Micro-, meso- and macro-scale analysis of composite laminates by unified theory of structures

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Our work

➢ Provide a unified methodology to systematically generate different classes of structural models

➢ Zoom into different scales by means of a unified formulation with no decoupling

➢ Reduce the computational size of composite simulation in such a way that several scales can be accounted
I. The unified formulation for beam analysis

II. Component-wise analysis: coupling macro-, meso, micro-scale modeling

III. Weak form solutions
    - Cross-ply beam

IV. Strong form solutions
    - Sandwich beam

V. Conclusions and perspectives
Carrera Unified Formulation 1D

**Euler-Bernoulli**

\[
\begin{align*}
    u_x(x, y, z) &= u_{x1}(y) \\
    u_y(x, y, z) &= u_{y1}(y) - x u_{x1,y}(y) - z u_{z1,y}(y) \\
    u_z(x, y, z) &= u_{z1}(y)
\end{align*}
\]

**Timoshenko**

\[
\begin{align*}
    u_x(x, y, z) &= u_{x1}(y) \\
    u_y(x, y, z) &= u_{y1}(y) - x u_{y2}(y) - z u_{y3}(y) \\
    u_z(x, y, z) &= u_{z1}(y)
\end{align*}
\]

**Saint Venant**

\[
\begin{align*}
    u_x(x, y, z) &= u_{x1}(y) - z u_{y2}(y) \\
    u_y(x, y, z) &= u_{y1}(y) - x u_{y2}(y) - z u_{y3}(y) + \psi(x,z)u_{y3}(y) \\
    u_z(x, y, z) &= u_{z1}(y) + x u_{z2}(y)
\end{align*}
\]

K. Washizu: "For a complete removal of the inconsistency and an improvement of the accuracy of the beam theory" -> enrich beam kinematics with higher-order terms

**Carrera Unified Formulation**

\[
\begin{align*}
    u_x(x, y, z) &= F_1(x, z) u_{x1}(y) + F_2(x, z) u_{x2}(y) + F_3(x, z) u_{x3}(y) + \ldots + F_M(x, z) u_{xM}(y) \\
    u_y(x, y, z) &= F_1(x, z) u_{y1}(y) + F_2(x, z) u_{y2}(y) + F_3(x, z) u_{y3}(y) + \ldots + F_M(x, z) u_{yM}(y) \\
    u_z(x, y, z) &= F_1(x, z) u_{z1}(y) + F_2(x, z) u_{z2}(y) + F_3(x, z) u_{z3}(y) + \ldots + F_M(x, z) u_{zM}(y)
\end{align*}
\]

\[
\mathbf{u}(x, y, x) = \mathbf{F}_\tau(x, z) \mathbf{u}_\tau(y) \quad \tau = 1, \ldots, M
\]
Component-wise analysis

- Using the unified formulation, **any class and order** of theory can be generated
  - FSDT, HOT, ESL, LW, ZZ
- Component-wise (CW): generalization of LW to **any kind of structural component**

![Assembled structural matrix](image)
Efficient structural solutions

- Each sub-component modeled by means of 1D or 2D refined elements

Multi-scale composite simulation

- ESL, LW and CW-type models can be generated for the same structural problem
- **Optimized analysis**: linking the scale to the class of theory
Hierarchical Legendre Expansions

➢ **Vertex expansions**

\[ F_\tau = (1 - r_\tau r)(1 - s_\tau s) \]

➢ **Side expansions**

\[ F_\tau = (1 - s)\varphi_p(r) \]

➢ **Internal expansions**

\[ F_\tau = \varphi_{pr}(r)\varphi_{ps}(s) \quad p_r + p_s = p \]

➢ **Hierarchical refinement** of the beam kinematics

➢ **Non-local distribution of unknowns** over the cross-section (CW)

➢ **Geometrically exact** curved sections by means of a non-isoparametric mapping
Weak form solutions

PVD for linear static
\[ \delta L_{\text{int}} = \int_L \int_{\Omega} \delta \varepsilon^T \sigma \, d\Omega \, dy = \delta L_{\text{ext}} \]

Displacement field (1D FEM)
\[ u(x, y, z) = F_\tau(x, z) N_i(y) u_{\tau i} \]

Internal work
\[ \delta L_{\text{int}} = \delta u_{\tau i}^T K^{\tau s i j} u_{s j} \]

External work
\[ \delta L_{\text{ext}} = F_\tau \dot{N}_i P \delta u_{\tau i}^T \]

Governing equations
\[ K^{\tau s i j} U^{s j} = P^{\tau i} \]

Fundamental nucleus
\[ K_{x x}^{i j r s} = \tilde{C}_{22} \int_{\Omega} F_{\tau, x} F_{s, x} \, d\Omega \int_{l} N_i N_j \, dy + \ldots \]
Cross-ply beam

- $L = 40 \text{ mm}$, $h = 0.6 \text{ mm}$
- $b = 0.8 \text{ mm}$, $d = 0.16 \text{ mm}$
- $L/h = 50$
- $[0/90/0]$ laminate

Proposed approaches:
1. **Meso-scale**: layer-wise model, precision at the layer scale
2. **Micro-scale**: direct numerical model, precision at the component level
3. **Meso-micro scale**: global-local model, precision at the component level in areas of interest over the cross-section

<table>
<thead>
<tr>
<th>Component</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_3$</th>
<th>$G_{12}$</th>
<th>$G_{13}$</th>
<th>$G_{23}$</th>
<th>$\nu_{12}$</th>
<th>$\nu_{13}$</th>
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<td>12.134</td>
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<td>8.358</td>
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</tbody>
</table>

Table 4: $1 \equiv$ longitudinal, $2 \equiv$ orthogonal and $3 \equiv$ transverse.
Cross-ply beam

Layer-wise approach
- 10 B4 beam elements
- 3 HLE expansions
- 1,674 (HL2) – 7,068 (HL6)

Direct numerical approach (1D)
- 10 cubic B4 beam elements
- 44 HLE curved expansions
- 22,506 (HL3) – 73,563 (HL6)

Glocal-local approach
- 10 B4 beam elements
- 16 HLE curved expansions
- 9,486 (HL3) – 29,295 (HL6)

Direct numerical approach (3D Nastran)
- 540,000 HEX8 brick elements
- 1,579,653 DOFs

November 6, 2017, Tampa Convention Center, Tampa (FL), USA
Cross-ply beam

Loadcase: clamped-free + point load

\[ L = 40 \text{mm} \]

\[ P = 1 \text{N} \]

Convergence analysis

<table>
<thead>
<tr>
<th>Hexa8</th>
<th>( v_z \times 10^3 \text{ m} )</th>
<th>( \sigma_{yy} \times 10^{-8} \text{ Pa} )</th>
<th>( \sigma_{yx} \times 10^{-8} \text{ Pa} )</th>
<th>DOFs</th>
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<td></td>
<td>[b/2, L/2]</td>
<td>[b/2, L/2, 0.0]</td>
<td>[b/2, L/2, 0.2]</td>
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<td>-5.659</td>
<td>-2.408</td>
<td>29295</td>
</tr>
</tbody>
</table>
Cross-ply beam

Layer wise

Longitudinal stresses

Shear stresses

Cost

~ 5,000

~ 70,000

~ 20,000

Model

Direct approach

Global-local

~ 20,000
Strong form solutions

Internal work
\[ \delta L_{\text{int}} = \int_I (\delta u_\tau)^T K^{\tau_\text{s}} u_s \, dy + [ (\delta u_\tau)^T \Pi^{\tau_\text{s}} u_s ] \big|_{y=l} \]

External work
\[ \delta L_{\text{ext}} = \left( \delta L_{p_{xx}}^{n_\pm} + \delta L_{p_{xy}}^{n_\pm} + \delta L_{p_{xz}}^{n_\pm} + \delta L_{p_{zy}}^{n_\pm} + \delta L_{p_{zz}}^{n_\pm} \right) \]

Navier-type solution
\[
\begin{align*}
    u_{xs}(y) &= U_{xs} \sin(\alpha y) \\
    u_{ys}(y) &= U_{ys} \cos(\alpha y) \\
    u_{zs}(y) &= U_{zs} \sin(\alpha y)
\end{align*}
\]

\[
P_{ij}^{n_\pm} = \begin{cases} 
    p_{xx}^{n_\pm} \sin(\alpha y), p_{xy}^{n_\pm} \cos(\alpha y), p_{xz}^{n_\pm} \sin(\alpha y), \\
    p_{zx}^{n_\pm} \sin(\alpha y), p_{zy}^{n_\pm} \cos(\alpha y), p_{zz}^{n_\pm} \sin(\alpha y) \end{cases}
\]

with \( \alpha = \frac{m \pi}{l} \)

Governing equations
\[ K^{\tau_\text{s}} U^{\text{s}} = P^{\tau} \]
**Sandwich beam**

\[ q = q_0 \sin(\eta y)/\eta \]

\[ u_y \text{ at } (b/2,0,z) \]

\[ u_z \text{ at } (b/2,L/2,z) \]

\[ \sigma_{yy} \text{ at } (b/2,L/2,z) \]

\[ \sigma_{yz} \text{ at } (b/2,0,z) \]

\[ b = 0.04 \text{m}, \ h = 0.12 \text{m}, \ L/h = 10 \]

**DOFs (Navier vs FE):**

- LW (975 vs 18,525)
- GL (7,509 vs 142,671)

---

**Table:**

<table>
<thead>
<tr>
<th>Component</th>
<th>( E_1 ) (Pu)</th>
<th>( E_2 ) (Pu)</th>
<th>( G_{12} ) (Pu)</th>
<th>( G_{13} ) (Pu)</th>
<th>( G_{23} ) (Pu)</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>4.41x10^11</td>
<td>1.30x10^10</td>
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<td>8.47x10^8</td>
<td>4.10x10^8</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>Matrix</td>
<td>6.55x10^9</td>
<td>8.55x10^9</td>
<td>5.55x10^9</td>
<td>3.25x10^9</td>
<td>3.25x10^9</td>
<td>3.25x10^9</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Core</td>
<td>2.04x10^7</td>
<td>2.04x10^7</td>
<td>2.04x10^7</td>
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<td>1.01x10^7</td>
<td>1.01x10^7</td>
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<td>Top face</td>
<td>2.04x10^7</td>
<td>9.50x10^6</td>
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<td>4.77x10^6</td>
<td>3.92x10^6</td>
<td>0.26</td>
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</tbody>
</table>

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*Y. Yan et al. Exact solutions for the macro-, meso- and micro-scale analysis of composite laminates and sandwich structures*
Sandwich beam

Longitudinal stresses at midspan

Transverse shear stresses at edge

*b0 = 0.04m, h = 0.12m, L/h = 10*

<table>
<thead>
<tr>
<th>Component</th>
<th>$E_1$ (Pa)</th>
<th>$E_2$ (Pa)</th>
<th>$E_3$ (Pa)</th>
<th>$G_{12}$ (Pa)</th>
<th>$G_{13}$ (Pa)</th>
<th>$G_{23}$ (Pa)</th>
<th>$\nu_{12}$</th>
<th>$\nu_{13}$</th>
<th>$\nu_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>4.44x10^10</td>
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<td>1.48x10^9</td>
<td>0.21</td>
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<tr>
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<td>8.55x10^9</td>
<td>3.29x10^9</td>
<td>3.29x10^9</td>
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<tr>
<td>Top face</td>
<td>2.29x10^9</td>
<td>9.65x10^7</td>
<td>9.65x10^7</td>
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<td>3.92x10^7</td>
<td>0.26</td>
<td>0.20</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Cross-section (DOFs Navier vs FE):

➢ LW: 32 L16 (975 vs 18,525)
➢ GL: 272 L16 (7,509 vs 142,671)

*Y. Yan et.al. Exact solutions for the macro-, meso- and micro-scale analysis of composite laminates and sandwich structures*
Conclusions

➢ The unified formulation is used as a generator of structural theories to provide efficient solutions for composite problems

➢ The component-wise (CW) method is presented as an extension of the traditional approaches (ESL, LW,...) and applied to the accurate analysis of composite structures

➢ A 3M (macro-, meso- and micro-scale) framework is proposed. Objects from the component to the fiber level are accounted in a unified manner without the need of changing the model paradigms from one scale to the other nor the use of artificial coupling techniques

➢ Low cost exact solutions can also be obtained through a strong formulation of the CW for particular cases. This tool can be used for benchmarking.

Future work

➢ Global-local framework in which FSDT, ESL, LW and CW theories can be axiomatically placed over the finite element space -> Node Dependent Kinematics (NDK)

➢ Investigation of damage and failure
FULLCOMP - FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures

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