Layerwise beam elements based on the Carrera's unified formulation and the Reissner's mixed variational theorem

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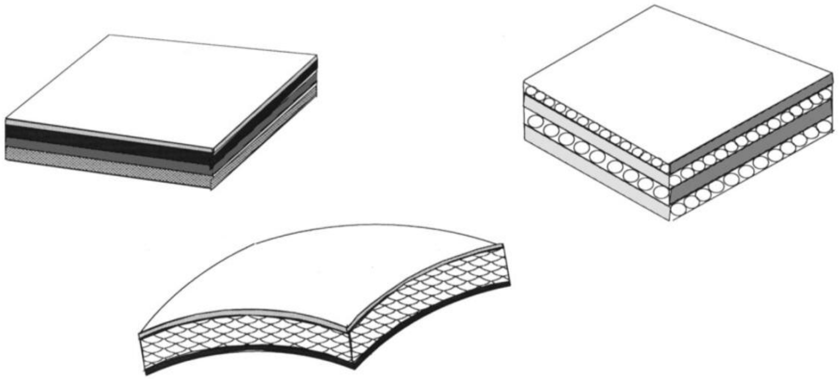
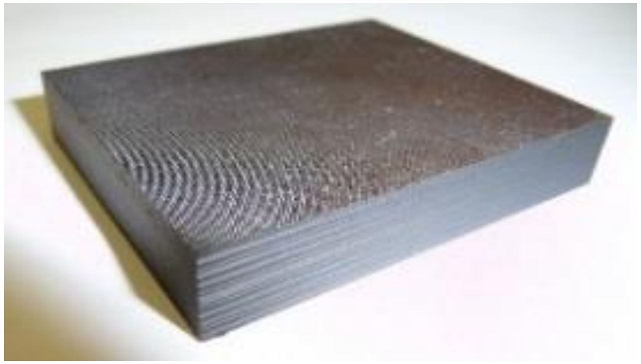
Overview



- Laminates: simulation challenges
- Reissner's mixed variational theorem (RMVT)
- Implementation of higher-order mixed beam elements
- Numerical assessment
- Advanced capabilities
- Conclusions

Laminated structures

- Multi-layered structures are built by adding plies of the same, or different materials, in a certain stacking sequence
- Optimized performance of the component
- Advantages: good specific properties



Transverse anisotropy: sudden change of mechanical properties in the through-the-thickness direction

Transverse stresses not neglible due to the high ratio between the elastic moduli (E_L/E_T= 5 – 40) and low transverse shear moduli (G_{LT} and G_{TT})

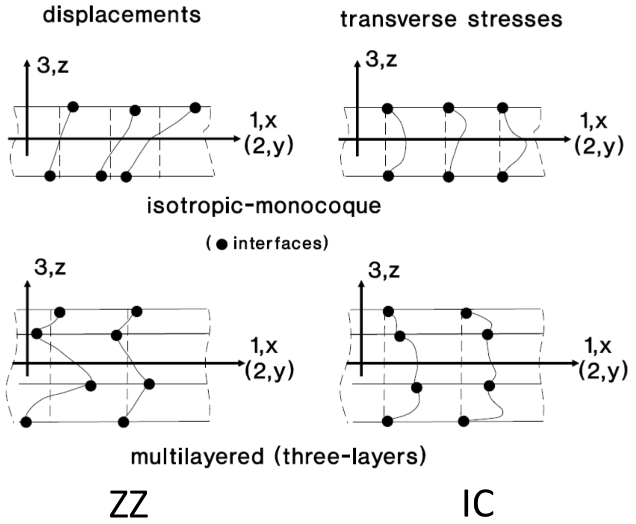
Compatibility: sudden changes in the slope of the displacement fields (u_x, u_y, u_z) across the thickness of the laminate

Equilibrium: continuity of transverse stresses (σ_{zz}, σ_{yz} , σ_{xz})

<u>Kinematics: C^0_z requirements</u>

- C^0 for displacements -> zig-zag effect (ZZ)
- C^0 for transverse stresses -> interlaminar continuity (IC)

Major challenge in the modelling of laminated structures: trasverse deformation must be included



RMVT

A posteriori approaches

- Classical Laminate Theories (CLT) and First-order Shear Deformation Theories do not fulfill the C^0_z requirements -> first derivatives constant
- Higher-order theories and ZZ theories -> IC not necessarily satisfied
- Layer-wise (LW) models account for independent kinematics at each layer
- Stress recovery methods: integration of stress solutions in the 3D equilibrium equations

Reissner's Mixed Variational Theorem (RMVT)

A priori fulfillment of the C^0_z requirements through the use of independent displacement and stress assumptions

Stress assumptions are *restricted to the transverse components* in laminates

PVD

$$\int_V (\delta\varepsilon_p^T\, \boldsymbol{\sigma}_p + \delta\varepsilon_n^T\, \boldsymbol{\sigma}_n)\, dV = \delta L_e$$

RMVT

$$\int_V (\delta\varepsilon_{pG}^T\, \boldsymbol{\sigma}_{pH} + \delta\varepsilon_{nG}^T\, \boldsymbol{\sigma}_{nM} + \boxed{\delta\boldsymbol{\sigma}_{nM}^T (\varepsilon_{nG} - \varepsilon_{nH})})\, dV = \delta L_e$$

Geometrical relations

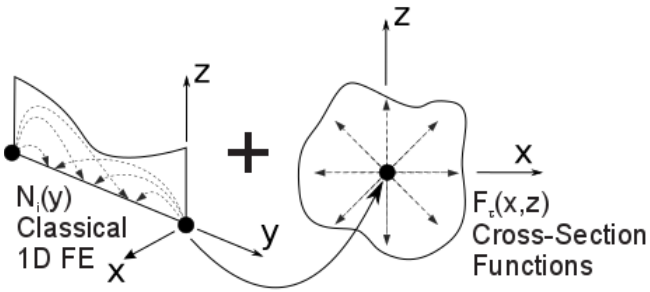
$$\varepsilon_{pG} = \boldsymbol{D}_{pG}\, \boldsymbol{u}$$
$$\varepsilon_{nG} = \boldsymbol{D}_{nG}\, \boldsymbol{u}$$

Constitutive equations

$$\boldsymbol{\sigma}_{pH}^k = \mathbf{C}_{pp}^k\, \varepsilon_{nG}^k + \mathbf{C}_{pn}^k\, \boldsymbol{\sigma}_{nM}^k$$
$$\varepsilon_{nH}^k = \mathbf{C}_{np}^k\, \varepsilon_{pG}^k + \mathbf{C}_{nn}^k\, \boldsymbol{\sigma}_{nM}^k$$

Mixed beam elements

Carrera Unified Formulation (CUF)



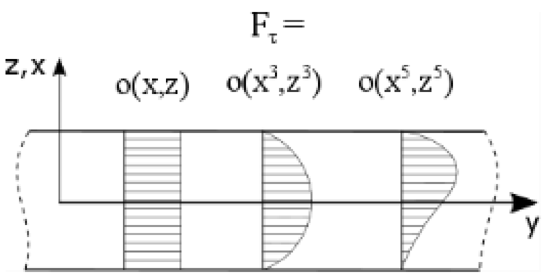
Beam kinematics:

$$\boldsymbol{u}(x,y,z) = N_i(y)\, F_\tau(x,z)\, \boldsymbol{u}_{\tau i}$$

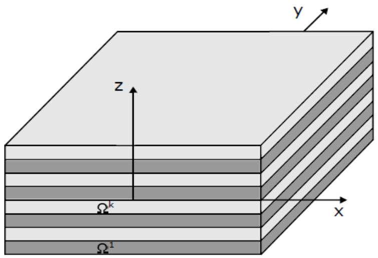
Fundamental Nucleus:

$$K_{xx}^{ij\tau s} = \tilde{C}_{22} \int_\Omega F_{\tau,x} F_{s,x}\, d\Omega \int_l N_i N_j dy + \dots$$

- Various solutions over the section



Mixed refined beams



LW beam kinematics:

$$\boldsymbol{u}^k(x,y,z) = N_i(y)\, F_\tau(x,z)\, \boldsymbol{u}^k{}_{\tau i}$$

LW stress assumptions:

$$\boldsymbol{\sigma}^k{}_n(x,y,z) = N_i(y)\, G_\tau(x,z)\, \boldsymbol{\sigma}^k{}_{n\tau i}$$

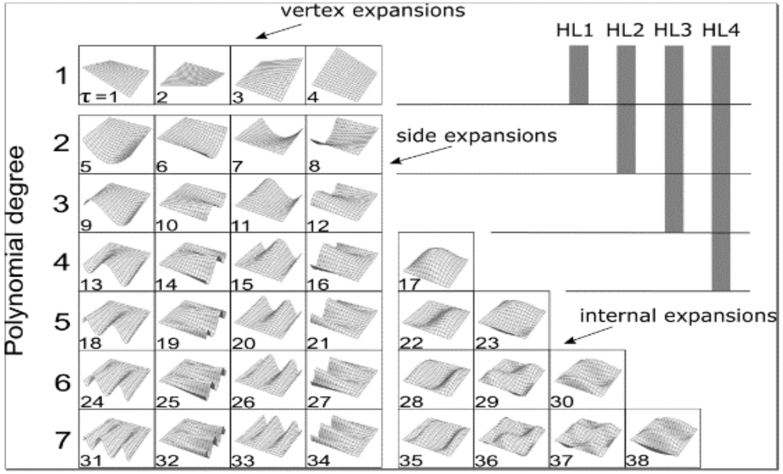
- Increase of number of unknowns
- No ZZ assumed functions required
- C^0_z requirements and IC satisfied at the interfaces between plies

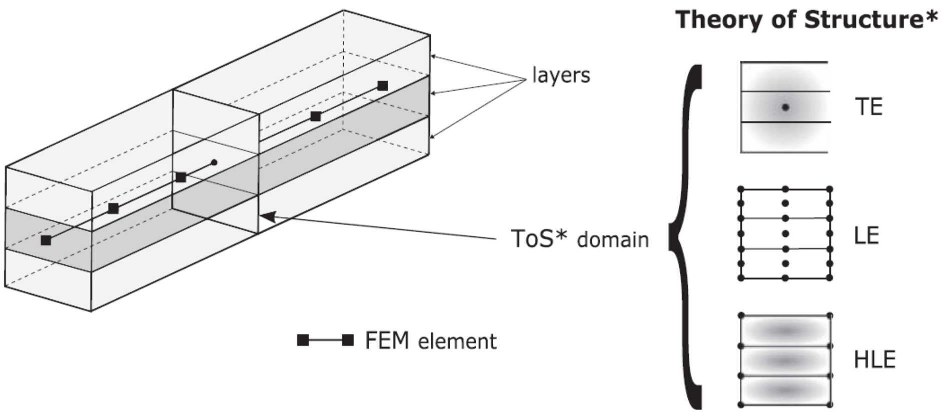
- Compatibility of displacements $\mathbf{u}_t^k = \mathbf{u}_b^{k+1}$
- IC of transverse stresses $\boldsymbol{\sigma}_{nt}^k = \boldsymbol{\sigma}_{nb}^{k+1}$

Beam theory

Hierarchical Legendre Expansions (HLE)

- Nodal, side and internal unknowns
- Hierarchical p-refinement of the expansion assumptions
- Layer-wise distributions of unknowns
- Strightforward assemby of displacements and stresses at the interfaces between layers

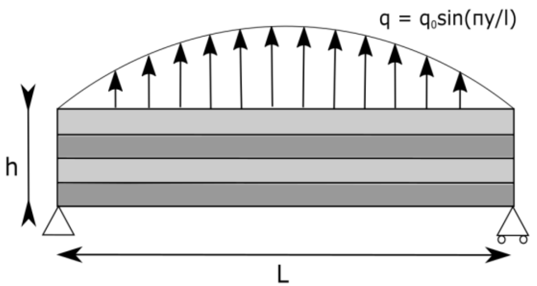




* Carrera, E., de Miguel, A., and Pagani, A., "Hierarchical theories of structures based on Legendre polynomial expansions with finite element applications," *International Journal of Mechanical Sciences*, Vol. 120, 2017, pp. 286 – 300

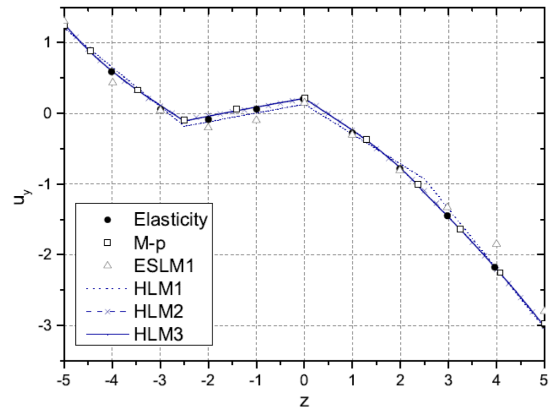
Assessment: linear static

*Pagano, 1969

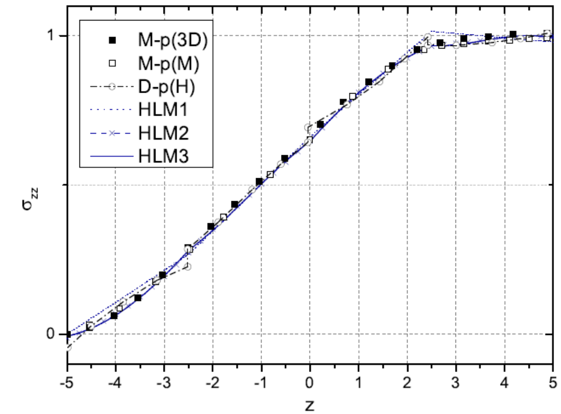


- Thick laminate: L/h = 4
- [0,90,0,90]
- E_1=25MPa E_2=1MPa
  G_{12}=0.5MPa G_{23}=0.2MPa
  $V_{12} = V_{23} = 0.25$
- Cylindrical bending
- 1D model
  4 cubic mixed elements
  4 HLE domains (LW)

Longitudinal displacements

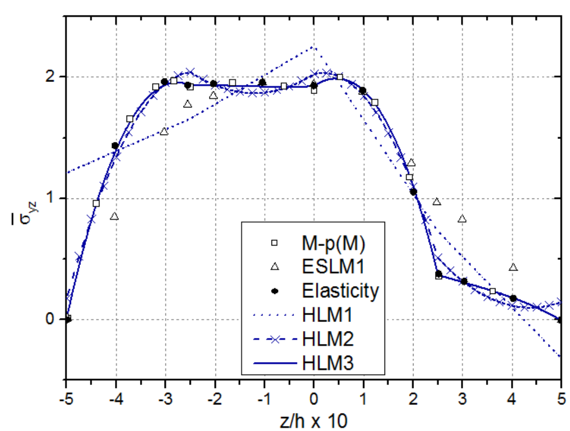


Normal stresses

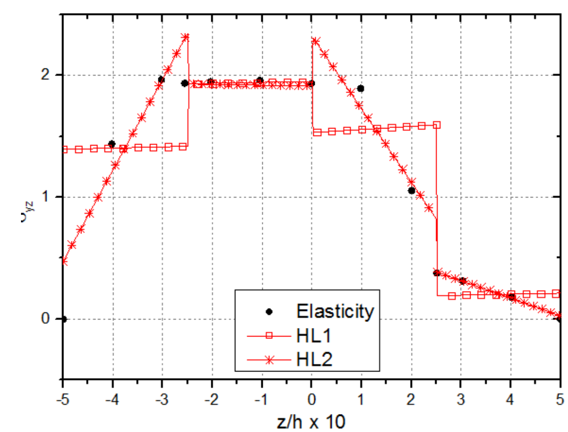


Shear stresses

RMVT



PVD

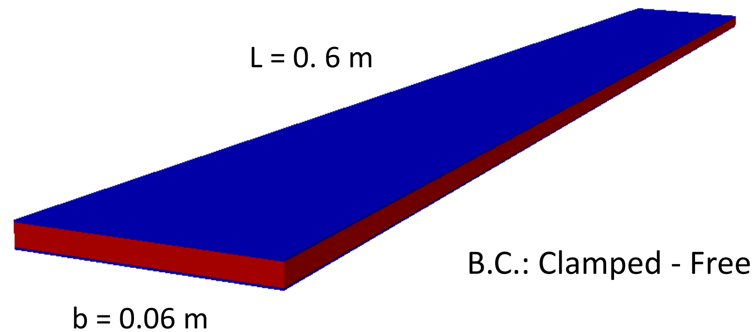


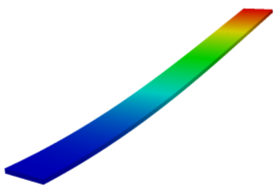
Assessment: free vibration

Sandwich beam

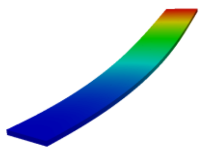
- Faces: **Al** (E=75 GPa, ν=0.33), 0.5 mm
- Core: **Foam** (E=0.1063 Gpa, ν=0.32), 5 mm

1D model:
- 10 cubic beam elements
- 3 HLE domains

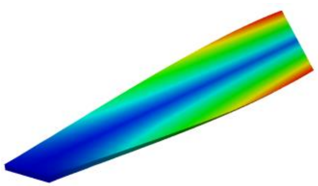




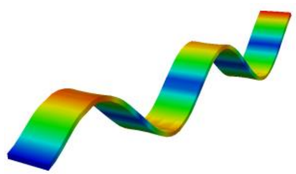
Mode 1



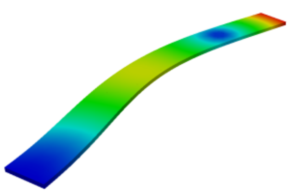
Mode 3



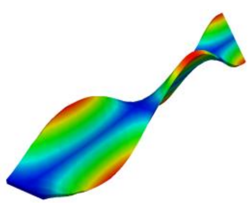
Mode 4



Mode 8



Mode 9



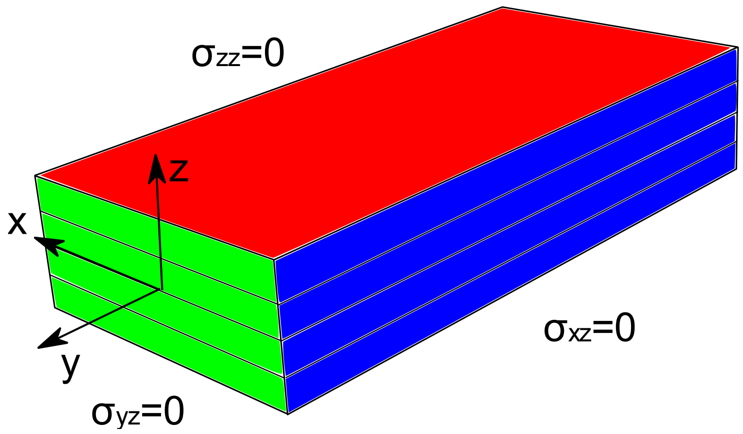
Mode 10

Approach	ESL - PVD		LW - PVD			LW - RMVT			3D Nastran
model	EBBT	TBT	HL1	HL2	HL3	HL1	HL2	HL3	HEX8
DOFs	93	155	744	1,674	2,604	1,416	3,186	4,956	251,937
1st	20.27	20.27	20.24	20.11	20.10	20.23	20.11	20.10	20.05
2nd	127.51	126.95	117.10	116.42	116.21	117.04	116.42	116.21	115.90
3rd	127.00	126.90	134.86	127.63	127.50	134.85	127.63	127.50	127.05
4th	355.41	354.74	196.25	196.13	190.42	196.23	196.11	190.44	190.06
5th	695.90	693.50	295.45	293.90	292.73	295.26	293.87	292.75	292.09
6th	787.18	764.44	513.57	511.05	507.98	513.09	510.96	508.05	507.11
7th	1149.25	1142.95	587.36	586.79	568.84	587.29	586.74	568.89	567.74
8th	1714.95	1701.33	752.65	749.12	743.57	751.68	748.87	743.65	742.49
9th	1978.47	1978.47	809.14	768.25	764.18	809.13	768.24	764.18	761.65
10th	2153.80	2022.60	972.56	971.14	938.81	972.40	971.06	938.90	936.94

Advanced capabilities

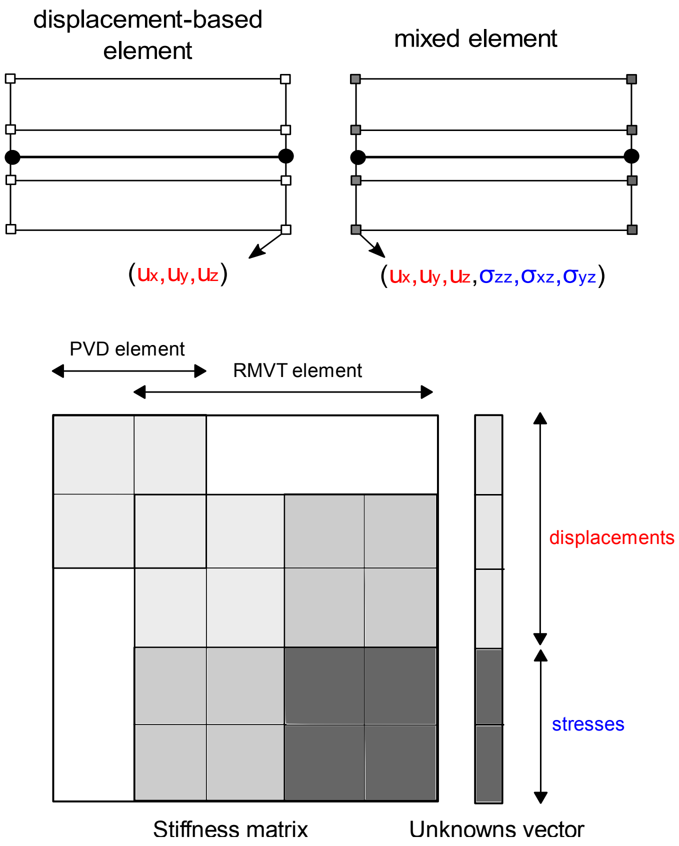
Stress boundary conditions

- Displacement-based models lack do not assure fulfillmet of the **equilibrium conditions** over the boundaries of the structure
- RMVT-based elements include **stress unknowns** which can be prescribed in the system
- Increased **accuracy** and **reliability**



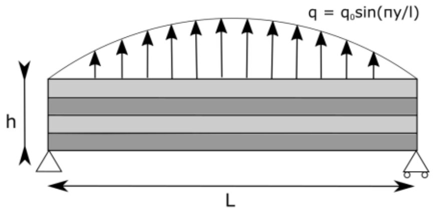
Global local analysis

- Efficient stress analysis using RMVT-based elements only in particular zones of interest.



Stress solutions

*Pagano, 1969



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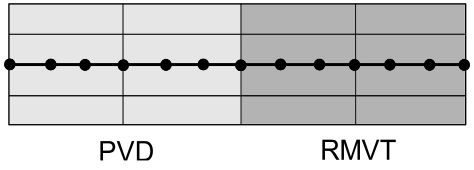
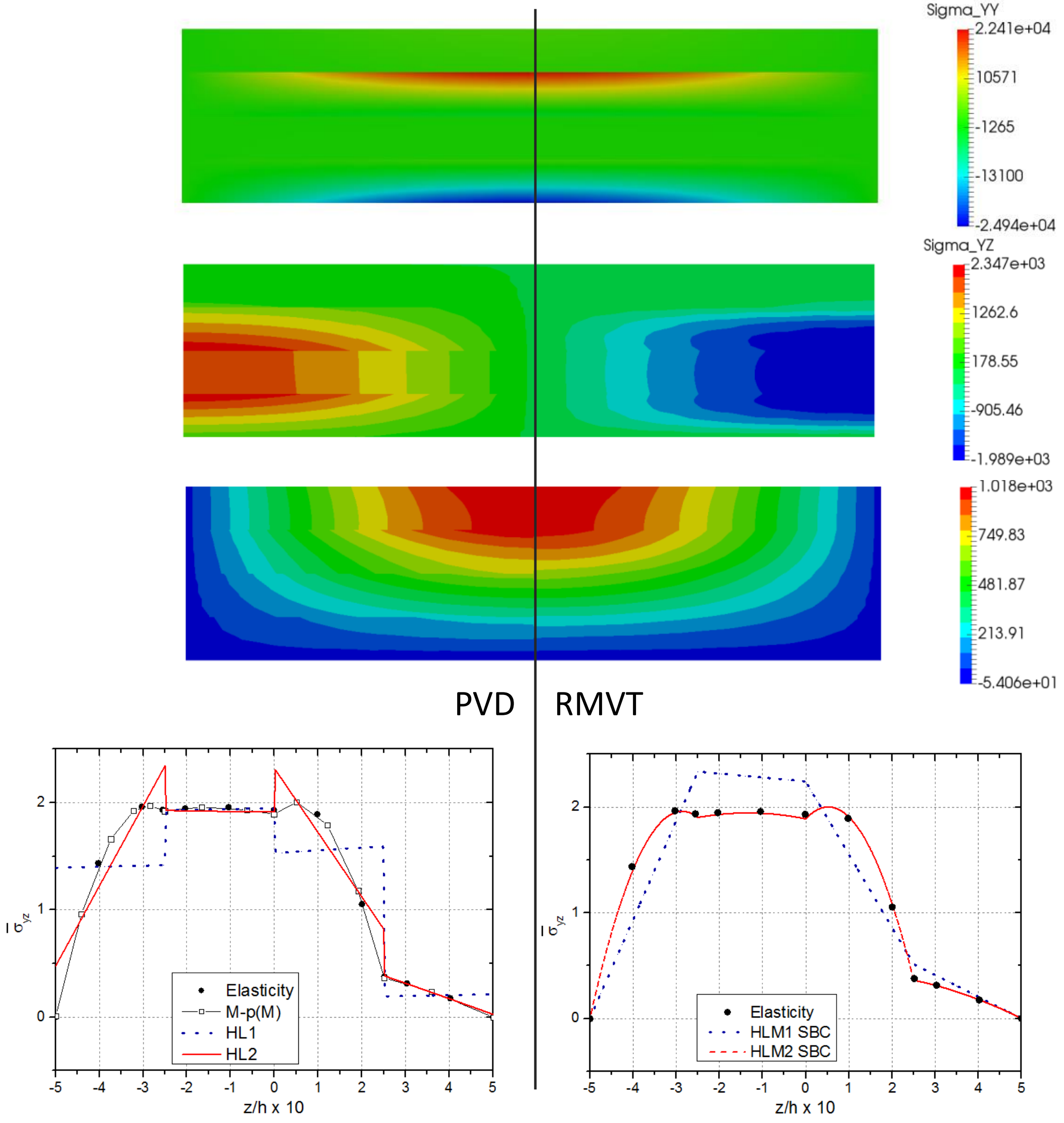
  G_{12}=0.5MPa G_{23}=0.2MPa
  $V_{12} = V_{23}$ = 0.25
- Cylindrical bending
- 1D model
  2 PVD elements + 2 RMVT elements





Free edge effects in laminates

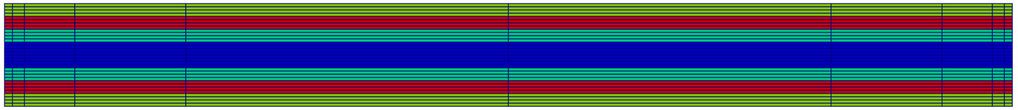
Tensile test

Uniaxial extension: F=6350N

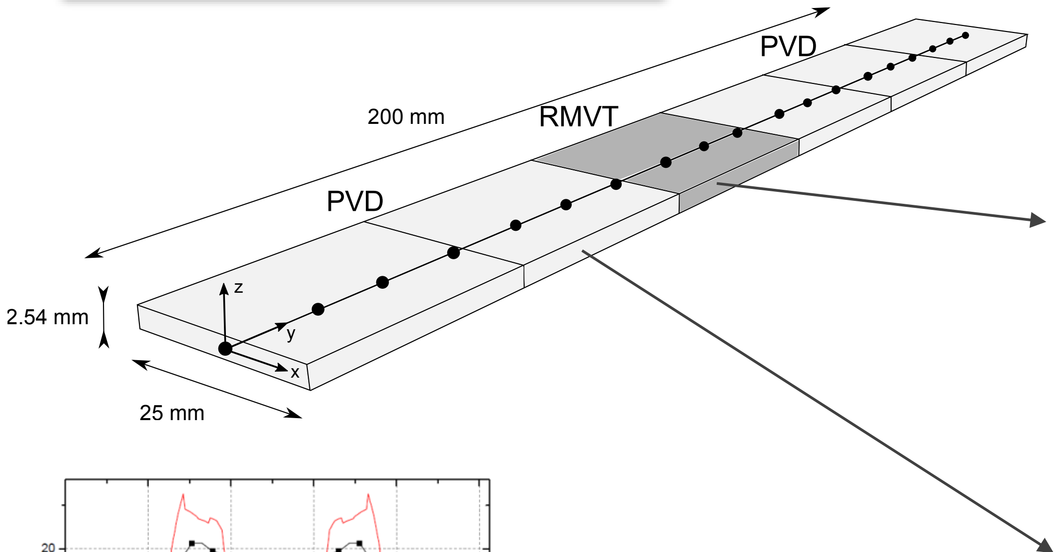
Model: 6 beam elements + 320 LE

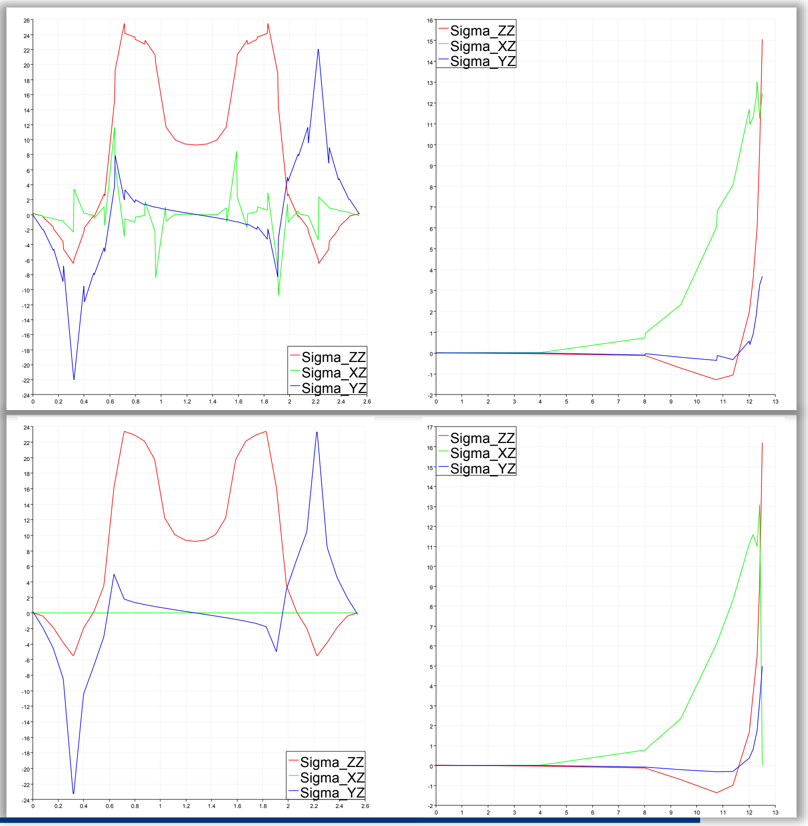
- Full PVD: 77805 DOFs
- 5 PVD + 1 RMVT: 94185 DOFs

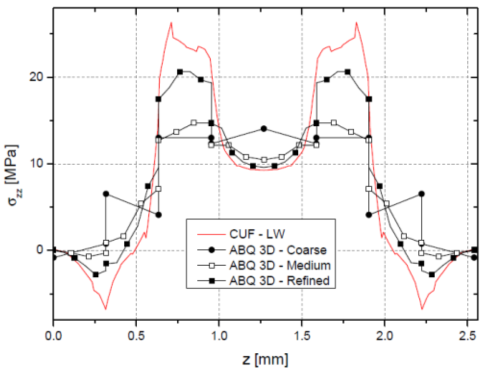
Cross-section



IM7/8552 - $[45,-45,90,0]_s$



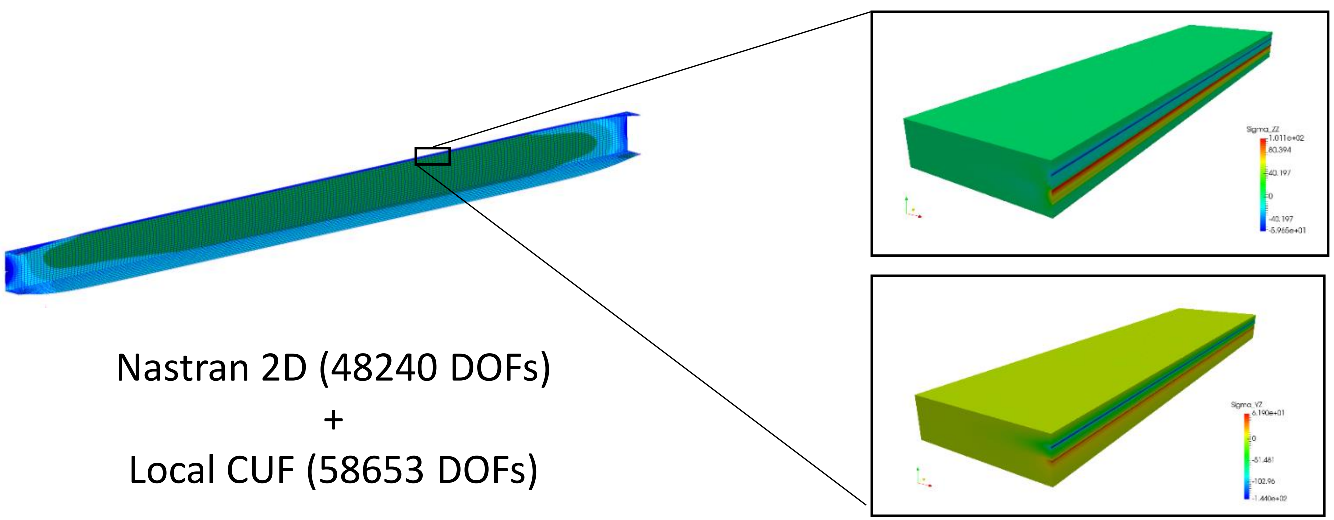




* A.G. de Miguel, M. Nagaraj, I. Kaleel, M. Petrolo, A. Pagani and E. Carrera. Accurate evaluation of failure indices of composite layered structures via various FE models. Submitted.

Conclusions

- Refined beam elements based on RMVT can be used for the **accurate stress analysis** of composite structures at the meso-macro scale
- Interlaminar continuity and stress boundary conditions are satisfied **a priori**
- Increased fidelity in the stress solutions at the expense of extra degrees of freedom. Total **computational costs are highly reduded** due to the use of 1D framework
- Its use is recommended for the **damage analysis** of laminated structures
- Further developments:
  - Variable displacement-stress assumptions (different polynomial orders)
  - **Global-local tool** for commercial FEM softwares



Nastran 2D (48240 DOFs)
+
Local CUF (58653 DOFs)

Acknowledgements

- **FULLCOMP** (FULLy integrated analysis, design, manufacturing and health-monitoring of COMPosite structures)

- Partners:
    i. Politecnico di Torino (Italy)
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    iv. Leibniz Universitaet Hannover (Germany)
    v. LIST (Luxemburg)
    vi. ELAN-AUSY GmbH (Germany)
    vii. Universidade do Porto (Portugal)
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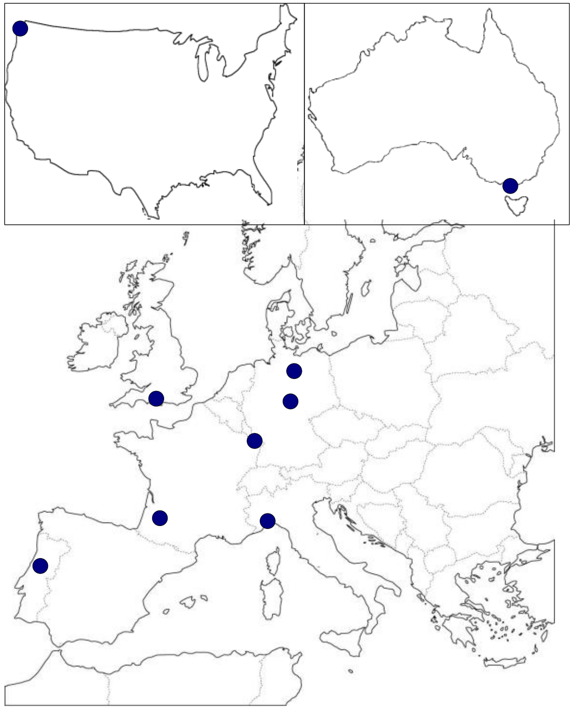

Thank you for the attention,
Any questions?