Micro-, meso- and macro-scale analysis of composite laminates by unified theory of structures

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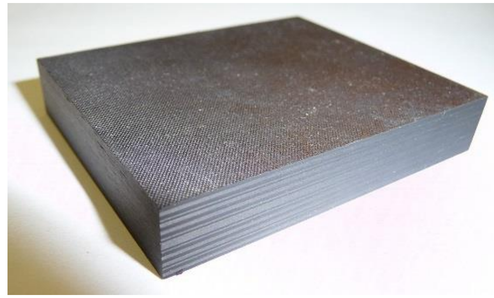
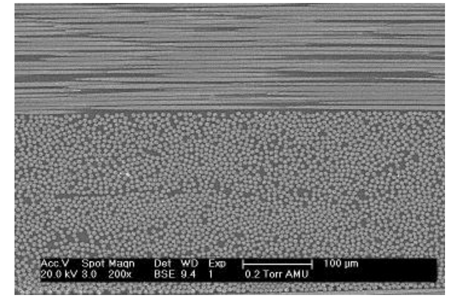
Our work

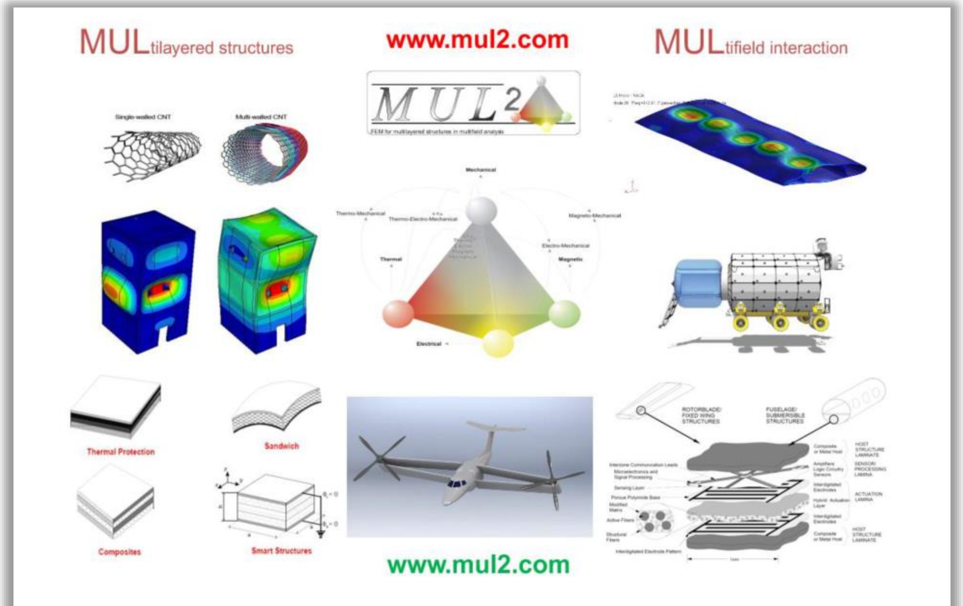
Structural body



Composite laminate



Multi-phase materials



- Provide a unified methodology to systematically generate different classes of structural models
- Zoom into different scales by means of a unified formulation with no decoupling
- Reduce the computational size of composite simulation in such a way that several scales can be accounted

Overview

I. The unified formulation for beam analysis

II. Component-wise analysis: coupling macro-, meso, micro-scale modeling

III. Weak form solutions

- Cross-ply beam

IV. Strong form solutions

- Sandwich beam

V. Conclusions and perspectives

Carrera Unified Formulation 1D

Refinement of the theory

Euler-Bernoulli

$$u_x(x,y,z) = u_{x_1}(y)$$
$$u_y(x,y,z) = u_{y_1}(y) - x\, u_{x_{1,y}}(y) - z\, u_{z_{1,y}}(y)$$
$$u_z(x,y,z) = u_{z_1}(y)$$

Timoshenko

$$u_x(x,y,z) = u_{x_1}(y)$$
$$u_y(x,y,z) = u_{y_1}(y) - x\, u_{y_2}(y) - z\, u_{y_3}(y)$$
$$u_z(x,y,z) = u_{z_1}(y)$$

Saint Venant

$$u_x(x,y,z) = u_{x_1}(y) - z\, u_{x_2}(y)$$
$$u_y(x,y,z) = u_{y_1}(y) - x\, u_{y_2}(y) - z\, u_{y_3}(y) + \psi(x,z)\, u_{y_3}(y)$$
$$u_z(x,y,z) = u_{z_1}(y) + x\, u_{z_2}(y)$$

K. Washizu: " *For a complete removal of the inconsistency and an improvement of the accuracy of the beam theory*" -> enrich beam kinematics with higher-order terms

Carrera Unified Formulation

$$u_x(x,y,z) = F_1(x,z)\, u_{x_1}(y) + F_2(x,z)\, u_{x_2}(y) + F_3(x,z)\, u_{x_3}(y) + \dots + F_M(x,z)\, u_{x_M}(y)$$
$$u_y(x,y,z) = F_1(x,z)\, u_{y_1}(y) + F_2(x,z)\, u_{y_2}(y) + F_3(x,z)\, u_{y_3}(y) + \dots + F_M(x,z)\, u_{y_M}(y)$$
$$u_z(x,y,z) = F_1(x,z)\, u_{z_1}(y) + F_2(x,z)\, u_{z_2}(y) + F_3(x,z)\, u_{z_3}(y) + \dots + F_M(x,z)\, u_{z_M}(y)$$

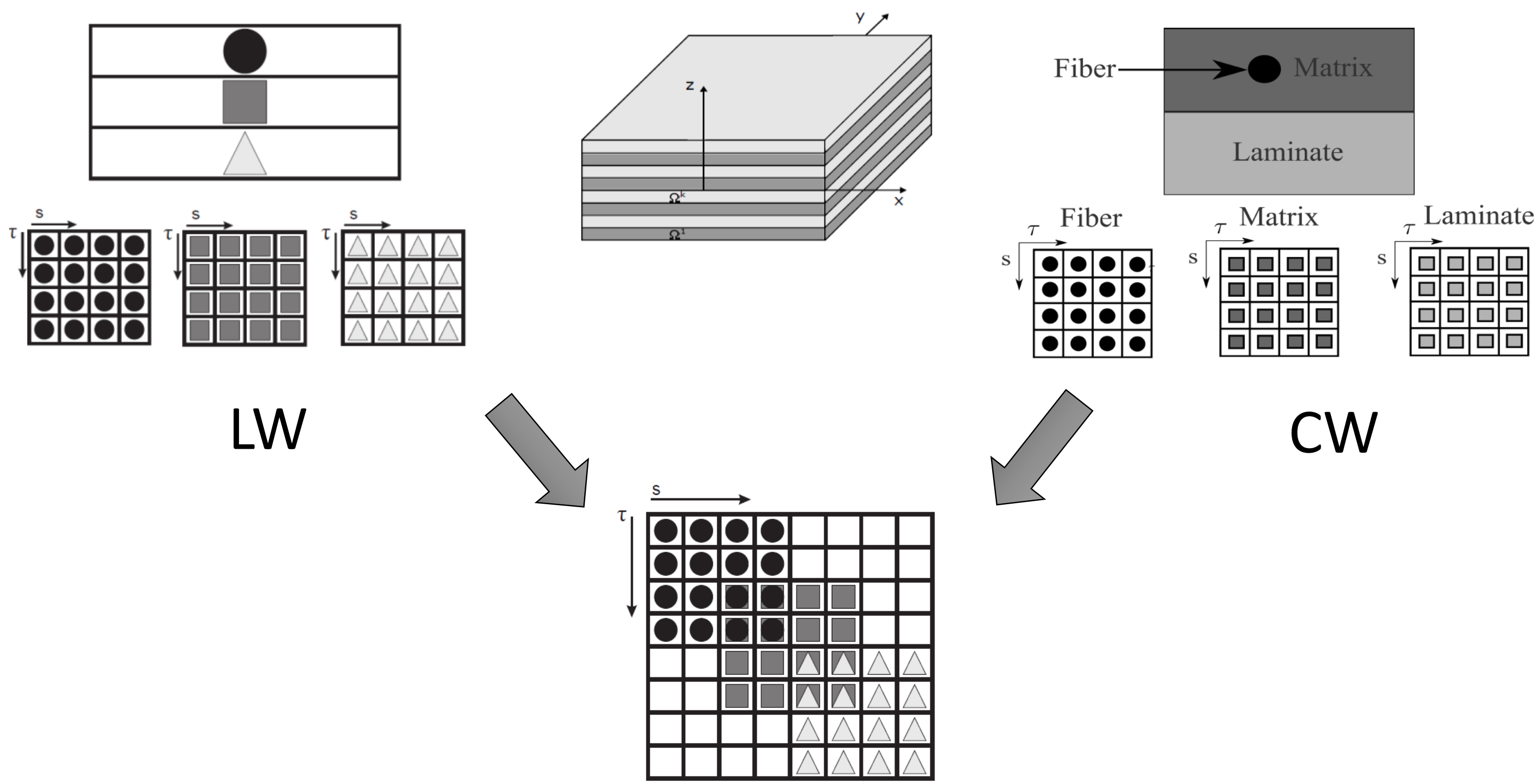
$$\boxed{\mathbf{u}(x,y,x) = F_\tau(x,z)\, \mathbf{u}_\tau(y) \qquad \tau = 1, \dots, M}$$

Component-wise analysis

- Using the unified formulation, **any class and order** of theory can be generated

  FSDT, HOT, ESL, LW, ZZ

- Component-wise (CW): generalization of LW to **any kind of structural component**

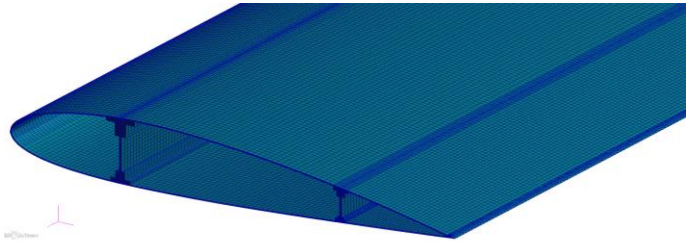


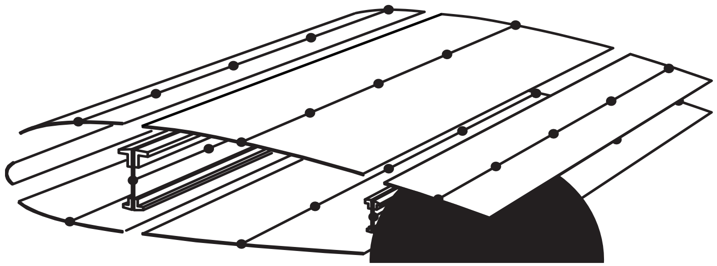
Assembled structural matrix

Component-wise 3M

Efficient structural solutions

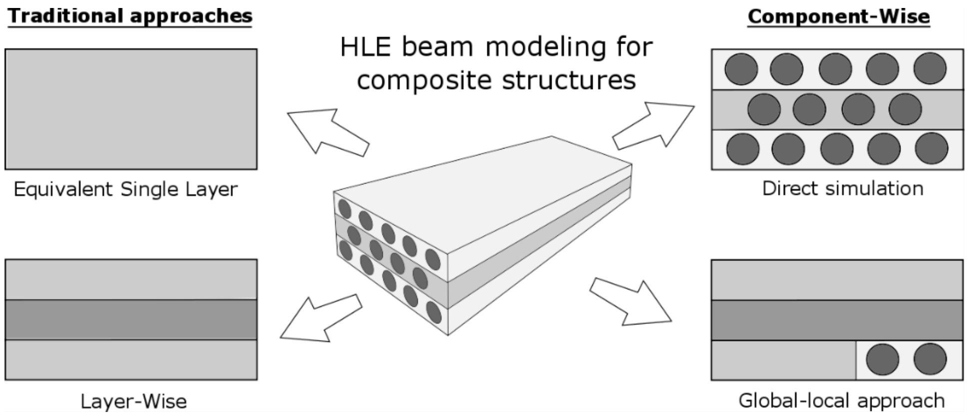
- Each sub-component modeled by means of 1D or 2D refined elements





Multi-scale composite simulation

- ESL, LW and CW-type models can be generated for the same structural problem
- **Optimized analysis**: linking the scale to the class of theory



Hierarchical Legendre Expansions

- Vertex expansions

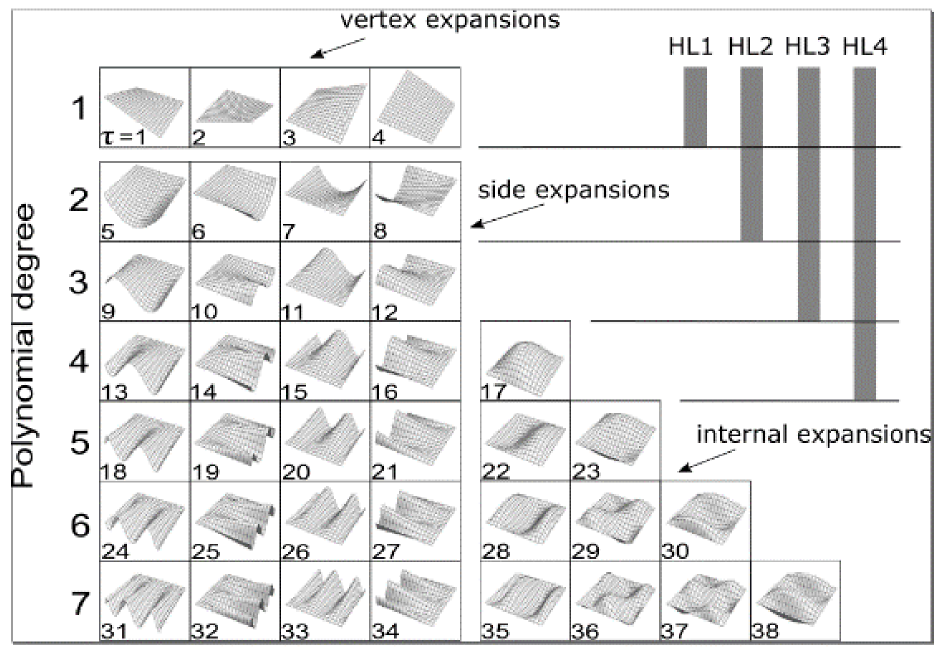
$$F_\tau = (1 - r_\tau r)(1 - s_\tau s)$$

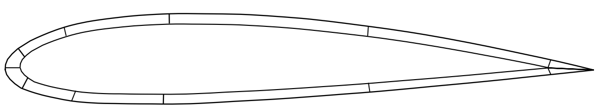
- Side expansions

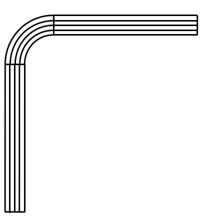
$$F_\tau = (1 - s)\varphi_p(r)$$

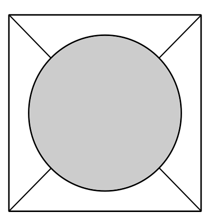
- Internal expansions

$$F_\tau = \varphi_{p_r}(r)\varphi_{p_s}(s) \qquad p_r + p_s = p$$









- **Hierarchical refinement** of the beam kinematics
- **Non-local distribution of unknowns** over the cross-section (CW)
- **Geometrically exact** curved sections by means of a non-isoparametric mapping

Weak form solutions

PVD for linear static

$$\delta L_{int} = \int_L \int_\Omega \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma}\, d\Omega\, dy = \delta L_{\text{ext}}$$

Dispacement field (1D FEM)

$$\mathbf{u}(x,y,z) = F_\tau(x,z) N_i(y) \mathbf{u}_{\tau i}$$

Internal work

$$\delta L_{\text{int}} = \delta \mathbf{u}_{\tau i}^T \mathbf{K}^{\tau s i j} \mathbf{u}_{sj}$$

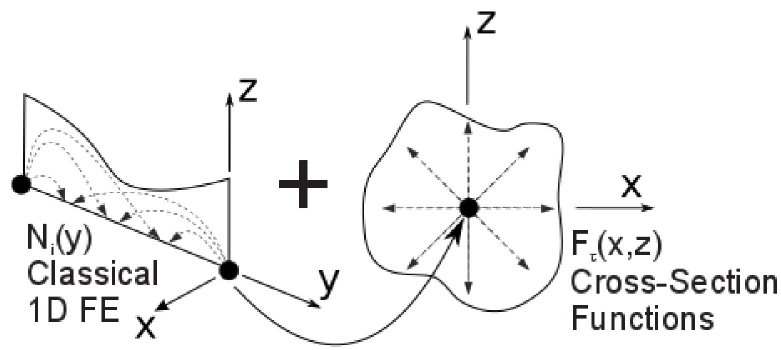
External work

$$\delta L_{\text{ext}} = F_\tau N_i \boldsymbol{P} \delta \mathbf{u}_{\tau i}^T$$

Governing equations

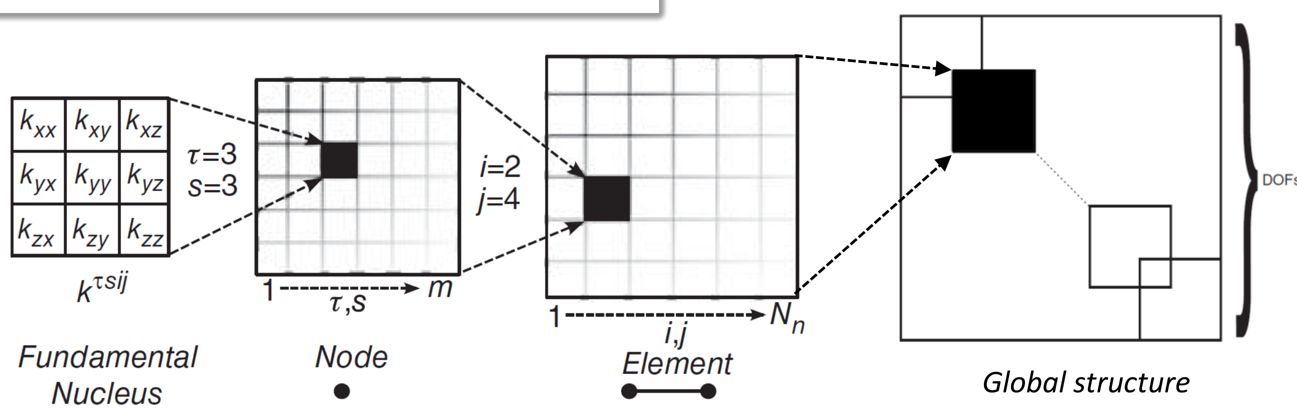
$$\mathbf{K}^{\tau s i j} \boldsymbol{U}^{sj} = \mathbf{P}^{\tau i}$$

1D FEM

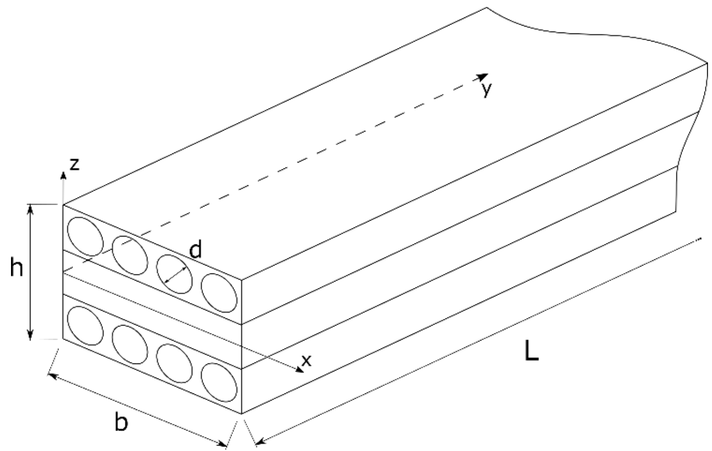


Fundamental nucleus

$$K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_\Omega F_{\tau,x} F_{s,x}\, d\Omega \int_l N_i N_j\, dy + \dots$$



Cross-ply beam



- L = 40 mm, h = 0.6 mm

  b =0.8 mm, d = 0.16 mm
- L/h = 50
- [0/90/0] laminate

Component	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
Fiber	202.038	12.134	12.134	8.358	8.358	47.756	0.2128	0.2128	0.2704
Layer	103.173	5.145	5.145	2.107	2.107	2.353	0.2835	0.2835	0.3124
Matrix		3.252			1.200			0.355	

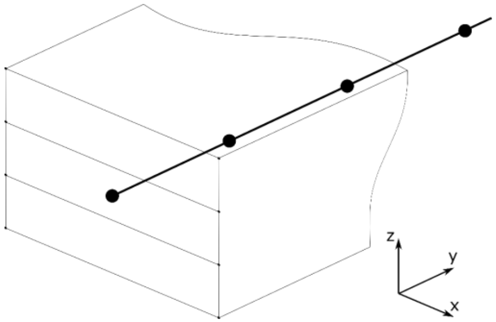
Table 4: 1 ≡ longitudinal, 2 ≡ orthogonal and 3 ≡ transverse.

Proposed approaches:

1. Meso-scale: layer-wise model, precision at the layer scale
2. Micro-scale: direct numerical model, precision at the component level
3. Meso-micro scale: global-local model, precision at the component level in areas of interest over the cross-section

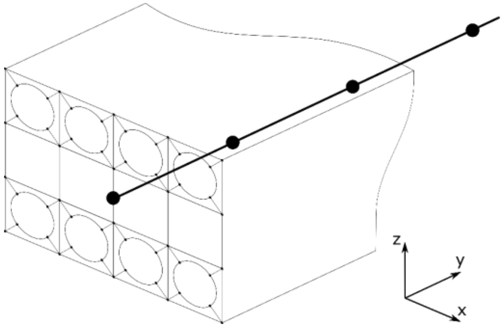
Cross-ply beam

Layer-wise approach



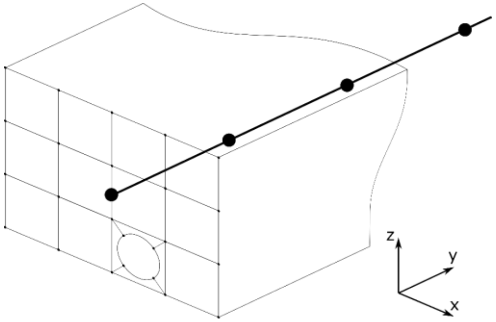
- 10 B4 beam elements
- 3 HLE expansions
- 1,674 (HL2) – 7,068 (HL6)

Direct numerical approach (1D)



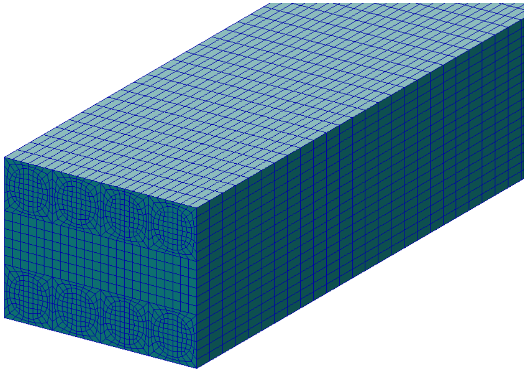
- 10 cubic B4 beam elements
- 44 HLE curved expansions
- 22,506 (HL3) – 73,563 (HL6)

Glocal-local approach



- 10 B4 beam elements
- 16 HLE curved expansions
- 9,486 (HL3) – 29,295 (HL6)

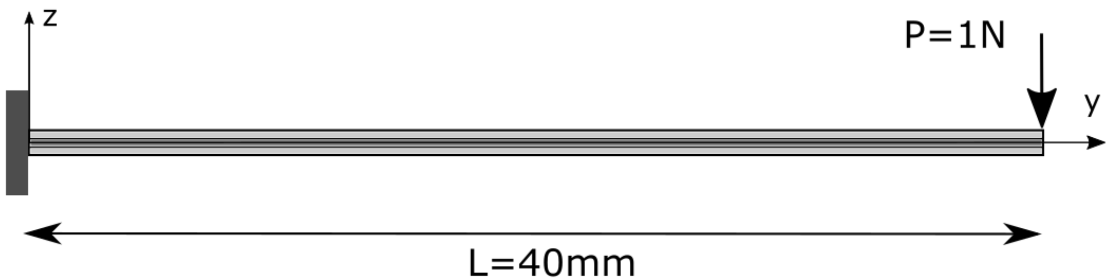
Direct numerical approach (3D Nastran)

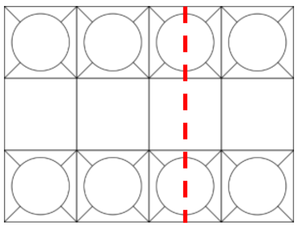


- 540,000 HEX8 brick elements
- 1,579,653 DOFs

Cross-ply beam

Loadcase: clamped-free + point load

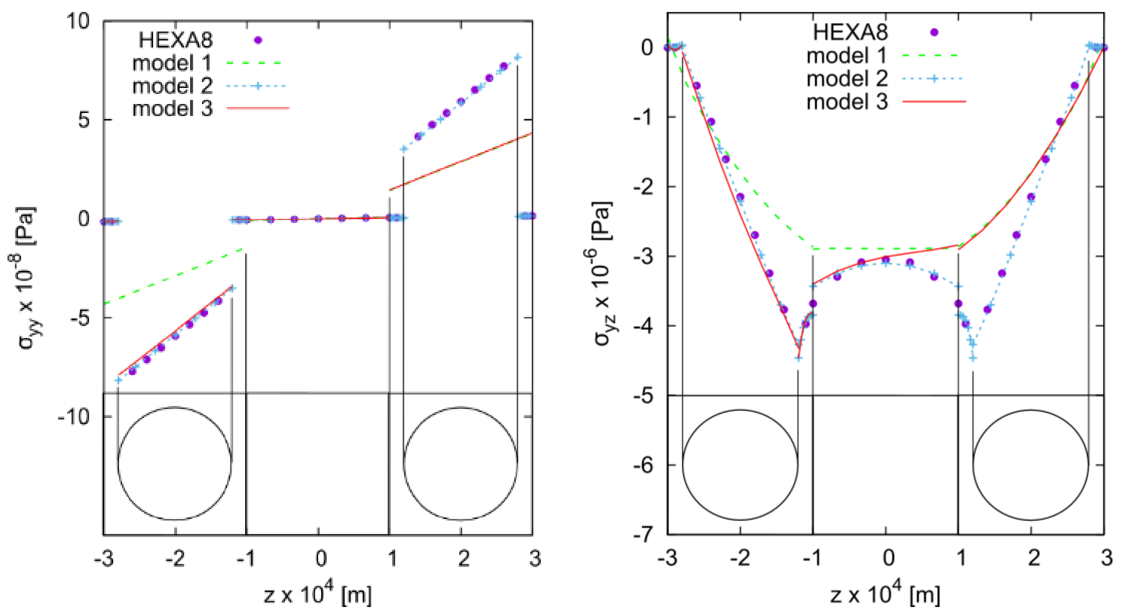




Convergence analysis

	$u_z \times 10^2$ m [b/2, L, 0]	$\sigma_{yy} \times 10^{-8}$ Pa [5b/2, L/2, -0.2]	$\sigma_{yz} \times 10^{-6}$ Pa [5b/2, L/2, -0.2]	DOFs
		MSC Nastran model		
HEXA8	-1.569	-5.928	-2.147	1579653
		HLE model 1		
HL2	-1.491	-2.880	-1.577	1674
HL3	-1.491	-2.880	-1.710	2604
HL4	-1.491	-2.879	-1.712	3813
HL5	-1.491	-2.879	-1.655	5031
HL6	-1.491	-2.880	-1.654	7068
		HLE model 2		
HL2	-0.348	-0.838	-43.008	13671
HL3	-1.547	-5.849	-2.142	22506
HL4	-1.548	-5.848	-2.169	35433
HL5	-1.548	-5.848	-2.211	52452
HL6	-1.548	-5.848	-2.212	73563
		HLE model 3		
HL2	-1.046	-3.717	-124.390	5859
HL3	-1.498	-5.661	-2.384	9486
HL4	-1.498	-5.659	-2.381	14601
HL5	-1.498	-5.659	-2.412	21204
HL6	-1.498	-5.659	-2.408	29295

Stress solutions, y=L/2



Cross-ply beam

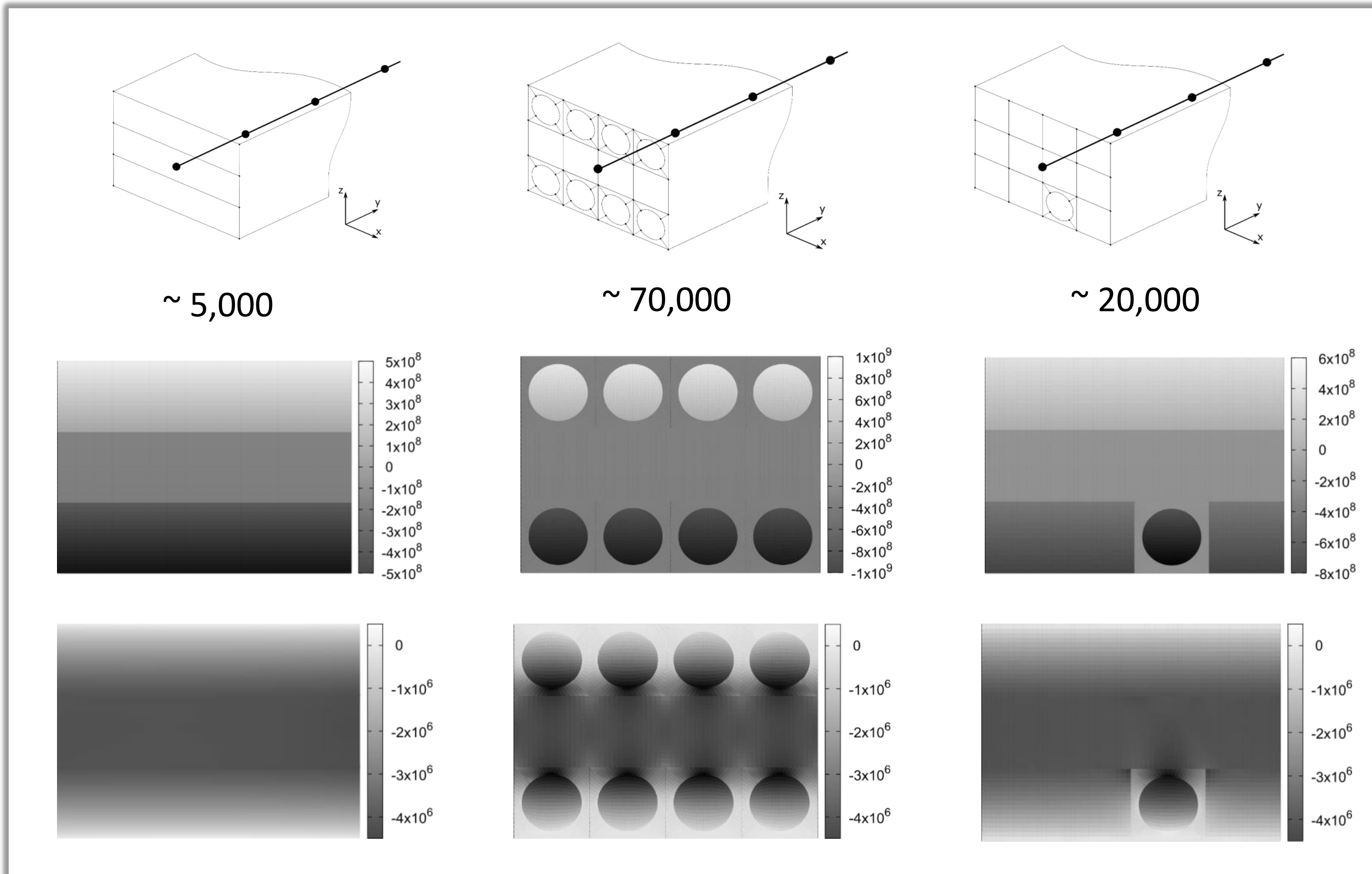
Layer wise | Direct approach | Global-local

Model

Cost

Longitudinal stresses

Shear stresses



Strong form solutions

Internal work

$$\delta L_{\text{int}} = \int_l (\delta \boldsymbol{u}_\tau)^{\text{T}} \mathbf{K}^{\tau s} \boldsymbol{u}_s \text{d}y + [(\delta \boldsymbol{u}_\tau)^{\text{T}} \boldsymbol{\Pi}^{\tau s} \boldsymbol{u}_s] \,|_{y=0}^{y=l}$$

External work

$$\delta L_{\text{ext}} = \left(\delta L_{\mathbf{p}_{\mathbf{xx}}^{\mathbf{n}\pm}} + \delta L_{\mathbf{p}_{\mathbf{xy}}^{\mathbf{n}\pm}} + \delta L_{\mathbf{p}_{\mathbf{xz}}^{\mathbf{n}\pm}} + \delta L_{\mathbf{p}_{\mathbf{zx}}^{\mathbf{n}\pm}} + \delta L_{\mathbf{p}_{\mathbf{zy}}^{\mathbf{n}\pm}} + \delta L_{\mathbf{p}_{\mathbf{zz}}^{\mathbf{n}\pm}} \right)$$

Navier-type solution

$$u_{xs}(y) = U_{xs} \sin(\alpha y)$$

$$u_{ys}(y) = U_{ys} \cos(\alpha y)$$

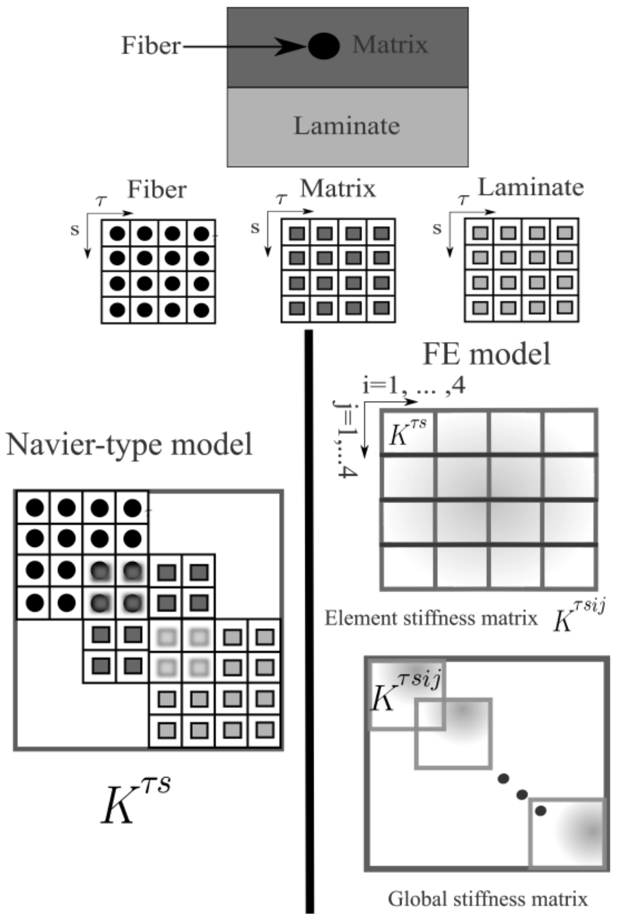
$$u_{zs}(y) = U_{zs} \sin(\alpha y)$$

$$\mathbf{p}_{\mathbf{ij}}^{\mathbf{n}\pm} = \left\{ \begin{array}{l} p_{xx}^{n\pm} \sin(\alpha y), p_{xy}^{n\pm} \cos(\alpha y), p_{xz}^{n\pm} \sin(\alpha y), \\ p_{zx}^{n\pm} \sin(\alpha y), p_{zy}^{n\pm} \cos(\alpha y), p_{zz}^{n\pm} \sin(\alpha y) \end{array} \right\}$$

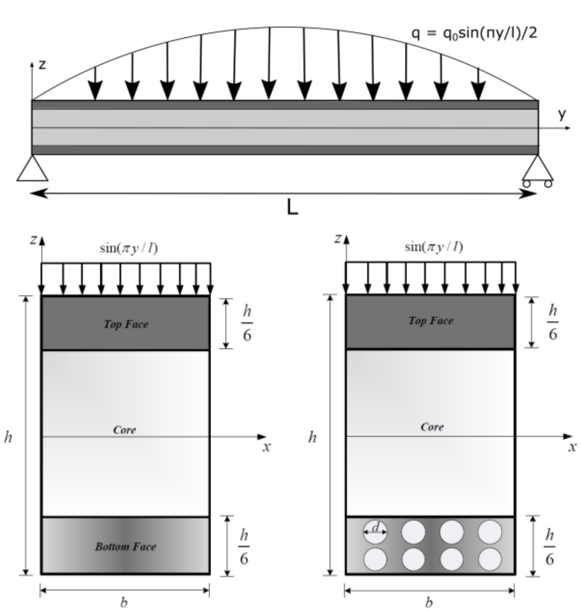
with $\alpha = \dfrac{m\pi}{l}$

Governing equations

$$\mathbf{K}^{\tau s} \boldsymbol{U}^{s} = \mathbf{P}^{\tau}$$



Sandwich beam



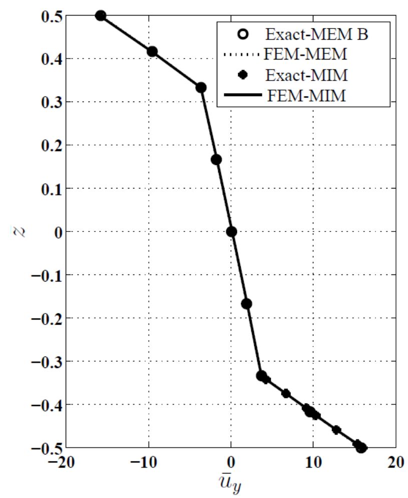
b = 0.04m, h = 0.12m, L/h = 10

Component	E_1 (Pa)	E_2 (Pa)	E_3 (Pa)	G_{12} (Pa)	G_{13} (Pa)	G_{23} (Pa)	ν_{12}	ν_{13}	ν_{23}
Fiber	4.44×10^{11}	1.16×10^{10}	1.16×10^{10}	8.67×10^{9}	8.67×10^{9}	4.80×10^{9}	0.21	0.21	0.21
Matrix	8.55×10^{9}	8.55×10^{9}	8.55×10^{9}	3.28×10^{9}	3.28×10^{9}	3.28×10^{9}	0.31	0.31	0.31
Core	2.21×10^{5}	2.00×10^{5}	2.76×10^{9}	1.66×10^{6}	5.45×10^{8}	4.55×10^{8}	0.99	3.00E-05	3.00E-05
Top face	2.28×10^{11}	9.85×10^{9}	9.85×10^{9}	4.77×10^{9}	4.77×10^{9}	3.92×10^{9}	0.26	0.26	0.26

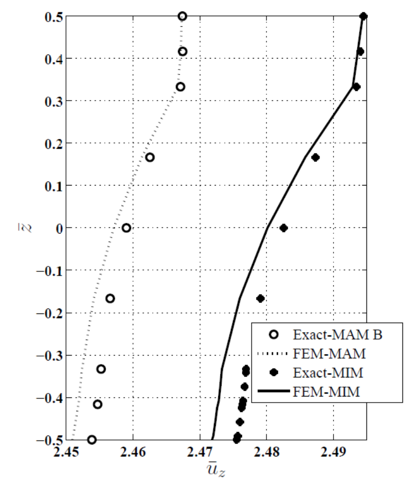
DOFs (Navier vs FE):

- ➢ LW (975 vs 18,525)
- ➢ GL (7,509 vs 142,671)

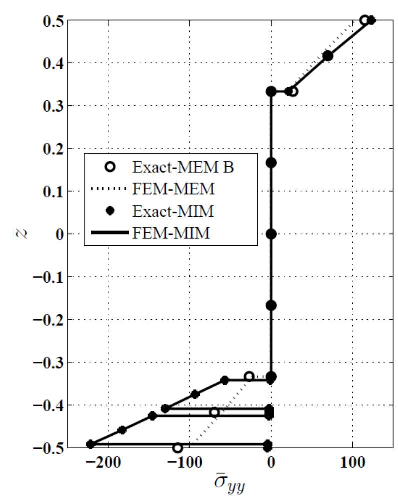
u_y at (b/2,0,z)



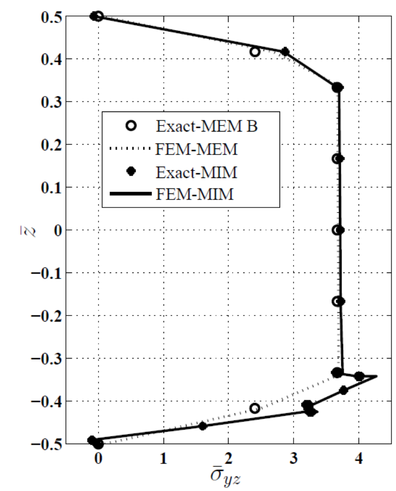
u_z at (b/2,L/2,z)



σ_{yy} at (b/2,L/2,z)

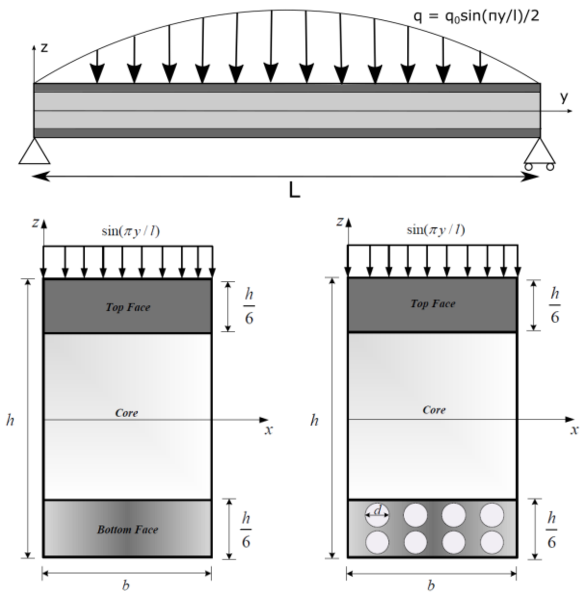


σ_{yz} at (b/2,0,z)



*Y. Yan *et.al. Exact solutions for the macro-, meso- and micro-scale analysis of composite laminates and sandwich structures*

Sandwich beam



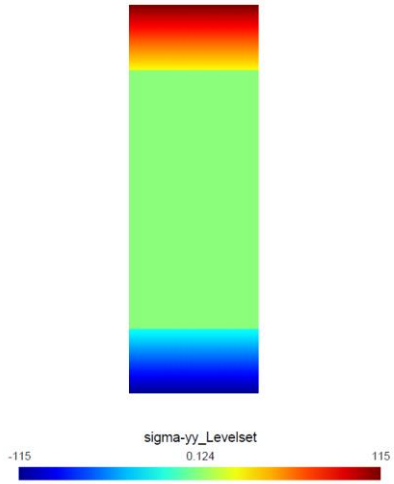
b = 0.04m, h = 0.12m, L/h = 10

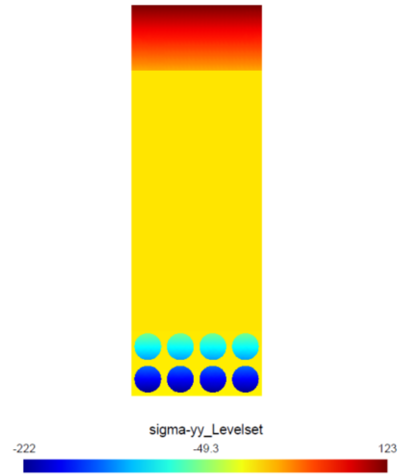
Component	E_1 (Pa)	E_2 (Pa)	E_3 (Pa)	G_{12} (Pa)	G_{13} (Pa)	G_{23} (Pa)	ν_{12}	ν_{13}	ν_{23}
Fiber	4.44×10^{11}	1.16×10^{10}	1.16×10^{10}	8.67×10^{9}	8.67×10^{9}	4.80×10^{9}	0.21	0.21	0.21
Matrix	8.55×10^{9}	8.55×10^{9}	8.55×10^{9}	3.28×10^{9}	3.28×10^{9}	3.28×10^{9}	0.31	0.31	0.31
Core	2.21×10^{5}	2.00×10^{5}	2.76×10^{9}	1.66×10^{6}	5.45×10^{8}	4.55×10^{8}	0.99	3.00E-05	3.00E-05
Top face	2.28×10^{11}	9.85×10^{9}	9.85×10^{9}	4.77×10^{9}	4.77×10^{9}	3.92×10^{9}	0.26	0.26	0.26

Cross-section (DOFs Navier vs FE):

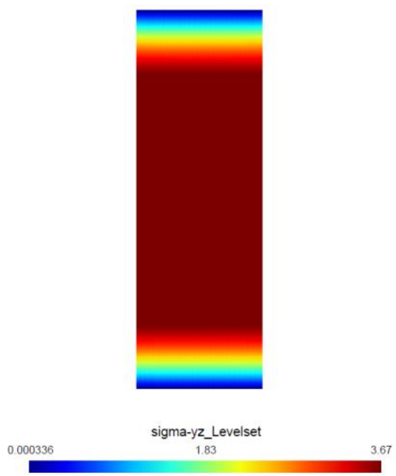
- ➢ LW: 32 L16 (975 vs 18,525)
- ➢ GL: 272 L16 (7,509 vs 142,671)

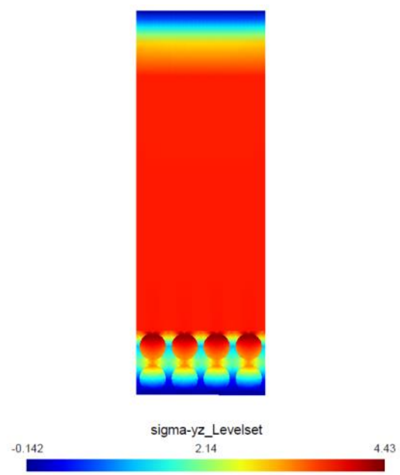
Longitudinal stresses at midspan





Transverse shear stresses at edge





*Y. Yan *et.al. Exact solutions for the macro-, meso- and micro-scale analysis of composite laminates and sandwich structures*

Conclusions

- The unified formulation is used as a generator of structural theories to provide efficient solutions for composite problems
- The component-wise (CW) method is presented as an extension of the traditional approaches (ESL, LW,…) and applied to the accurate analysis of composite structures
- A 3M (macro-, meso- and micro-scale) framework is proposed. Objects from the component to the fiber level are accounted in a unified manner without the need of changing the model paradigms from one scale to the other nor the use of artificial coupling techniques
- Low cost exact solutions can also be obtained through a strong formulation of the CW for particular cases. This tool can be used for benchmarking.

Future work

- Global-local framework in which FSDT, ESL, LW and CW theories can be axiomaticaly placed over the finite element space -> Node Dependent Kinematics (NDK)
- Investigation of damage and failure

Acknowledgements

FULLCOMP - FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures

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  4. Leibniz Universitaet Hannover (Germany)
  5. LIST (Luxemburg)
  6. ELAN-AUSY GmbH (Germany)
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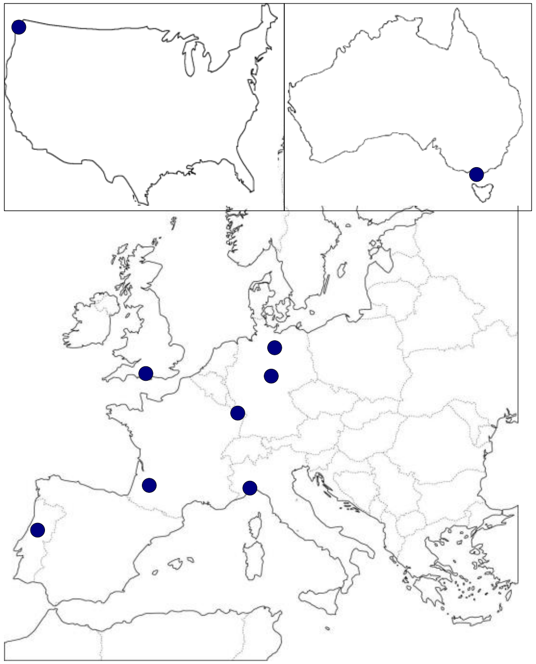

Thank you for the attention,
Any questions?