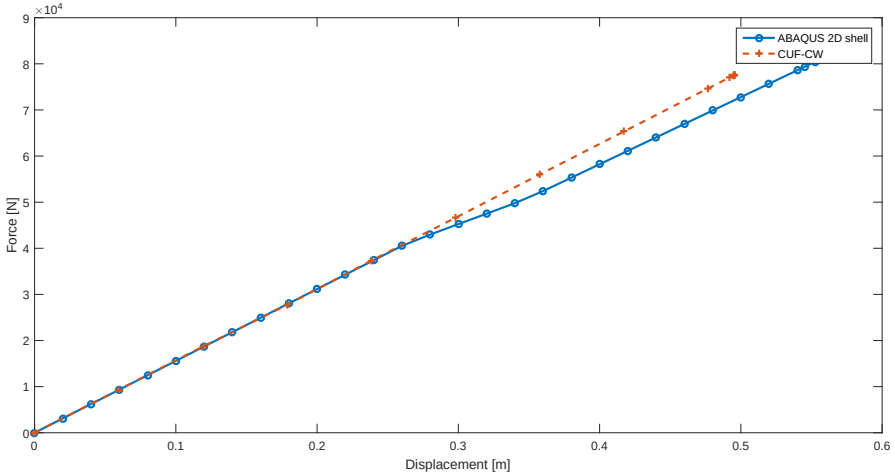
90^0 lamina damaged

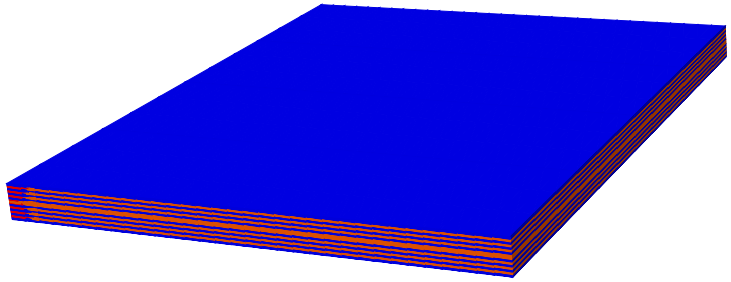
Fully damaged composite

	Experimental	*DDM*[1]	CUF-CW	ABAQUS-2D shell
Failure Strength	1060	1107.6	1037	1088
DOF	-	900*	17,391	40,000

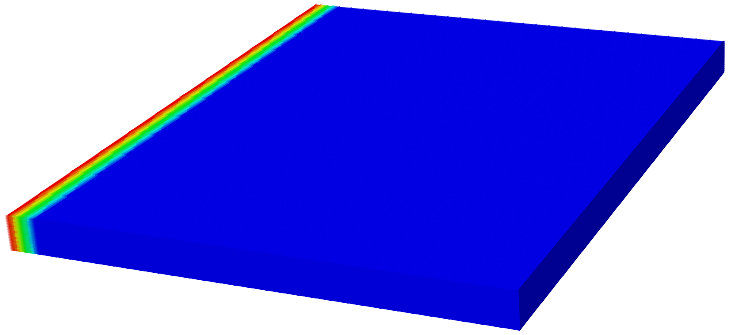
1. Moure et al., Damage evolution in open-hole laminated composite plates subjected to in-plane loads, 2015

Stacking Sequence : $[90/0]_{4s}$





σ_{yy}



Tensile Matrix Damage (d_{mt})

	Experimental	*DDM*[1]	CUF-CW	ABAQUS-2D shell
Failure Strength (MPa)	1110	1118.1	1036.29	1067.73
DOF	-	900*	27,621	40,000

1. Moure et al., Damage evolution in open-hole laminated composite plates subjected to in-plane loads, 2015

Main Conclusions and Perspectives

- Accurate 3D stress fields obtained through CUF is successfully employed for damage propagation analysis
- Framework is able to predict the failure strength of composites (un-notched) within acceptable limits
- Computationally efficient higher-order non-linear finite element framework is developed

Future extensions

- Failure strength estimation of notched composites
- Component wise (fiber and matrix) damage propagation
- Extension of damage analysis for explicit scheme
- Inclusion of other damage initiation criteria

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