H2020
Marie Skłodowska-Curie actions
European Training Networks

Excellence Course on Geometrically Exact Composite Shell Elements for Multifield Problems through Sampling Surfaces Formulation (01RJKRO)

Gennady Kulikov

Tambov State Technical University, Russia

The Doctoral School is pleased to announce an Excellence Course on Composite Shell Elements for Multifield Problems. This course deals mainly with the linear elastic analysis of laminated, composite plate and shell structures. Mechanical, thermal and electrical loadings are considered. Particular attention is paid to the use of the Finite Element Method for this kind of structural problems.

Introduction and description of the course

In recent years, considerable work has been carried out on three-dimensional (3D) continuum-based finite elements that can handle analyses of thin laminated composite shells subjected to mechanical, thermal and electric loading, satisfactorily. These elements are typically defined by two layers of nodes at the bottom and top surfaces of the shell with three displacement degrees of freedom per node and known as isoparametric solid-shell elements. In the isoparametric solid-shell element formulation, initial and deformed geometry are equally interpolated allowing one to describe rigid-body motions precisely. The development of isoparametric solid-shell elements is not straightforward. To overcome element deficiencies such as shear, membrane, trapezoidal and thickness locking, assumed natural strain, enhanced assumed strain, and hybrid-mixed finite element formulations were utilized. Still, the isoparametric solid-shell element formulation is computationally inefficient because stresses and strains are analyzed in the global and local orthogonal Cartesian coordinate systems, although the normalized element coordinates represent already curvilinear convected coordinates.

Another promising approach is to develop the exact geometry or geometrically exact (GeX) solid-shell elements for solving the multifield problems through the use of curvilinear surface coordinates. The term "GeX" reflects the fact that the parametrization of the middle surface is known a priori and, therefore, the coefficients of the first and second fundamental forms are taken exactly at element nodes. Such finite elements are very promising because in the geometric modelling of modern CAD systems the surfaces are usually generated by non-uniform rational B-splines (NURBS). Allowing for that surfaces are conventionally produced by the position vector with the representation of two parameters, one can connect the geometric modeling of the shell surface generated in the CAD system to the finite element analysis of shell structures. So, it is advantageous to use the NURBS shell surface functions directly in the shell calculations, and GEX solid-shell finite elements are best suited for this purpose. They also have the two-parameter representation in surfaces and all geometric computations can be done by using NURBS surface representations in the CAD system.

It is well known that a conventional way of developing the higher-order shell formulation consists in the expansion of displacements into power series on the transverse coordinate referred to the direction normal to the middle surface. For the approximate presentation of the displacement field, it is possible to use finite segments of power series because the principal purpose of the shell theory consists in the derivation of approximate solutions of elasticity. The idea of this approach can be traced back to Cauchy and Poisson. However, its implementation for thick shells is not straightforward since it is necessary to retain a large number of terms in corresponding expansions to obtain the comprehensive results.

An alternative way of developing the shell theory is to choose inside each layer a set of not equally spaced sampling surfaces (SaS) $\Omega^{(n)1}, \Omega^{(n)2}, ..., \Omega^{(n)I_n}$ parallel to the middle surface in order to introduce the displacements, temperatures and electric potentials of these surfaces as basic shell variables, where I_n is the number of SaS of the *n*th layer ($I_n \geq 3$) and $n = 1, 2, ..., N$, where N is the total number of layers. Such choice of shell unknowns with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer allows the presentation of governing equations of the SaS formulation in a very compact form. The feature of the SaS shell theory is that it is based on the strain-displacement relationships, which precisely represent all rigid-body motions of a shell in any convected curvilinear coordinate system. This fact has a great importance since one may read in the open literature that the shell theory is an absolute academic exercise due to the difficulties of representing the rigid body shell modes.

It is important that the SaS shell formulation with equally spaced SaS does not work properly with the Lagrange polynomials of high degree because of the Runge's phenomenon. This phenomenon can yield the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equispaced nodes is increased, then the oscillations become even larger. However, the use of the Chebyshev polynomial nodes inside the shell body can help to improve significantly the behavior of the Lagrange polynomials of high degree because such a choice permits one to minimize the error uniformly due to the Lagrange interpolation. Thus, the solutions based on the SaS technique asymptotically approach the 3D exact solutions of elasticity, thermoelasticity and electroelasticity as the number of SaS tend to infinity. This in turn gives an opportunity to derive the stress/strain state with a prescribed accuracy employing the sufficient number of SaS.

To avoid shear and membrane locking and have no spurious zero energy modes the hybrid-mixed GeX solid-shell element formulation can be utilized in which the SaS are located inside each layer at Chebyshev polynomial nodes. In the proposed hybrid-mixed four-node finite element formulation, the Hu-Washizu variational principle is invoked to introduce the assumed interpolations of displacements, displacement-dependent and displacement-independent strains and stress resultants. Such a finite element formulation for multifield problems exhibits excellent performance in the case of coarse mesh configurations and has computational advantages compared to conventional isoparametric hybrid-mixed solid-shell element formulations because it reduces the computational cost of numerical integration in the evaluation of the element stiffness matrix. This is due to the facts that all element matrices require only direct substitutions, i.e. no expensive numerical matrix inversion is needed. It is impossible in the framework of the isoparametric hybrid-mixed shell element formulation. Second, the GeX four-node solid-shell element formulation is based on the effective analytical integration throughout the finite element employing author's enhanced ANS method. The latter has great meaning for the numerical modelling of doubly-curved composite shell structures with variable curvatures

H2020
Marie Skłodowska-Curie actions
European Training Networks

Executive Summary of Prof Kulikov

Prof. Gennady M. Kulikov was born in Tambov on April 23, 1953. After earning a degree in Applied Mathematics and Mechanics (1975) at the Novosibirsk State University, he began working as a researcher at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences before joining Tambov State Technical University in 1981. He had been working for ten years as an Assistant Professor and an Associate Professor of the Department of Mathematics. From 1991 to 2015 he was a Professor and Head of the Department of Applied Mathematics and Mechanics. Since 2015, he is a Head of the Laboratory of Intelligent Materials and Structures. Prof. Kulikov received his first doctoral degree in Solid Mechanics in 1981 at the Moscow State University and his second degree of the Doctor of Physics and Mathematics in Solid Mechanics in 1991 at the Kazan State University. He was a Visiting Professor at the Institute of Mechanics, Technische Universität Berlin (1995, 1999, 2002) and the Department of Aeronautics and Aerospace Engineering, Politecnico di Torino (2007, 2016). He won international grants under INTAS/RFBR (1997-1999), DFG/RFBR (1998-2000) and RSF (2015-2017) foundations. He is the author of three books and more than hundred refereed journal articles on these topics published in international journals. He is a member of the editorial board of some international journals Journal of Applied Mathematics, Curved and Layered Structures and Coupled Systems Mechanics and many international conferences on computational mechanics and structural mechanics. He serves as a referee for more than twenty international journals.

Dates and Venue

24-27 October 2016

Monday, Tuesday, and Wednesday: 9:00-10:30; 11:00 -12:30.

Thursday: 9:00-10:30.

Room: Sala Ferrari, DIMEAS (II Floor)

Acknowledgements and Contacts

This short course is organized by SCUDO in cooperation with the Marie Curie project FULLCOMP (www.fullcomp.net). For more info about this course and the FULLCOMP project, please contact Prof. Erasmo Carrera and Dr. Marco Petrolo (erasmo.carrera@polito.it, marco.petrolo@polito.it, www.mul2.com).