Reissner's mixed variational theorem for layer-wise beam models based on the unifed formulation

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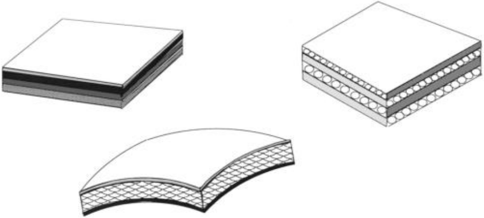
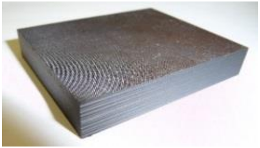
Overview



1. Laminates: simulation challenges
2. Reissner's mixed variational theorem (RMVT)
3. Derivation of a mixed beam finite element using a layer-wise approach and higher-order expansions
4. Numerical assessment
5. Structural applications: tensile and bending tests
6. Conclusions

Laminates

- Advantages: good specific properties
- Multi-layered structures are built by adding plies of the same, or different materials, in a certain stacking sequence
- Optimized performance of the component



Transverse anisotropy: sudden change of mechanical properties in the through-the-thickness direction

Transverse stresses not neglible due to the high ratio between the elastic moduli (E_L/E_T= 5 – 40) and low transverse shear moduli (G_{LT} and G_{TT})

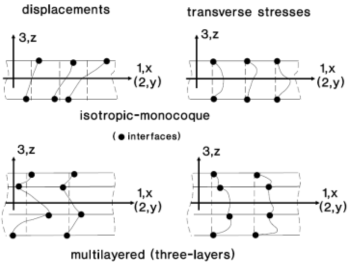
Compatibility: sudden changes in the slope of the displacement fields (u_x, u_y, u_z) across the thickness of the laminate

Equilibrium: continuity of transverse stresses (σ_{zz}, σ_{yz} , σ_{xz})

Kinematics: C^0_z requirements

- C^0 for displacements -> zig-zag effect (ZZ)
- C^0 for transverse stresses -> interlaminar continuity (IC)

Major challenge in the modelling of laminated structures: trasverse deformation must be included



ZZ IC

RMVT

A posteriori approaches

- Classical Laminate Theories (CLT) and First-order Shear Deformation Theories do not fulfill the C^0_z requirements -> first derivatives constant
- Higher-order theories and ZZ theories -> IC not necessarily satisfied
- Layer-wise (LW) models account for independent kinematics at each layer
- Stress recovery methods: integration of stress solutions in the 3D equilibrium equations

Reissner's Mixed Variational Theorem (RMVT)

A priori fulfillment of the C^0_z requirements through the use of independent displacement and stress assumptions

Stress assumptions are *restricted to the transverse components* in laminates

$$\int_V (\delta\boldsymbol{\varepsilon}_p^T \boldsymbol{\sigma}_p + \delta\boldsymbol{\varepsilon}_n^T \boldsymbol{\sigma}_n)\, dV = \delta L_e$$

PVD

→

$$\int_V (\delta\boldsymbol{\varepsilon}_{pG}^T \boldsymbol{\sigma}_{pH} + \delta\boldsymbol{\varepsilon}_{nG}^T \boldsymbol{\sigma}_{nM} \boxed{+ \delta\boldsymbol{\sigma}_{nM}^T (\boldsymbol{\varepsilon}_{nG} - \boldsymbol{\varepsilon}_{nH})})\, dV = \delta L_e$$

RMVT

Geometrical relations

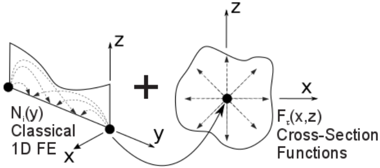
$$\boldsymbol{\varepsilon}_{pG} = \boldsymbol{D}_{pG}\, \boldsymbol{u}$$
$$\boldsymbol{\varepsilon}_{nG} = \boldsymbol{D}_{nG}\, \boldsymbol{u}$$

Constitutive equations

$$\boldsymbol{\sigma}_{pH}^k = \mathbf{C}_{pp}^k\, \boldsymbol{\varepsilon}_{nG}^k + \mathbf{C}_{pn}^k\, \boldsymbol{\sigma}_{nM}^k$$
$$\boldsymbol{\varepsilon}_{nH}^k = \mathbf{C}_{np}^k\, \boldsymbol{\varepsilon}_{pG}^k + \mathbf{C}_{nn}^k\, \boldsymbol{\sigma}_{nM}^k$$

Mixed beam elements

Carrera Unified Formulation



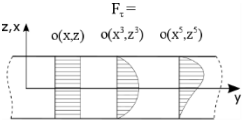
Beam kinematics:

$$\boldsymbol{u}(x,y,z) = N_i(y)\, F_\tau(x,z)\, \boldsymbol{u}_{\tau i}$$

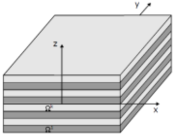
Fundamental Nucleus:

$$K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_\Omega F_{\tau,x} F_{s,x}\, d\Omega \int_l N_i N_j dy + \dots$$

- ➢ Field solutions over the section



Mixed refined beams



LW beam kinematics:

$$\boldsymbol{u}^k(x,y,z) = N_i(y)\, F_\tau(x,z)\, \boldsymbol{u}^k{}_{\tau i}$$

LW stress assumptions:

$$\boldsymbol{\sigma}^k{}_n(x,y,z) = N_i(y)\, G_\tau(x,z)\, \boldsymbol{\sigma}^k{}_{n\tau i}$$

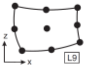
- ➢ Increase of number of unknowns
- ➢ No ZZ assumed functions required
- ➢ C^0_z requirements and IC satisfied at the interfaces between plies

- Compatibility of displacements $\mathbf{u}_t^k = \mathbf{u}_b^{k+1}$
- IC of transverse stresses $\boldsymbol{\sigma}_{nt}^k = \boldsymbol{\sigma}_{nb}^{k+1}$

Cross-section expansions

Lagrange Expansions (LE)

- Nodal unknowns
- Hp-refinement of the expansion assumptions
- Layer-wise distributions of unknowns



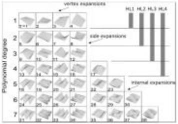
$$F_\tau = \frac{1}{4}(r^2 + r\, r_\tau)(s^2 + s\, s_\tau) \quad \tau = 1,3,5,7$$

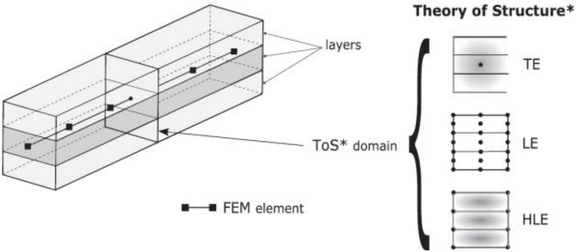
$$F_\tau = \frac{1}{2}s_\tau^2(s^2 + s\, s_\tau)(1 - r^2) + \frac{1}{2}r_\tau^2(r^2 + r\, r_\tau)(1 - s^2) \quad \tau = 2,4,6,8$$

$$F_\tau = (1 - r^2)(1 - s^2) \quad \tau = 9$$

Hierarchical Legendre Expansions (HLE)

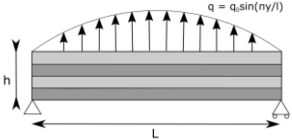
- Nodal, side and internal unknowns
- Hierarchical p-refinement of the expansion assumptions
- Layer-wise distributions of unknowns





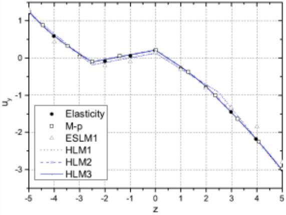
Laminate [0/90/0/90]

*Pagano, 1969

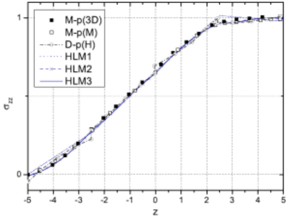


- Thick laminate: L/h = 4
- E_1=25MPa E_2=1MPa
  G_{12}=0.5MPa G_{23}=0.2MPa
  V_{12} = V_{23} = 0.25
- Cylindrical bending
- 1D model
  4 cubic mixed elements
  4 HLE domains (LW)

Longitudinal displacements

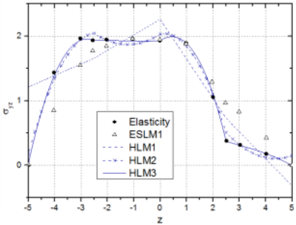


Normal stresses

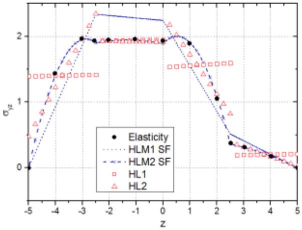


Shear stresses

RMVT convergence

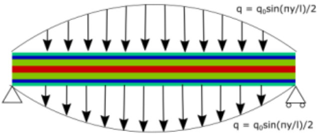


RMVT SF & PVD



Sandwich laminate

*Groh & Weaver, 2015

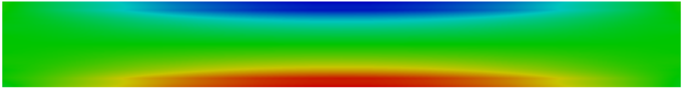


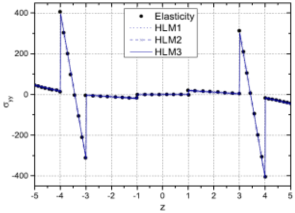
Skin: carbon-fiber

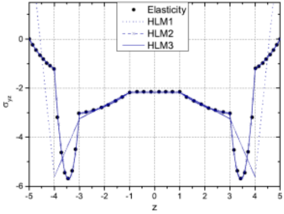
Core: polyvinyl chloride foam + honeycomb

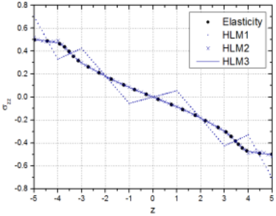
- Stacking sequence: [cf_{90}/cf_0/pvc/h/pvc/cf_0/cf_{90}]
- High *trasverse anisotropy*
- Model: 4 mixed beam elements + 7 HLE domains (max 4680 DOFs)

Longitudinal stress / Transverse shear stress / Transverse normal stress

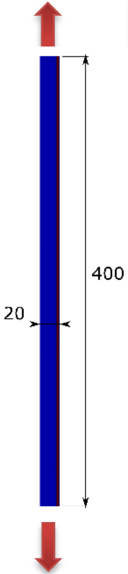




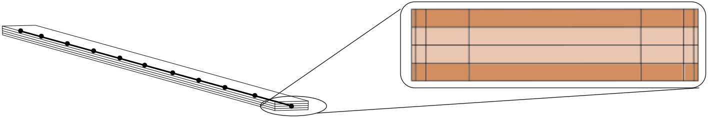


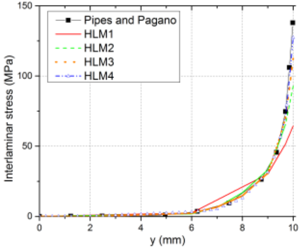


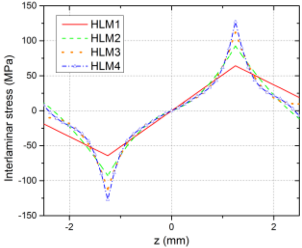
Tensile test



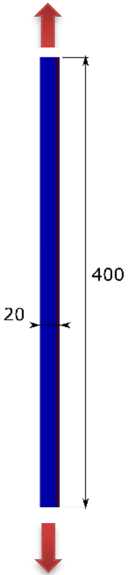
- *Pipes and Pagano 'Interlaminar stresses in composite laminates –An approximate elasticity solution', ASME J. App. Mech. 1974.
- Material : E1=137.9 GPa, E2=E3=14.5 GPa, G12=G13=G23=5.9 GPa, V12=v13=v23=0.21
- Stacking sequence: [45/-45]s, b/h = 4 -> h_0 = 1.25 mm
- Model: 10 cubic mixed elements + 28 HLE section domains



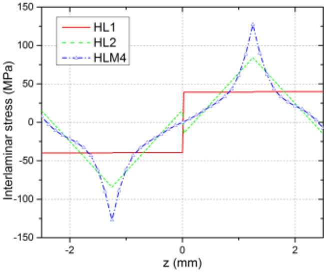




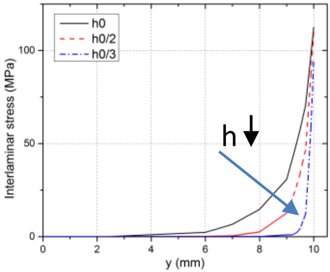
Tensile test



- Traditional approaches



- Thickness study (HLM3)



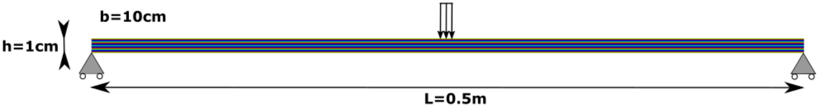
Accurate **3D stress fields** within the entire structure

Free edge effects on cross-ply laminates are accurately captured by increasing the polynomial order of the expansions over the cross-section

Computational effort (DOFs):

- **RMVT beam:** HLM1 (6,720), HLM2 (17,976), HLM3 (29,232), HLM4 (45192)
- **PVD beam:** HL1 (3,360), HL2 (8988) – **IC not fulfilled**

Three point test

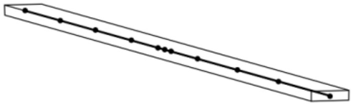


AS4/PEEK carbon-fiber : E1=122.7 GPa, E2=10.1 GPa, G12=5.5 GPa, G23=3.7 GPa, V12=0.25

20 plies **$[45/-45/0_2/90]_{2s}$**

Refined 1D model

- 10 cubic mixed beam elements

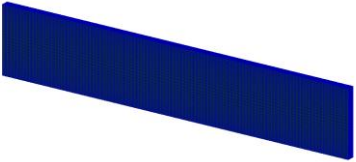


- LE second-order expansion
- 140 expansion domains
- Kinematics distribuited towards the edges

- 103,320 DOFs

Conventional 3D model

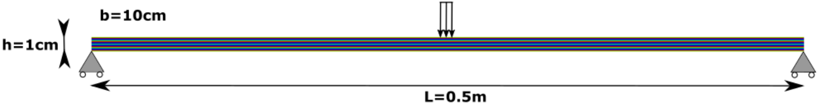
- 200,000 linear brick elements



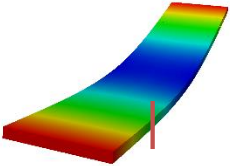
- Mesh refinement towards the edges

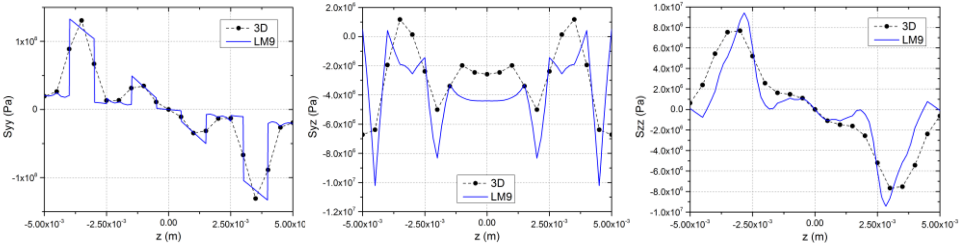
- 644,028 DOFs

Three point test



- For this stacking sequence, interlaminar stresses play a predominant role in the damage mechanisms -> **delamination**.
- In structural applications, delamination starts under mixed-mode loading. **Accurate transverse shear and normal stress** components needed.
- Mixed beam elements reduce the computational expenses of composite analysis, with **no loss in accuracy with respect to 3D models**.





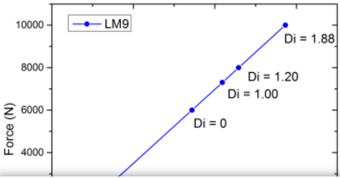
Three point test: delamination index

Mixed mode quadratic criteria

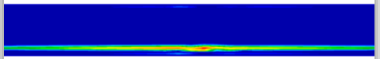
$$\left(\frac{\langle\sigma_n\rangle}{N}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 + \left(\frac{\sigma_t}{T}\right)^2 = 1$$

N = 80 Mpa   T = S = 100 MPa

- Linear elastic study -> goodness of stress fields
- Refined 3D model required (3 HEXA8 per layer)



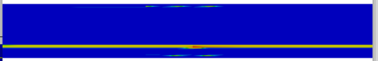
3D Nastran (644,028 DOF



3D Nastran refined

3D Nastran (1,508,103 DO

1D LM9 (103,320 DOFs,



1D LM9

Conclusions

- A novel class of **refined mixed beam elements** based on the RMVT and a LW approach is proposed
- The displacement and trasverse stresses are assumed over the cross-section by means of independent sets of Lagrange and Legrende polynomials
- IC of transverse stresses is satisfied *a priori* at the expense of extra DOFs for the stress assumptions. No stress recovery techniques are required
- The numerical assessment shows that in benchmark cases the elasticity solutions are captured with higher-order models
- Real applications (tensile and bending tests): 3D-like accuracy is obtained for the stress fields with significant reduction of the computational efforts. Free edge effects can be computed

Developments

- Great potential for damage analysis, in particular delamination
- Introduction in a decohesive element framework
- Global-local analysis: mixed 1D finite elements locally placed in areas of interest

Acknowledgements

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    - vi. ELAN-AUSY GmbH (Germany)
    - vii. Universidade do Porto (Portugal)
    - viii. University of Washington (USA)
    - ix. RMIT (Australia)

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any questions?